

टि.एस्. कुप्पण शास्त्रि विरचिता

ज्योतिषे शोधलेखावलि:



राष्ट्रिय संस्कृत संस्थानम् , नवदेहली
(तिरुपति केन्द्रीयसंस्कृत विद्यापीठग्रन्थमाला-५२)

ज्योतिषे शोधलेखावलिः

टि. एस्. कुप्पणशास्त्रिविरचिता

तिरुपतिस्थ-
केन्द्रीय संस्कृत विद्यापीठेन प्रकाशिता



केन्द्रीय संस्कृत विद्यापीठम्

तिरुपतिः

१९८९

Indological Truths

प्रकाशनवत्सरः—१९८९

सर्वेऽधिकाराः राष्ट्रिय संस्कृत-संस्थानेन स्वायत्तीकृताः

मानवसंसाधनविकासमन्त्रालयान्तर्गत-

नवदेहलीस्थ-राष्ट्रियसंस्कृत-संस्थानस्य कृते

तिरुपतिस्थ-केन्द्रीय-संस्कृत-विद्यापीठस्य

प्राचार्येण

डा. एन्. एस्. रामानुजताताचार्येण

प्रकाशिता

रत्नम् प्रेस्, मद्रास-1

Indological Truths

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Collected Papers on Jyotisha

By

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Indological Truths

FOREWORD

Jyotisha is the science of celestial luminaries. It deals with the twin fields of mathematics and astronomy. It is classified as one of the essential limbs (constituents) of the Veda (*Vedāṅgas*).

Vedic literature enjoins that *Yajñas* are to be performed ; these sacrifices are based upon a knowledge of appropriate time for their performance. The science of astronomy gives us a knowledge of time. Hence, indeed, it has been regarded as one of the six *Vedāṅgas*. This is stated in the ninth verse of the *Siddhānta-Sīromaṇi* (=SS) as under :

वेदास्तावत् यज्ञकर्मवृत्ताः
यज्ञाः प्रोक्ताः ते तु कालाश्रयेण ।
शास्त्रादस्मात् कालबोधो यतः स्यात्
वेदाङ्गत्वं ज्योतिषस्योक्तमस्मात् ॥

Sāyaṇa in his Preface to the *Rgvedabhāṣya* strikes a note on the use of Jyotisha by citing the *Pāṇinīya-sikṣā* (41-42) :

छन्दः पादौ तु वेदस्य हस्तौ कल्पोऽयं पठ्यते ।
ज्योतिषामयनं चक्षुर्निरुक्तं श्रोत्रमुच्यते ॥

Metre represents the feet of the *Veda-purusha*, Kalpa the hands, the science of Jyotisha its eyes, and Etymology its ears.

The use of the morphemic sequence यज्ञकालार्थसिद्धये occurring in the *Vedāṅgajyotisha* (3) justifies the view that astronomy arose to establish the times and seasons for conducting sacrifices. Sāyaṇa further refers to several passages of the Taittiriya School in support of the above view. The text, *saṁvatsaram...* indicates the periods of

years. The text, *vasante...* refers to the seasons. The text *māsi māsi...* refers to months. The text *Yam kāmayaeta...* refers to half months. The text *ekāṣṭakāyām...* refers to days. The text *prātar juhoti...* refers to part of days. The text *Kṛttikāsu* refers to nakṣatras. Hence is the need for *Jyotisha* for determining the times and seasons proper for performing sacrifices.

Occasional references are made in the Vedas to new moon and the full moon, the number of days in a year, the two halves of the year indicated by the terms, *Devayāna* and *Pitṛyāna*, the additional or intercalary month (*adhikamāsa*) and the deletory month (*kṣayamāsa*). The text — “These Kṛttikas (constellation of pleiades) do not deflect from the East” — suggests that the pleiades were observed to rise always at the east point. This is possible only when the first point of Aries was in the constellation of pleiades, since these are situated in the ecliptic. The point has, of course, shifted backwards now. From this observation, it emerges out that the stars of the zodiac were enumerated, commencing from the kṛttikas (Arka Somayaji, *A Critical Study of Ancient Hindu Astronomy*, p. 1, 1971).

The *Taittirīya Sam.* (=TS) at 4, 4, 10 lists the *Nakṣatras* starting from kṛttikas to Bharāṇi. This section in TS contains mantras for the nakṣatra bricks which are arranged in a circle round the naturally perforated brick, beginning on the south-east with kṛttikas and ending with Viśākha, then continuing on the north-west with Anurādhā and ending with Bharāṇi. The full moon brick is placed at the east point and the new moon at the west point (Keith's notes to TS 4, 4, 10). Vedic Astronomy cum mathematics shows the determination to count large numerical notations. The TS at 4, 4, 11, ii-iii (= *Vājasaneyi Mādhyandina Samhitā* XVII · ii) enumerates such numerical terms as *eka*, *daśa*, *śata*,

sahasra, *ayuta*-(10,000), *niyuta*-(100,000), *prayuta*-(1,000,000), *arbuda*-(10,000,000), *nyarbuda*-(100,000,000), *samudra*-(1,000,000,000), *madhya*-(10,000,000,000), *anta*-(100,000,000,000), and *parārdha*-(1,000,000,000,000). Mahīdhara's commentary at *Vāj. Mādh. Sam.* XVII-ii is worth quoting here :

अत्रैकादि परार्धपर्यन्तैः शब्दैरुत्तरोत्तरं
दशगुणिता संख्योच्यते ॥

Apart from division of the sphere into 27 or 28 nakṣatras, Vedic astronomy has contributed to our understanding of the conception of great yugas — of course, carrying out a radical change of the heavenly bodies — and that of the *Tithi*.

In the area of geometry, the *Śulba - sūtras* (c. 200 B.C.) which are concerned with the measurements of sacrificial altars, discuss the construction of squares and rectangles, the relation of the diagonal to the sides, the equivalence of rectangles and squares and the construction of equivalent squares and circles.

The *Vedāṅga-Jyotisha* is a post-vedic development. The fourth verse of this text treats Mathematics as standing at the head of all Vedāṅgas ; and it reads :

यथा शिखा मयूराणां नागानां मणयो यथा ।
तद्वद्वेदाङ्गशास्त्राणां गणितं मूर्धनि स्थितम् ॥

Thus Mathematics and astronomy are twin disciplines, the one complementing the other. *Gaṇitam* includes astronomy, and geometry (*kṣetragaṇitam*) belongs to the science of *kalpasūtras*. Geometry included the scope of *gaṇitam*. *Gaṇitam* also includes fundamental operations (*parikrama*-), determinations (*Vyavahāra*) and so on. Subsequently arose the *Siddhānta*-literature.

A *Siddhānta* text is an astronomical treatise, dealing with various measures of time, ranging from a *Tṛtī* upto the duration of a Kalpa (which culminates in a deluge), planetary theory, arithmetical computations as well as algebraical processes, problems relating to intricate ideas and their solutions, location of the earth, the stars and the planets and description and usage of instruments (SS vs. 6). Of the eighteen *Siddhānta* works that are noticed, mention could be made of the following texts which have come down to us: *Sūryasiddhānta*, *Paitāmaha siddhānta*, *Romaka-siddhānta*, *Paulīśa-siddhānta*, *Vāsiṣṭhasiddhānta*, *Brahmasiddhānta* and *Vṛddhavāsiṣṭhasiddhānta*. Varāhamihira wrote the *Pañcasiddhāntikā* and *Paitāmahasiddhānta*.

Of these the *Sūryasiddhānta* and the *Brahmasiddhānta* deserve special mention here, since both these have received correction from time to time. At the same time the former work has shown “the process of adaptation of the new science to Indian ideas in its most pronounced state” (Keith, *Hist. of Skt. Lit.*, 518). It reveals in the theory of Kalpas, restores the pre-eminence of mount Meru at the north pole and deals with such astronomical concepts as Nakṣakras and others in the Indian context.

The astronomer who wrote the *Āryabhaṭīyam* (499 A.D.) is Āryabhaṭa (born in 478 A.D.) who introduced new ideas into Indian astronomy. He is the Sanskritist to write a distinct chapter on mathematics in relation to astronomy. It may not be an exaggeration to say that he was the only Hindu astronomer to propound the doctrine of diurnal rotation of the earth, as stated by Arka Somayaji in *A Critical Study of the Ancient Hindu Astronomy* (p. 2). The astronomers who followed him were Lalla (500 A.D.), Varāhamihira (505 A.D.), Brahmagupta, Mahāvīra (628 A.D.), Śrīdhara (750 A.D.), Muñjāla (932 A.D.), Śrīpati (1039 A.D.),

Bhāskara II, the author of *SS* (1150 A.D.), Makaranda (1478 A.D.) and Gaṇeśa (1520 A.D.). The names of others such as Garga, Vṛddhagarga and Nārada, who existed before Varāhamihira, may be added to this list.

The authorities on Ancient Sanskrit astronomy and mathematics are of the opinion that the last scientific work in Jyotisha is *SS*. A temple inscription quoted by O. E. SMITH, (*History of Mathematics*) runs as under :

Triumphant is the illustrious Bhāskarā-
charya whose feet are revered by the wise,
eminently learned.

Bhāskara II worked at the Astronomical observatory in Ujjain where Brahmagupta is said to have conducted certain experiments several centuries ago.

Actually there were two Bhāskarāchāryas. The first was a contemporary of the well-known astronomer, Brahmagupta. He wrote the *Mahābhāskarīya*, *Laghu bhāskarīya*, and the *Āryabhaṭīya* which are commentaries on the famous work of Āryabhaṭa. The Second Bhāskara (1114-1185 A.D.), as has been stated in *SS* (vs 3.) composed the crest-jewel of astronomical treatises, i.e. *SS*, after having mastered the science under his talented father, Maheśvara, a pioneer in astronomy, who championed the cause of Jyotisha in the eleventh century A.D.

The chief contribution made by Bhāskara II to mathematics *cum* astronomy consisted in realising the true nature of division by Zero, anticipating the modern theory on convention of signs, representing unknown quantities by phonemes, presenting solutions for quadriatic equations reduced to a single type taking into consideration only positive roots as genuine, solving a few cubic and bi-quadriatic equations and indeterminate

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equations of the first and second degree, computing elaborate tables of sines, studying regular polygons upto 384 sides, giving the value of π as 754/240 and anticipating kepler's method of determining the surface and volume of a sphere (N.N. Sachitanand's article in the *Hindu*, Madras, dated 1-7-1979 and M. D. Balasubrahmanyam's Foreword to the Annotation of SS by Arka Somayaji, Kendriya Sanskrit Vidyapeetha, Tirupati series No. 29, 1980). Furthermore, Bhāskara II gave a scientific exposition of the sidereal revolution of planets, circumference of the earth, lunar eclipses, measurement technique of celestial bodies, longitude of the stars and other astromical facts. Needless to say, the third and fourth parts of SS, — under the heading, *Gaṇitādhyāya* (or *Grahagaṇita*) and *Golādhyāya* — are exclusively devoted to astronomy.

After Bhāskara II, very little original work appeared in India in this field. Later scholars were content with the writing of some commentaries on the earlier standard treatises of stalwarts, simply to whet their appetite. But for the scholarly compositions of eminent Sanskritists like Nilakaṇṭha and the rest, belonging to the productive Kerala School of astronomy, Jyotisha Pandits concentrated their attention more on astrology than on studies and research in mathematical astronomy.

However, in recent times scholars have been attempting to examine astronomical theories in the light of western thought.*

Realising that specialisation in mathematical astronomy and other sciences has witnessed a decline, the Tirupati Vidyapeetha started a project entitled, 'Coordination of Sanskrit and Ancient India', so that unpublished and rare works on Sanskrit mathematics, astronomy, and other disciplines might be critically

* For a brief sketch of Astronomy, read *Collected Papers*, pp. 371-78.

edited with translation and annotation, besides monographs on historical and descriptive studies on Jyotisha might be brought out. Under this scheme, the Vidyapeetha has already brought out Dr. Arka Somayaji's Exposition in English and Annotation of Bhāskarācārya's SS. - (1980). Under the same project, the Vidyapeetha has now come forward to issue Professor T. S. KUPPANNA SASTRY'S *Collected papers* on Hindu astronomy, mathematics and other related disciplines. I record here that it is rather unfortunate for SASTRY and us that he could not live to see his outstanding publication—the last challenging *magnum opus* of SASTRY. That Professor SASTRY, an eminent scholar in almost all the branches of Sanskrit literature (including mathematics and Astronomy) was specially qualified to write the *collected papers*, will become evident, if we look at his *curriculum vitae* and publications.

Professor SASTRY (1900–1982) alias Srinivasan, was born in Tirumanilayoor (near Karur, Tiruchirapalli district of Tamilnadu) to Subrahmanya Iyer and Bhagirathi Ammal. A scion of Nīlakaṇṭha Dīkṣita, the celebrated Sanskrit polymath of the sixteenth century, Professor SASTRY devoted all his time to a critical study and appreciation of almost all the Sanskrit Shāstras including Gaṇita, Jyotisha, and modern astronomy. In boyhood he underwent training in the traditional recitation of Sāmavedic hymnology. Having completed his schooling in Karur, he passed the B.A. examination as a student of the famous St. Joseph's College, Tiruchirapalli. He worked as Headmaster of the High School at Tirumayam (erstwhile Pudukkottah State), and then joined the Maharaja's High School, Pudukkottah (later known as Brihadambal High School). Subsequently he worked as lecturer, Assistant Professor in Sanskrit at Maharaja's College, Pudukkottah,

Government Arts College, Kumbhakonam and in the Madras Presidency College from where he retired in 1955. Then he taught at the Madras Sanskrit College for about five years as Professor of comparative Philology and History of Sanskrit Literature. Even after retirement he served the college as Honorary Professor of Sanskrit.

Professor SASTRY critically edited six astronomical texts. He brought out a critical edition of the *Mahābhāskarīya* with the commentaries of Govindasvāmin and Paramēshvara with annotation and indices in 1957. Again he edited the *Vākyakaraṇa*, the basis of the Vākya almanacs of South India, with the commentary of Sundararāja in 1962. He also critically edited the *Vedāṅgajyotiṣa* with translation and notes. Subsequently he critically edited the *Pañcasiddhāntikā* with translation and notes.

Dr. K. V. Sarma (now Professor at the Adyar Library Research Centre) who informally collaborated with Professor SASTRY in editing the first two works mentioned above, writes (in the Bio-data of Professor T. S. Kuppanna Sastry) as under :

His (Prof. Sastry's) deep understanding of Indian astronomy... helped him in preparing a rational edition with detailed exposition in English of the *Vedāṅga Jyotiṣa* and the *Pañcasiddhāntikā*, both of which are master-pieces illustrative of forensic skill in presenting distended facts to prove his point. He prepared also a book on the computation of eclipses incorporating modern corrections, but couched in such a form that it could be used by Indian almanac makers.

His *collected papers* issued by the Vidyapeetha, is a collection of valuable and original papers—published in several learned Journals—numbering about twenty. The

author has made a systematic, thorough-going and comparative study of the Hindu and Western systems of astronomy. The book deals with such interesting and illuminating topics as the Vāsiṣṭha Sun and Moon, calender in Hindu Tradition, Varāhamihira's Śaka Era, Hindu astronomical processes, Vateśvara Siddhānta, Āryabhaṭa School of Astronomy, Hindu Astronomy in the age corresponding to pre-copernican European Astronomy, Tamil Astronomy, determining the date of Ādi Sankarāchārya (on astronomical grounds), the law of gravitation, the structure of atom and the theory of Relativity and others. Needless to say, among the astromers who have attempted a methodological and critical study of Jyotisha, Professor T. S. Kuppanna Sastry, the eminent scholar of ancient and modern astronomy, stands out as preeminent. I state in all humility that the development of astronomy, marshalled in its historical perspective in the *collected papers*, will furnish some definite criteria governing the relevancy and applicability of ancient Indian observations as enshrined in Jyotisha to modern astronomy.

It is now left for us to thank Dr. K. V. Sarma sincerely for his hearty cooperation and assistance in printing this book. He read through the proofs, compiled the Bibliography of Prof. SASTRY's writings and sent us the author's Bio-data. Special acknowledgement should be made to the Rathnam Press, Madras for setting the appropriate types for the book.

Lastly I pay homage to my *guru*, Professor T. S. Kuppanna Sastry for his excellent contribution to mathematics *cum* astronomy.

Kendriya Sanskrit
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M. D. BALASUBRAHMANYAM
Principal (1973-85)

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THE VĀSIṢṬHA SUN AND MOON IN VARĀHAMIHIRA'S PAÑCASIDDHĀNTIKĀ

(Reprinted from J.O.R., K.S.R.I., Madras, 1955-56)

The Vāsiṣṭha Sun and Moon are contained in Ch. II and in Ch. III. 4 of the Pañcasiddhāntikā.¹ II. 1 gives the true sun, II. 2-6 give the computation of the true moon, III. 4 gives a rule for the daily motion of the moon, II. 7 gives rules for the sun or moon's Nakṣatra and the Tithi, and II. 8-13 deal with topics related with the sun, like the duration of day-light, the length of the noon-shadow when the sun is known and *vice versa*, and lastly finding the Orient Ecliptic Point (Lagna) when the shadow is known and *vice versa*.

Obviously, the most important parts are II. 1, II. 2-6, and III. 4, which form the basis for the rest of the work. Also these parts are very interesting from a historical point of view. The methods given form a transition from the more ancient astronomy represented by the Paitāmaha Siddhānta condensed by Varāha, giving only mean sun and moon, to the later epicyclic astronomy represented by the Saura Siddhānta condensed by him. Of these, about II. 1, Dr. Thibaut (T) makes the remark, "a stanza of obscure import," and leaves it at that, without attempting to translate it, and Sudhakara Dvidevi (5) remarks: "अनेन श्लोकेन किं साधयतीति न ज्ञायते, अन्यशुद्धवत्." So much for the sun. About II. 2-6, T says (and S echoes him): "Of the above

1. The references are to the Pañcasiddhāntikā, edited by Dr. Thibaut and MM. Sudhakara Dvidevi 1889 and reprinted by Motilal Banarsi Das, Lahore, 1930.

stanzas we have succeeded in making out the sense in part only. They manifestly teach how to find the mean and perhaps also the true positions of the moon by means of a process more compendious than the one usually employed in Indian Astronomy.

“What preliminary operation is presented in stanza 1, we are altogether unable to say¹.....It is not apparent why stanza 5 directs us to add for that half-gati six Signs plus four minutes to the moon’s mean place, for the moon’s mean motion in one half-gati amounts to considerably more, viz. six Signs plus about 92 minutes. Nor are we at present able to throw light upon the meaning of the processes prescribed in stanza 6. They possibly refer to the operation of finding the moon’s true place, although we are more inclined to think that this latter part is treated in stanzas 4-9 of the next (i.e., III) chapter. And S says: “गत्यर्थे कलाचतुष्टयसंस्कारस्य तथा ६ श्लाकस्य इदं नीपयन्तं न बुद्धा उपपत्तिः” Thus T and S are both unaware that stanza 6 indeed gives the operation of finding the true moon. Further, their interpretation of II. 3-5 is wrong in several places, and by this they have shut out important necessary data. About III. 4. (including the next five stanzas as well), T remarks without being able to give any translation, “Six stanzas referring to the moon. The details, however, are obscure.” About the same S says, “अधेषां (i.e. 4-9) व्याख्यां अग्रे दक्ष्ये,” and he has left it there without coming back to it. In spite of this self-confessed ignorance, T remarks on p. li of his Introduction, “..... the methods are so crude and so completely omit to distinguish between mean and true astronomical quantities that the Vāsiṣṭha Siddhānta can hardly be included within scientific Hindu Astronomy”.

1. It is evident that Dr. Thibaut thinks that II. 1 also deals with the moon, while it really deals with the sun,

The Pañcasiddhāntikā was first printed in 1889 and during these nearly 70 years nobody, to my knowledge, has thrown light on the true nature of the Vāsiṣṭha Siddhānta.¹ In dealing with the topic in question, it is my desire also to discuss the readings of the text. We have only two manuscripts to go by, one badly vitiated and the other worse, both printed in the edition, one against the emended text and the other as footnotes. Frequent quotations, from the Pañcasiddhāntikā are found in Bhaṭṭotpala's commentary of the Bṛhatsamhitā, but the range of our topic is limited, the whole thing taking only 14 āryās, and these are not found quoted anywhere, as far as I know. But the subject matter being scientific, it is possible to fix the correct text fairly well in most places of doubt, taking for our guidance the āryā metre and the relics of the slaughtered words, provided we are certain of the intention of the author. I now proceed to the elucidation of the text. The understanding of the text every where comes first, as that is the basis for the correction of the text.²

The Sun

II. 1.

कृतगुणयषमृत्युतमैकतुम-
नुहन्तं षड्मेन्दुभिर्विमजेत् ।
शशिखलखयमस्वरकृत-
नवनववसुषट् कविषयोनेः ॥

कृतगुणमृत्युतमेक-
तुमनुहन्तं षड्मेन्दुभिर्विमजेत् ।
शशिखलखयमकृतस्वर-
नवनववसुषट् कविषयोनेः ॥

1. This is only one of many important topics in the Pañcasiddhāntikā needing enlightenment, like the Pauliśa sun, moon, solar eclipse, and 'star-planets', the last 18 verses dealing with 'star-planets' and their authorship, etc. where T and S have either professed ignorance or gone wrong.
2. The printed Ms. Text is given on the left and my own emended text, opposite to it on the right. T's readings and the footnote readings are mentioned when necessary.

II. 1. The days from epoch (*Ahargana*) are to be multiplied by 4, and 6 added to the product. This is to be divided by 1461 (and the remainder taken). From the remainder should be deducted successively 126 minus 1, 0, 0, 0, 2, 4, 7, 9, 9, 8, 6, 5, (i.e., the twelve quantities 125, 126, 126, 126, 124, 122, 119, 117, 117, 118, 120, 121 are to be deducted successively). (The sun's *Rāsis*, *Meṣa* etc., are successively got).

It is to be noted that *Ahargana* is not mentioned, that we should take the remainder for the operation is not mentioned, and what we get by this rule is not given. We have to presume all these. The word *Ahargana* can be easily understood, because that is the starting point of of computation. The rule given, as also the numbers, point to the necessity of taking the remainder, and to the true sun in *Rāsis*, as the object of the operation.

The rule is explained as follows: The days from the epoch by being multiplied by 4 are converted into quarter-days. 6 quarter days are added to this because the beginning of the year, (in this case the true year), is $1\frac{1}{2}$ days before the epoch, and by the addition of the 6 quarter-days we get the total quarter-days from the beginning of the true year. According to this *Siddhānta* the year contains $365\frac{1}{4}$ days or 1461 quarter days. So by dividing out the total quarter days by 1461 and taking the remainder we get the quarter days from the beginning of the current year. During the first 125 quarter days (i.e., $31\frac{1}{4}$ days) of the year the sun traverses the first *Rāśi*, i.e., *Meṣa*. During the next 126 quarter days (i.e., $31\frac{1}{2}$ days) he crosses the second, i.e., *Ṛṣabha Rāśi*, and so on. It is to be noted that there are 12 quantities for the 12 *Rāśis*, and that these add up to 1461. The sun's position within a *Rāśi* is expected to be found by proportion.

Thus if the days from epoch is 942, say, the quarter days from the beginning of the year just before the epoch is, $942 \times 4 + 6 = 3774$. Dividing out by 1461, we get 2 full solar years elapsed (which are not wanted), and 852 quarter days are left over in the third year. We can take from this 125, 126, 126, 126, 124 and 122, and 103 quarter days are left over, i.e. the sun has passed six full Rāsis, and in the seventh he has gone 103/119 parts or 26° .

At epoch, the sun is $6/125 \times 30^\circ = 1^\circ 26'$ in Meṣa, at sunrise at Ujjain on Sunday, (near the end of śaka 427). What is the epoch, and how we are to find the days from epoch, these two things are not mentioned here. But ch. I. 8-13 gives rules to find the days from epoch and we can adopt it, though given for Romaka and Pauliśa, for the interval between two points of time is invariant. Only we must take into account the time of day of epoch. Vāsiṣṭha epoch most probably is sunrise at Ujjain, Sunday, at the end of śaka 427. This matter will be discussed subsequently.

Now we are in a position to discuss the adopted readings, T and S, because they have not understood the stanza, have simply corrected the obvious scriptory errors, and so far as they go, they are correct. The text-reading यषमृनु has been corrected into षड्कतु, following the variant reading षट्कतु: युतमेकतुं is corrected into युतमेकतुं. स्वरकत is corrected as स्वरकृत्. I have made the following further corrections: (1) षड् has been deleted and only कतु retained, because the first foot clearly ends with मेकृत् and there are two mātras extra. It is better षड् is omitted because one Ms. has it, the other having a corruption, यष. But both Mss. have कतु. (2) In the third foot, I have interchanged स्वरकत and made it कृतस्वर, because order requires it. There must be a gradual and continuous fall from 126 to 117. The values indeed must have been obtained empirically, but it is too much to assume that the Siddhānta was not aware of the

gradual nature of the fall and rise, and gave what it saw as स्वरकृत. The error of observation also would be too great if it is स्वरकृत.

The Moon

II. 2-6

रसगुणनवेन्दुयुक्त
शशिगुणखगुणोद्धृतेद्यत, युगणे ।
शेषे नवभिर्गुणिते
गतयोऽष्टजिनैः पदं शेष ॥

घन[षोडश]हृतं शेष
प्रोज्झ्याद्यस्त्रिगुणितं चतुर्भक्तं ।
भादि कलाद्विगुणघना
शशिमुनि नवयमाश्वराशाद्या ॥

विषयधृत्वयो गतिघ्ना
गततिषष्टांशोनिता कलाः प्रोक्ताः ।
वेदाकाः पादसंख्या
गत्यर्थं धनमृणं परतः ॥

गत्यर्थं भगणार्धं
देयं लिप्ताश्चतुष्कसंयुक्तं ।
शेषपदसमाश्चंशा-
स्तद्व धनर्णात् फलं दन्त्यं ॥

व्येकपदमिन्द्रियघ्नं
कृतनवदशसंयुतं वियुक्तं च ।
मनुवेद्यमेभ्यः पद-
गुणे त्रिषष्ट्यो धृते लिप्ता ॥

नगात्पदाद्दशघ्नात्
सप्तांशः सश्वि सांख्योभुक्तिः ।
गत्यर्थान्ता छोध्यो
लिप्ताभ्यो वसुमुनिनवभ्यः ॥

2 रसगुणनवेन्दुयुक्ते
शशिगुणखगुणोद्धृते घना युगणे ।
शेषे नवभिर्गुणिते
गतयोऽष्टजिनैः पदं शेषम् ॥

3 घन[षोडश] हृतं शेषं
प्रोज्झ्याद्यास्त्रिगुणितं चतुर्भक्तम् ।
भादि कलाद्विगुणघना
शशिमुनि नवयमाश्व राश्याद्याः ।

4 विषयधृत्वयो गतिघ्ना
गतिषष्टांशोनिताः कलाः प्रोक्ताः
वेदार्काः प. संख्या
गत्यर्थं धनमृणं परतः ॥

5 गत्यर्थं भगणार्धं
देयं लिप्ताचतुष्कसंयुक्तम् ।
शेषपदसमाश्चंशा-
स्तैश्च धनर्णात् फलं देयम् ॥

6 व्येकपदमिन्द्रियघ्नं
कृतनवदशसंयुतं वियुक्तं च ।
मनुवेद्यमेभ्यः पद-
गुणे त्रिषष्ट्योद्धृते लिप्ताः ॥

III. 4

[वि]नगात्पदाद्दशघ्नात्
सप्तांशससाश्विखसरो भुक्तिः ।
गत्यर्थान्ताच्छोध्यो
लिप्ताभ्यो नवमुनिवसुभ्यः ॥

II. 2. 1936 is added to the days from epoch, and the total divided by 3031. The quotient are called Ghanas. The remainder is to be multiplied by 9 and divided by 248. The quotient are called Gatis. The remainder are called Padas.

11. 3. The Ghanas are to be divided by 16 and the remainder taken. This should be multiplied by 3 and divided by 4. This is Rāśi etc. It should be deducted from 12 Rāśis (and the remainder written down). Minutes of arc equal to twice the total Ghanas (are to be added). One Rāśi, 7 degrees and 29 minutes are (also to be added).

II. 4. 185 multiplied by the Gatis minus a tenth of the Gatis are minutes (which are also to be added). The (first) 124 Padas are designated a half-gati. (If the Padas are less than 124) they are called plus-padas. If more than 124, 124 is taken from it to form a half-gati, and the remainder are called minus-padas.

II. 5. If there is a half-gati, (for the sake of that half-gati) 6 Rāśis and 4 minutes are to be added. Also degrees equal in number to the plus-padas or minus-padas (are to be added). With these plus or minus padas respectively, the minutes got by the plus operation or minus operation respectively (in II. 6) are to be added.

II. 6. Deduct one from the plus-pada or minus-pada, and multiply this by 5. (If plus-pada) add the product to 1094, multiply the sum by the plus pada and divide by 63. The resulting minutes (are to be added). If minus-pada, the product is to be subtracted from 2414, the remainder multiplied by the minus-pada, and divided by 63. The resulting minutes (are to be added). (Thus the true moon is got).

III. 4. Deduct 9 from the number of plus or minus padas, multiply this by 10 and divided by 7. Add this

to 702 if plus-padas. Deduct this from 879 if minus-padas. The resulting minutes are the daily motion (for the day ending with the padas).

The following is the explanation of the process. The true moon at a given time t , is, (i) the mean moon at t plus (ii) the equation of the centre for t . (i) is given here in five parts. We shall call them a , b , c , d , e which are to be added up to get the total mean moon. a (usually called the Mūla-dhruva) is the position of the mean moon at a point of time 1936 days before the epoch, when the moon's apogee and the mean moon exactly coincided, according to this Siddhānta. This is given as शशिसुनिनवयमाश्च राश्याद्याः, i.e. 1 Rāśi, 7 degrees, 29 minutes. b is the mean motion during a whole numbers of cycles of 3031 days, each cycle equal to 110 anomalistic revolutions of the moon, from that point of time. This b is found by multiplying the mean motion per cycle, (110 revolutions; 11 Rāśis, 7 degrees, 32 minutes), by the number of cycles, called Ghanas, obtained as quotient by dividing the Ahargaṇa plus 1936 days, by 3031. As full revolutions may be neglected, it is enough if we multiply the Ghanas by 11 Rāśis, 7 degrees, 32 minutes, which may be done as $\text{Ghanas} \times 2' + \text{Ghanas} \times 11^{\circ} 7' 30''$. $\text{Ghanas} \times 2'$ is given by द्विगुणघनाः कलाः (योऽयम्). Because $16 \text{ Ghanas} \times 11^{\circ} 7' 30''$ equals 15 full revolutions, it is enough if we divide out the Ghanas by 16 and take the remainder alone for multiplication, which we are asked to do by घनषड्यद्वतशेषम्. As $11^{\circ} 7' 30''$ is $\frac{3}{4}$ Rāśi less than a full revolution, we can multiply the remaining Ghanas by $\frac{3}{4}$ Rāśi and take this as subtractive, which we are instructed to do by प्रोद्भवा-घस्त्रिगुणितं चतुर्भक्तं भदि. Thus b is disposed of, c is the mean motion during the subsequent full anomalistic revolutions called Gatis, which form the quotient got by dividing the remaining days by the anomalistic period, 248/9 days; (multiplying the days by 9 and dividing by

248 is only dividing by 248/9). For each Gati the mean motion is 1 revolution and 184 9/10 minutes. Hence the rule to multiply the Gatis by 185' and deduct minutes equal to 1/10 of the Gatis. This is given by विषयधृतयो गतिग्रा गतिकाष्ठांशोनिताः कलाः (योऽयः). What are now left over are 9ths of days called Padas (and these obviously would be less than 248). The mean motion per pada is $1^{\circ}27'.843$, and so Padas $\times 1^{\circ}27'.843$ should be added to complete the mean motion till t . Of this the Siddhanta asks us to add 1° per Pada first which is given by शेषपदसमाश्चांशः (योऽयः). This forms d . The residue $27'.843$ per Pada forming e is combined with the equation of the centre (ii) and given by the two equations of II. 6. If the Padas contain a half-gati, the value of $d+e+(ii)$ for the half-gati part are combined together and given as $180^{\circ}4'$. This $180^{\circ}4'$ is got as follows: The half-gati is equal to 124 padas. So $d=124^{\circ}$. $e+(ii)$ given by the equation is $\{(1094+5 \times (124-1)) \times 124/63 = 3364' = 56^{\circ}4'; 124^{\circ}+56^{\circ}4' = 180^{\circ}4'$. Thus it is that we get $180^{\circ}4'$ for a half-gati, given by गत्यर्थे भगणार्थे देयं लिप्ताचतुःकसंयुक्तम्, which instruction has so much puzzled T and S . But of course this is incorrect, and the defect lies in the equation of the centre part of the formulae in II, 6, which give the zero value for the equation of the centre not at 124 Padas, but at 133 Padas, as I shall show presently.

I shall first explain II, 6, and show how the formulae here combine the residual mean motion, viz. Padas $\times 27'.843 (=e)$, with what is identifiable with the equation of the centre ($=ii$). If P =plus or minus Padas, the formula for plus-padas can be written down as $\{1094+5(P-1)\}P/63$, and the formula for minus-padas, as $\{2414-5(P-1)\}P/63$. Let us first take the plus-pada formula. As we have said $e+(ii)=\{1094+5(P-1)\}P/63$, and $e=27'.843 P$.

$$\begin{aligned}
\text{Therefore (ii)} &= \{1094 + 5(P-1)\} P/63 - 27'.843 P \\
&= (1089 + 5P) P/63 - 63 \times 27'.843 P/63. \\
&= (1089 - 1754 + 5P) P/63. \\
&= (5P - 665) P/63.
\end{aligned}$$

Taking the minus-pada formula,

$$\begin{aligned}
\text{(ii)} &= \{2414 - 5(P-1)\} P/63 - 27'.843 P \\
&= (2419 - 5P) P/63 - 63 \times 27'.843 P/63 \\
&= (2419 - 1754 - 5P) P/63 \\
&= (665 - 5P) P/63
\end{aligned}$$

Now, the plus-pada representing the original Padas lying within 0 to 124 Padas, correspond to the anomaly lying between 0 and 180°. The minus-padas correspond to the anomaly lying between 180° and 360°. So for the plus-pada the equation of the centre must be negative.¹ We see, $(5P-665) P/63$ is indeed negative for all values of plus-pada, and $(665-5P) P/63$ is positive for all values of minus-pada. It is to be noted that the one is the negative of the other. Again, the former starts from the value 0 for $P=0$, gradually goes to a minimum for $P=66\frac{1}{2}$ and again gradually, increases reaching 0 for $P=133$, (but P stops with 124), while the latter starting from 0 value for $P=0$, goes to a maximum for $P=66\frac{1}{2}$ and falls gradually to 0 in a similar manner. Thus the equations roughly behave like the term of the equation of the centre of modern astronomy, $-k \sin \theta$, and therefore identifiable with the equation of the centre. This is noteworthy, as forming a transition from a stage of no equation of the centre to the stage of epicyclic astronomy giving the equation of the centre in the form,

1. Negative, because unlike in modern astronomy, moon minus apogee is taken as anomaly. This is the practice in Hindu astronomy.

$-k \sin \theta$. Substituting $66\frac{1}{2}$ for P we get the maximum or minimum equation of the centre = 351 .¹

I now proceed to the explanation of III.4, giving the formulae for the daily motion. They can be written down thus :

For plus-padas, $702' + 10' (P-9)/7$. For minus-padas, $879' - 10' (P-9)/7$. We see, these being linear equations, that the increase or decrease in the daily motion is uniform, from $702'$ to $879'$ and back again from $879'$ to $702'$. $702'$ is the minimum and $879'$ is the maximum, the former being $88'.5$ less and the latter $88'.5$ more² than the mean motion $790'.6$. Though mathematically we can get values less than $702'$ and more than $879'$ for P less than 9, this is not envisaged by the Siddhānta. I shall now derive these rules from those of II. 6 and thus show that these belong to the Vāsiṣṭha, to whatever other Siddhānta also they belong.

The daily motion for the day ending P padas is clearly, the true moon for P minus the true moon for $(P-9)$. So for plus-padas the daily motion is,

$$\begin{aligned}
 & [(a+b+c+P^2+\{1094+5(P-1)P\}P/63)] \\
 & - [(a+b+c+(P-9)^2+\{1094+5(P-1-9)\} \\
 & \quad (P-9)/63)] \text{ in minutes} \\
 & = 540 + 1089 P/63 + 5P^3/63 - 1089 (P-9)/63 - 5 (P-9)^3/63 \\
 & = 540 + 1089 (P-9+9)/63 + 5 (P-9+9)^3/63 - 1089 (P-9)/63 \\
 & \quad - 5(P-9)^3/63 \\
 & = 540 + 1089 \times 9/63 + 5 \times 9^3/63 + 90 (P-9)/63 \\
 & = 540 + 162 + 10 (P-9)/7 \\
 & = 702 + 10 (P-9)/7.
 \end{aligned}$$

1. Note how close this is to the modern value, $377'$ and how different from the circa = $301'$ of later Hindu astronomy.
2. Modern astronomy gives = $86'$ on the average. See how the Vāsiṣṭha value = $88\frac{1}{2}$ is far better than the $C. 68'$ of later Hindu astronomy.

In the same way for the minus-padas,

$$\begin{aligned}
 & [(a+b+c+P^0+\{2414-5(P-1)\}P/63)] \\
 & - [(a+b+c+(P-9)^0+\{(2414-5(P-1-9))\}(P-9)/63] \\
 & \hspace{15em} \text{(in minutes)} \\
 & = 540+2419P/63-5P^2/63-2419(P-9)/63+5(P-9)^2/63 \\
 & = 540+2419(P-9+9)/63-5(P-9+9)^2/63-2419 \\
 & \hspace{15em} (P-9)/63+5(P-9)^2/63. \\
 & = 540+2419 \times 9/63-5 \times 9^2/63-90(P-9)/63 \\
 & = 540+339-10(P-9)/7 \\
 & = 879-10(P-9)/7.
 \end{aligned}$$

It is also possible to establish the connection, by summing the two expressions of III.4, and arriving at the formulae of II. 6. For the matter of that, there are reasons to surmise that II. 6. was got from III. 4. by summation. The difference from the mean position must have been first noticed. This must have been accounted for by a variation in the motion from a minimum to a maximum and *vice versa*, postulating the variation to be constant as in III. 4. The factor (p-9) must have been introduced because the minimum 702' must be obtained for the first day ending 9 Padas, and so on for the others, (though $P-4\frac{1}{2}$ would have done better). Then by summation formulae, II. 6. must have been got. That is why the 0 value of the equation of the centre results for 133 padas instead of 124, and for a half-gati we get 180°4' instead of the correct 181°32'. Otherwise, it is easy to have given the 0 value for 124 padas, and the maximum or minimum for 62 padas, by making the equation of the centre formulae equal to $+(5P-620) \times P/63$, and combining these with $27'.843P$. The factor (P-1) in the formulae of II. 6, seems to form a relic of a prior summation, which Varāha seems to have retained out of respect for the original Siddhānta, for the same result will be got by the more simplified forms, $(1089+5P)P/63$ and $(2419-5P)P/63$.

I shall now point out some of the errors committed by *T* and *S*. They have seriously gone wrong in their interpretation of the second half of II. 3, and this after making two uncalled for emendations of the Ms. text. (See discussion of the text). In this part, as we have already seen, we are asked to add minutes equal to twice the Ghanas, and also add the *Mūla-Dhruva*, (or *Kṣepa*), 1 Rāśi 7°29'. *T* and *S* interpret it as, "Multiply the Ghanas by 2 and divide by 2971. The resulting Rāśis etc. are to be added". This means, instead of 2' per Ghana we add 1' 13'', and we do not add the *Kṣepa* 1 Rāśi 7° 29' at all. As for the 1' 13'', this is unwarranted when both the Mss. say 2' per Ghana. An emendation is called for only when a quantity given is so far removed from the actual that it is not likely to be the quantity given by the Siddhānta. Now the value 2' agrees better with the mean motion (siderial) per Ghana, viz. 110 revolutions 11 Rāśis 7° 32' 33''.5 given by modern astronomy, 7° 32' 15" for the time of Varāha. It agrees almost perfectly with the value of Siddhānta Śiromaṇi of Bhāskara II. While it is a matter for wonder and admiration how the ancient Vāsiṣṭha achieved a thing not achieved by most later Siddhāntas, *T* and *S* come in and spoil the whole thing by their emendations.

As for the *Kṣepa* which *T* and *S* have obliterated, it is essential, as it supplies the *Mūladhruva*. This can be seen from the mean moon I give according to different systems for sun-set at Yavanapura, (Alexandria), i.e., for 37 nādis, 20 vinādis after mean sun-rise at Ujjain, Sunday, close to the end of śaka 427.

Modern astronomy (this from the Vernal equinox of epoch)	354° 48'
Vāsiṣṭha (with the <i>Kṣepa</i> , taking Ujjain sunrise for the Vāsiṣṭha epoch)	355° 6'

Vāsiṣṭha (without the Kṣepa, taking Ujjain sunrise for the Vāsiṣṭha epoch)	317° 37'
Siddhānta Śiromaṇi	355° 49'
Romaka	356° 12'
Saura (with Ujjain noon for epoch, given)	355° 6'
Vāsiṣṭha (with the Kṣepa, taking Ujjain 37-20 nāḍikas for epoch for Vāsiṣṭha also)	346° 54'
Vāsiṣṭha (without the Kṣepa, taking Ujjain 37-20 nāḍikas for Vāsiṣṭha epoch also)	309° 25'

We see that with the Kṣepa in tact Vāsiṣṭha mean moon agrees with the other Siddhāntas and modern astronomy. With the Kṣepa gone, there is a defect of 37°.

Incidentally we must discuss another point here, viz. what time of the day is the Vāsiṣṭha epoch. Varāha gives different times of day for different Siddhāntas, and sometimes even for the same Siddhānta. For the Saura sun, moon, moon's apogee and Rāhu, midday at Ujjain (Sunday) is given as the time of epoch, and for the planets, the midnight following. For the Paitāmaha the time of day of epoch is morning (though the year is different). For the Romaka the epoch time of day is sunset at Yavanapura (Alexandria) which is equal to 37 nāḍikas 20 vināḍikas from sunrise at Ujjain (Sunday), excepting for the moon's apogee, for which sunset at Ujjain is given as the time. In the case of the Pauliśa the time is not mentioned, unless we strain नाऽतिचिरे ढौलिशेऽप्येवं (I. 10.) a bit, and make it mean that the Pauliśa time of epoch also is sunset at Yavanapura like the Romaka. But we can infer that it is indeed so, from III. 13-15. For the Vāsiṣṭha also the time is not mentioned. Is it sunset at Yavanapura, because it has been given as a general instruction? Or is it sunrise at Ujjain, as in the Paitāmaha? If it is sunrise at Ujjain,

the Vāsiṣṭha moon agrees closely with those of other Siddhāntas as also modern astronomy. If it is sunset at Yavanapura it is defective by about 8° . (If the Kṣepa is not taken into account the defect will be about 45°). Now 8° difference is too much, especially when we consider the accuracy of the Vāsiṣṭha moon's mean motion. So the time of day of the epoch for Vāsiṣṭha must be mean sunrise at Ujjain (Sunday). If we emend II. 3. into, ".....शशिमानु नवयमाश्चराद्याद्याः," then the Kṣepa is $17-14^\circ 29'$. In this case the Vāsiṣṭha moon will be $353^\circ 54'$ at Ujjain $37-20$ Nādikās, which agrees approximately with other Siddhāntas. So we can take the Kṣepa as $1-14^\circ 29'$, and keep sun-set at Yavanapura. But here there is the weight of the emendation.

We shall now continue our main discussion. In their interpretation of II. 4-5 also. *T* and *S* have fallen into material errors. In II. 4, two terms, plus-padas and minus-padas are defined, to be used in II. 6, which have been missed by them.

In II. 5, *T*'s translation wants us to add degrees equal to the number of Padas, only in the case of the Padas left over after deducting 124, i.e., in the case of what amounts to the minus-padas only. On the other hand this applies to the plus-padas as well. But *S*'s commentary gives the correct interpretation. In the second part of this stanza, *T*'s translation is non-committal. *S*'s interpretation is positively erroneous. He says : "अर्थात् वेदाकल्पपदेषु ऋणं, अधिकेषु धनमिति बुद्धिमद्भिः स्वयमेवोक्तम्". i.e., if the original padas are less than 124, the result of II. 6. is subtractive, if more than 124, additive. Evidently he thinks that II. 6. gives the equation of the centre, pure and simple, and it must be subtractive for padas less than 124, and additive otherwise. But as I have already explained, there are two rules in II. 6, the first requiring the addition of two terms, and the second requiring the subtraction of one

term from another. In the first, Padas less than 124 are to be used. But the result of either rule is to be added to the mean moon so far obtained. There is no question of subtraction here at all.

As for II. 6. *T* and *S* have said in so many words they do not understand it, but nevertheless given an interpretation, which naturally is wrong. III. 4, they have not touched.

Now we are in a position to discuss the readings of II. 2-6. and III. 4.

In II. 2. there are the scriptory errors, युक्तशशिगुण for युक्तेशशिगुण, and घना for घना which *T* and *S* have corrected. I have adopted these corrections.

In II. 3, the scribal errors घन for घन, हृतं शेषं for हतशेषं, छस्त्रिगुणितं for धस्त्रिगुणितं, घना for घना and श्वराशयाद्या for श्र राश्याद्याः, which *T* and *S* have corrected, I have adopted. *T* and *S* have corrected प्रोज्ज्य into प्रोह्य, while I have made it प्रोज्ज्य, as giving the sense better, and also more likely. There is a broken part in the Ms. text which *T* has correctly guessed as षोडश, and printed within brackets, and I have adopted this.

Apart from these small errors the Ms. is alright. But *T* and *S* have carried out two uncalled for emendations which seriously affect the subject matter, The first is the changing of कला into फलं, (when both Mss. give कला), simply because if the second emendation is made, कला would be troublesome. The second is the conversion of शशिमुनिनवयमाश्च into शशिमुनिनवयमहताश्च, when neither Ms. has हत, and the āryā metre is sinned against by the addition of the two mātrās, thus giving 18 mātrās to the

1. The one mātrā extra as it is, is excusable according to some commentators on Prosody. Or else, राश्याद्या must be read as राश्यादि.

fourth foot and making it a Gīti, which Varāha nowhere uses in his text¹. Further they have done this thinking they are improving the text. On the other hand they have spoiled the already correct and better text, and also shut out a necessary data (viz., the Mūladhruva), as we have shown before.

In II. 4, *T* and *S* have corrected नितः कडाः into नितः कलाः, which I have also done. Metre requires that पादसंख्या, must be changed into पदसंख्या and पद will be a better word also. *T* and *S* have failed to notice this, and retained पादसंख्या. षष्टांश, (a likely corruption for षष्ठांश or षष्ठ्यंश,) does not agree with the value given for the Ghana, and *T* and *S* have rightly amended it into काष्ठांश. *T* and *S* have corrected परतः into पदतः, not understanding the text properly, and परतः may stand.

In II. 5. लिप्ताश्चतुष्क for लिप्ताचतुष्क, समाश्वांशाः for समाश्वांशा, तश्च for तैश्च, and दन्त्ये for देयं, are all scribal errors corrected by *T* and *S* and adopted by me. In II. 6, षष्ठ्यो धृते for षष्ठ्योद्धृते is the only error, and that too scribal. *T* and *S* have made the correction and I have taken it.

In III. 4, गत्यन्तार्धाच्छोध्यो for गत्यन्तार्धाच्छोध्यो, is a scribal error corrected by *T* and *S* and adopted by me. As for साश्विसांवरो, corrected by *T* and *S* into साश्विखाचलो, साश्विखाचलो is a better correction, as retaining the ओ at the end. The masculine gender that results is not ungrammatical, for making the word qualify सप्तांशः is better than making it qualify भुक्तिः. An alternative correction would be साश्वि वस्वरो, which seems to be better still as it retains वरो (रो is given by both Mss.) instead of making it चलो, and I have adopted this. All three readings give 702 which is required in the formula. नगात् and वसुमुनिनवभः being apparently correct, *T* and *S* have

1. The one mātrā extra as it is, is excusable according to some commentators on Prosody. Or else, राश्याद्या must be read as राश्यादि.

not touched it, which is what all they could have done, because they have refrained from interpreting the stanza, saying the meaning is obscure. But both need correction. First we shall take up नगात् . A mātrā is wanting in the foot. I have shown the rules here are derivable from II. 6. The rules require "Padas minus nine" to be used. So I have made नगात् पदात् into [वि] नवात् पदात्, which gives the required meaning, supplying the wanting mātrā. As for वसुमुनिवसुभ्यः the second rule requires 879, and the numbers are given in the wrong order by the manuscript text. So I have emended it into नवमुनिवसुभ्यः.

Tithi and Nakṣatra

II. 7

शशादलं त्रिकुतिघ्नम्-
क्षमंशस्थिता मुहूर्ता स्युः ।
व्यर्केन्दुदलं विषयो-
हतं तिथिस्तद्वद्वचोक्तः ॥

शश्यर्धदलं त्रिकुति-
घ्नमुक्षमंशस्थिता मुहूर्ता स्युः ।
व्यर्केन्दुदलं विषया-
हतं तिथिस्तद्वद्वचोक्तः ॥

III. 7. The quarter part of the true moon multiplied by 9 gives the Nakṣatra (in the Rāśi column), and what is in the degree column are the Muhūrtas. Half the sun minus moon, multiplied by 5 gives the Tithis, in the same manner, (i.e., in the Rāśi column, and 30ths of Tithis in the degree column).

The rules are simple and based on the fact that there are $2\frac{1}{2}$ Nakṣatras for a Rāśi and $2\frac{1}{2}$ Tithis. Two things are noteworthy here. The word Muhūrta now only a measure of time, (through the time of staying of the moon in a Nakṣatra being used as a measure of time), is used for the 30th of a Nakṣatra segment. Even as applied to the 30th part of the duration of a Nakṣatra, it is different from the period of 2 Nāḍikās, in which sense it is usually used.

Now for the text: The scriptory errors शस्थिता for शस्थिता and मुहूर्ता स्युः for मुहूर्तास्युः, are corrected by *T* and

S and *I* have adopted their correction. शशादलं in the manuscript is corrupt and has been emended into शश्वर्धदलं by *T* and *S* because the rule requires it, and two mātrās are wanting. Also, the foot should end with त्रिकृति. I have adopted their emendation. राश्वर्धदलं would be an alternative emendation, which would extend the scope of the Nakṣatra rule to the sun as well.

Daylight

II. 8

मकरादौ गुणयुक्तो
मेखादौ तिथियुतो रवेर्दिवसः ।
कर्कटकादिषु सत्सु
त्रयस्त्रिकाः शर्वरीमानम् ॥

मकरादौ गुणयुक्तो
मेषादौ तिथियुतो रविर्दिवसः ।
कर्कटकादिषु षट्सु
त्रयस्त्रिकाः शर्वरीमानम् ॥

II. 8. When the sun is in Makara etc. (upto Meṣa), the sun (in Rāśis) plus 3 is the duration of daytime (in Muhūrtas). When the sun is from Meṣa etc. (upto Karkāṭa), the sun plus 15 is the duration of daytime, When the sun is in the 6 Rāśis from Karkāṭa, the sun plus 9, is the measure of the night-time.

The rule is easy to understand. According to this Siddhānta the shortest daytime is 12 Muhūrtas (i.e., 24 Nāḍikās), which occurs when the sun is at the beginning of Makara. The longest day is 18 Muhūrtas with the sun at the beginning of Karkāṭa. Between Makara and Karkāṭa, there is increase in daylight in Muhūrtas equal to the motion of the sun in Rāśis, and from Karkāṭa to Makara, there is increase of the night-time in the same manner.

It is to be noted that the rule gives the same result as the Paitāmaha Siddhānta and the Vedāṅga Jyautiṣa, with this difference that here the variation of daylight is with the *true* sun. It is also to be noted that really the daylight increases or decreases not with the sun as given here, but with the tangent of the declination of the sun. The

maximum-minimum daylight gives points to the extreme north of India, lat. 34°45'.

Now, the meaning easily follows with two small changes made in the text, रविः for रवेः and षट्सु for सत्सु (which latter *T* and *S* have also made). But not understanding the Stanza (or perhaps finding it disagreeing with II. 9-10) *T* and *S* have made a drastic change in the text, viz. भूस्वर्गतिथिमितो for मेषादौ तिथियुतो. The letters of भूस्वर्गति are quite unrelated with the letters in मेषादौ. I cannot understand how भूस्वर्गतिथिमि means 1591, i.e., how स्वर्गति means 9. (It is not a misprint, for the commentary also uses the expression.) Again त्रयस्त्रिकाः is taken to mean three by three, while it can only mean three threes, i.e., 9. Even if we allow these things, the meaning given by them is self-condemnatory. At the rate of 3 palas a day the measure of the day is 1865 palas when the sun is at the beginning of Meṣa, and 2139 when the sun is at the beginning of Karkāṭa, which means the night is 1461 palas; and this disagrees with 1591, because the duration of the day at Makarādi is the duration of the night at Karkāṭādi. The equinoxes according to this interpretation would occur 22 days before the sun reaches Meṣa and 22 days after he leaves Kanyā, i.e., from Vernal equinox to Autumnal equinox, there would be 226 days and from Autumnal equinox to Vernal equinox, there would be 139 days !

I have incidentally discussed the readings here. One remains, मेखादौ, which I have changed into मेषादौ, as ख being written for ष is common in certain parts of India.

The noon - day shadow and the sun

II. 9-10

9. कर्कटादिषु भुक्तं
द्विगुणं मध्यदिनी भवेच्छाया ।
मकरादिषु चाप्येव
किंचस्मिमण्डला लोध्यं ॥

9. कर्कटकादिषुभुक्तं
द्विगुणं माध्यन्दिनी भवेच्छाया ।
मकरादिषुचाप्येव
किन्त्वस्मिन्मण्डलाच्छोध्यम् ॥

10. मध्याह्नछायार्धं

सत्रिभमर्कोऽयने भवे द्याम्ये ।

उदगयने संशोध्यं

पञ्चदशभ्यो रविर्भवति ॥

10. मध्याह्नछायार्धं

सत्रिभमर्कोऽयने भवेद्याम्ये ।

उदगयने संशोध्यं

पञ्चदशभ्यो रविर्भवति ।

II. 9. When (the Sun is) in the (6) Rāsis beginning with Karkaṭa, the number of Rāsis passed beginning with Karkaṭa, multiplied by 2, is the noon-shadow (in Aṅgulis). The same (rule should be followed) when (the Sun is) in the (6) Rāsis from Makara. But in this case the result is to be deducted from 12 Rāsis (to get the shadow).

II. 10. When the Sun is in the south-ward course, (i.e. in the 6 Rāsis from Karkaṭa), half the noon-shadow plus three Rāsis is the Sun (in Rāsis etc.) When he is in the north-ward course, (i.e. in the 6 Rāsis from Makara) half the noon-shadow is to be subtracted from 15 and the Sun is got.

It is supposed that the shadow increases proportionately with the Sun from a starting value zero at the beginning of Karkaṭa. At the beginning of Makara the maximum, 12 aṅgulis, is reached. Then it decreases as it increases, and comes back to zero at the beginning of Karkaṭa. The length of the gnomon is not mentioned, but most probably it is 12 aṅgulis, commonly given in Hindu astronomy. Zero at the beginning of Karkaṭa would give about 24° Lat. to the region where the rule holds, and 12 aṅgulis maximum would give 21°. Thus there is contradiction within the stanza itself. This contradiction may be resolved by taking the gnomon as 11 aṅgulis, but this is not likely. Neither of these agrees with Lat. 34°45' got from the maximum and minimum daylight. The uniform variation is wrong, which means that the rule is not based on observation even at intervals. Therefore, the rules must have been taken from different sources and pieced together, and the remark of Brahma-

gupta, “एतान्येव गृहीत्वा वासिष्ठो विष्णुचन्द्रेण (कृतःकथा) seems to hold here, II. 10 is the reverse of II. 9.

As for the readings, कर्केटादिषु for कर्कटकादिषु, मध्यंदिनी for माध्यन्दिनी, भवेच्छाया for भवेच्छाया, वाप्येव for चाप्येव, मण्डलाच्छोध्यं for मण्डलाच्छोध्यं, are scribal errors corrected by T and S, and adopted by me. T and S have corrected किंचस्मिन् into किंवास्मिन्, while I have made it किंत्वस्मिन्, because तु is better for the meaning than च, and the likelihood is greater. All this in II. 9. There is no error in II. 10, worth the name.

Lagna from shadow and shadow from Lagna

II. 11-13

- | | |
|------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------|
| 11. द्वाशभिः सञ्जयै-
मध्याह्नौ नैर्भजेद्रसजताशं ।
अपराह्णे चन्द्रार्द्ध-
द्विशोध्य साकं भवति लग्नं ॥ | 11. द्वादशभिः सच्छायै-
मध्याह्नौ नैर्भजेद्रसहुताशम् ।
अपराह्णे चक्रार्द्ध-
द्विशोध्य साकं भवति लग्नम् ॥ |
| 12. व्यक्तं लग्ने लिताः
प्राक्पश्चाच्छोचितास्तु चक्रार्द्धात् ।
कायछेदः शून्यां
वराष्ट्रलवणोदषट्कानां ॥ | 12. व्यक्तं लग्ने लिताः
प्राक्पश्चाच्छोचितास्तु चक्रार्द्धात् ।
कार्यछेदः शून्या-
स्वराष्ट्रलवणोदषट्कानाम् ॥ |
| 13. लब्धं द्वादशहीनं
मध्याह्नच्छायया समायुक्तं ।
सा विज्ञेया छाया
वासिष्ठसमाससिद्धान्ते ॥ | 13. लब्धं द्वादशहीनं
मध्याह्नच्छायया समायुक्तम् ।
सा विज्ञेया छाया
वासिष्ठसमाससिद्धान्ते ॥ |

II. 11. The shadow at any moment is to be added to 12 and the noon shadow (of date) is to be deducted from that. By this, 36 is to be divided. This result is to be added to the Sun in Rāśis to get the Lagna, (i.e. Orient Ecliptic Point) (if it is forenoon). If afternoon, the result is to be subtracted from 6 Rāśis and this added to the Sun to get the Lagna.

II. 12-13. The sun is to be deducted from the Lagna and the remainder converted into minutes of arc, if it is forenoon. If afternoon, the remainder should be deducted from 6 Rāśis and then converted into minutes. 64800 is to be divided by the minutes. The result is to be added to the noon-shadow (of date) and 12 deducted from this. This is the shadow (at the time of the given Lagna).

This according to the brief Vāsiṣṭha Siddhānta.

Rule II. 11. can be explained as follows. (There is no question of proving it because it is rough and has been constructed empirically, its justification being its serviceability). As in the case of the rough rules current to find the time of the day from the shadow, and because Lagna follows the time roughly, the equation for Lagna, elapsed after sunrise, or to elapse till sun-set, can be expressed in the form, $a/(s+b)=L$, where a and b are constants, and s is a function of the shadow. If as a first approximation we take the latitude of the place as zero, and the ecliptic as coinciding with the celestial equator, it follows that there is a rise of 3 lagnas from sun-rise to noon, and 3 more from noon to sunset, that at noon, when $L=3$, the shadow is zero, and that midway between sunrise and noon L is $1\frac{1}{2}$, and the shadow then is $12 \times \tan 45^\circ$, i.e. 12. If really there is no such uniform rise of Lagna, and noon does not correspond to rise of 3 lagnas, but a little more or less, it is because the latitude may not be zero, and the ecliptic does not coincide with the celestial equator. These would also cause shadow at noon where otherwise there would be no shadow, and make the shadow at other times differ correspondingly. Now it is supposed that by deducting the noon-shadow from the shadow the first mentioned ideal condition is established, (because then s would become zero for noon, and the difference in the shadow at other times would be roughly righted). So it is

thought that by taking the shadow minus noon-shadow instead of the shadow itself, everything would be all right (though strictly they would not be). So we may write "shadow minus noon-shadow" for s . As its value, as we have already shown, is 0 for $L=3$, and 12 for $L=1\frac{1}{2}$, we have the two equations, $a/(0+b)=3$, $a/(12+b)=1\frac{1}{2}$, to determine a and b ; from which we obtain $a=36$, and $b=12$. Thus we get the expression $36/(\text{shadow minus noon-shadow} + 12)$ as the value of the Lagna elapsed from sun-rise in the forenoon, and the Lagna to elapse till sunset in the afternoon. As at sun-rise, the longitude of the sun is the Lagna, this is to be added to the sun. As at sun-set the longitude of the sun plus 6 Rāśis is the Lagna, the result is deducted from the sun plus 6 Rāśis, or which is the same, deducted from 6 Rāśis and the sun added. Thus the rule is explained. T's statement in his explanation, "In order to establish a workable proportion... 12 is added to the first and to the third members..." makes the rule appear more arbitrary than necessary.

The rule II. 12-13. is the inverse operation of the rule II. 11. and that explains it. But because the divisor which is in Rāśis in II. 11, is converted into minutes here, the dividend, 36, is multiplied by the number of minutes in a Rāśi and given as 64800.

One thing is to be noted. Except for finding when an auspicious lagna begins or ends by taking the shadow, II. 12-13 can only serve as a mathematical exercise as it is, for no rule has been given by the Siddhānta for the Lagna other than by using the shadow, and if the Lagna is got from the shadow, what is the meaning of getting the shadow back from the Lagna? At night the Lagna can be got by observation, but then there will be no shadow caused by the sun, to which alone the rule applies.

The concluding word, वासिष्ठसमाससिद्धान्ते, though forming a part of the last sentence giving the rule II. 12-13 may be taken to mean the whole of chapter II, and even III. 4, which we have shown to be Vāsiṣṭha's.

Now for the readings : the scribal errors द्वाशभिः for द्वादशभिः, सछायैः for सच्छायैः, रसजताशं for रसहुताशं चन्द्रार्द्धात् for चक्रार्द्धात् in II. 11, प्राक्पञ्च छोधितान्न for प्राक्पञ्चाच्छोधितान्न, कामछेदः for कार्यछेदः शून्यांबराष्ट for शून्याम्बराष्ट in II. 12, लब्धं for लब्धं, वासिष्ठ for वासिष्ठ in II. 13, have been corrected by T and S, which I have also adopted.

The place of the Vāsiṣṭha in the history of Hindu Astronomy

The Vāsiṣṭha marks a stage of development in Hindu Astronomy which is intermediate between those represented by the Paitāmaha (*P*) and the Saurasiddhāntas condensed by Varāha. The system we find in the *P* is the same as that we find in the Vedāṅga Jyotiṣa (*V.J.*). The five-year-yuga, the beginning of the yuga from Māgha Śukla, the treatment of the Sun and the moon alone, the absence of the equation of the centre, so that all reckoning is done using only the mean sun and the mean moon, the maximum and minimum duration of daylight and its uniform increase and decrease, are all found in both. Only the methods are different, the *V. J.* giving the ending moments of Tithis, lunar and solar, Nakṣatras etc., by an ingenious method which avoids computation using large numbers, while the *P* gives the *Ahargana* first, and uses this for computation, like the other Siddhāntas. The Saura, on the other hand, represents the fully developed stage of Hindu Astronomy, extending the field of operations to the planets as well, using epicycles for the equation of the centre and the equation of conjunction, and using spherical trigonometry to solve various problems. In between comes the Vāsiṣṭha. Methods for the true sun and moon are given.

Though the maximum and minimum daylight and its uniform increase and decrease is the same as in *P*, the *Vāsiṣṭha* deals with other topics also like computing the noon-shadow from the sun and vice versa, and the Lagna from the shadow and vice versa.

The *Vāsiṣṭha* gives a solar year of $365\frac{1}{4}$ days, which is fairly correct. As for the true sun, the method is crude, and is valuable only as indicating a distinction made between mean and true astronomical quantities, and the ability to express the same. Periods of $31\frac{1}{4}$, $31\frac{1}{2}$, $31\frac{1}{2}$, $3\frac{1}{2}$, 31, $30\frac{1}{2}$, $29\frac{3}{4}$, $29\frac{1}{4}$, $29\frac{1}{4}$, $29\frac{1}{2}$, 30, $30\frac{1}{4}$ days are given for the sun to traverse the *Rāśis* Meṣa etc., from which we can roughly say that the slowest motion is in the middle of Mithuna, and the quickest in the middle of Dhanus. All these are fairly accurate for the time of the *Siddhanta*.¹ Only, the days for Meṣa should be 31 and for Mīna $30\frac{1}{2}$. Even here the error is less than $\frac{1}{8}$ day, though it appears to be $\frac{1}{4}$ day.

The *Vāsiṣṭha* must have obtained these values empirically, most probably by an analysis of eclipses. It cannot be that it is *Varāha* that gives the values in this empirical form, computing them from the equation of the centre given by the original *Vāsiṣṭha*, for *Varāha* faithfully gives not merely the values, but also the method of the original. Or else why should he give different methods for the different *Siddhāntas*? In the case of the *Vāsiṣṭha* he gives the sun in periods of days, correct to quarter days. In the case of the *Pauliśa* he gives a correction in

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1. The periods of days agree well with the equation of the centre for the sun, about $132'$ given by Hindu Astronomy. At about 200 A.D. the equation of the centre was nearly $120'$, but with this was combined the moon's Annual Equation, about $12'$, with the Opposite sign, and this made the equation of the centre $132'$.

minutes to the mean sun, the correction depending upon Rāśis of "Anomaly", the term anomaly here being used in a peculiar sense. In the case of the Romaka he gives the correction to the Sun in the proper form of the equation of the centre, "k sine anomaly". In the case of the Saura he gives the regular epicyclic theory. It is also reasonable to suppose that before the epicyclic theory was formed, the previous three stages, just mentioned were gone through in the development of Hindu Astronomy.

As regards the moon, we have seen that the mean motion of the moon according to Vāsiṣṭha is more accurate than that of other Siddhāntas, as also its maximum equation of the centre, and its maximum-minimum daily true motion. This accuracy is possible only if observations of the moon with reference to the stars are made, instead of merely depending on analysis of eclipses, which is also necessary. Analysis of eclipses can give the mean motion correctly, but the maximum or minimum equation of the centre got would be only about $= 301'$, considerably less than the true values $\pm 377'$, because at syzygies where alone eclipses occur, the second inequality called the *evection* is reduced to the same form as the equation of the centre with the opposite sign, and a maximum numerical value $76'$, and thus reduces $377'$ to $301'$. It should not be argued that such accuracy is against Varāha's statement in I.4. There he compares only the Tithis of different Siddhāntas (because that is the most essential thing required, and one of the few things that are comparable), and Vāsiṣṭha - Tithi is spoiled not by the moon but by the sun got through using the incorrect mean solar year of $365\frac{1}{4}$ days¹ and also by the roughness of the rules in II.6. giving the moon's equation of the centre. As already explained (vide

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1. *T* and *S* have emended Tithi into Kṛta, when both Mss. give Tithi, and condemned Vāsiṣṭha beyond Varāha's intention,

above) these rules must have been based on observation of the daily motions, and the theory that they increase or decrease uniformly. Thus the equation of the centre was expressed first in an algebraic form as in the Vāsiṣṭha. Then the trigonometrical expression, "k sine anomaly," was discovered and used as agreeing better with observation. After this the epicycle theory was propounded, for, everywhere in science theory comes later to explain observed facts. Thus the Vāsiṣṭha is historically important as marking a stage of the development of Hindu Astronomy, where we see the first dawn of the true notion of heavenly bodies.

THE BĪJOPANAYA: IS IT A WORK OF BHĀSKARĀCĀRYA?

(Reprinted from J.O.I., M.S.U., Baroda, June 1959)

Introduction

The *Bījopanaya* is a short work on Indian astronomy which enunciates two corrections, to be applied to the value of the longitude of the Moon got by applying the usual *Equation of the Centre*, to make it more accurate. This work with a commentary called *Vāsanābhāṣya* was first published in 1876 by two pioneers in the field of *Dyḷ - almanacs* in South India, the late Chintamani Raghunathacharya of Nungambakkam and Taḍhakamalla Venkatakrishna Raya of Triplicane.¹ A short work called *Tithinirṇayakārikā* by Śrinivāsa Yajvan is also found added as an Appendix to this edition. Fifty years later, in 1926, another edition with an Introduction was brought out by Dr. Ekendranath Ghosh.² Neither edition speaks anything about the manuscripts used by the editors and it is not definitely known whether the 1926 edition was based on independent manuscript material or only on the older edition, which is not improbable especially in view of the fact that here too the *Tithinirṇayakārikā* appears as an Appendix. In both editions the text and the commentary are ascribed to the famous Indian astronomer Bhāskarācārya II of the 12th cent., author of *Siddhānta-śiromaṇi*, as stated in the colophons of the editions.³ But the work seems to be a much later production and an analytical examination tends to show that it cannot be a

1. Pub. Graves Cookson and Co., Popham's, Broadway, Madras.
2. Pub. Motilal Banarsidass, Punjab Sanskrit Book Depot, Lahore.
3. इति श्रीमहेश्वरोपाध्यायसुतभास्कराचार्यविरचिते वासनाभाष्ये भिताक्षरे बीजोप-
नयाधिकारः संपूर्णः ।

work of Bhāskarācārya, and that it is fathered upon him by its later author. It is intended to study the question from this light.

In passing, it may be noted that the present work, *Bijopanaya*, seems to be little-known, and no Indian astronomer or commentator who followed Bhāskara mentions this work or quotes from it, the only exception being the *Tithinirṇayakārikā* of Śrīnivāsa Yajvan, which is given as an Appendix to both the editions of the *Bijopanaya*. Also modern historians of Indian astronomy like MM. Sudhakara Dvivedi and S. B. Dikshit have not noticed this work in their histories.

The two corrections enunciated in the *Bijopanaya* are asked to be applied after the usual *Equation of the Centre*¹ has been applied. Of these, the first makes up for the deficiency in the *Equation of the Centre* when compared with the correct one, and the second is the equivalent of the Inequality called *Variation* which, when applied to the Moon will take it nearer to its true position in longitude.

A brief recapitulation of the history of the discovery of the three principal Lunar Inequalities will help in the proper evaluation of the *Bijopanaya*. In the West, the first Inequality, viz. the *Equation of the Centre*, was discovered by the Greek astronomer Hipparchus in c. 140 B.C. Ptolomy of Egypt (c. 140 A.D.) discovered the second Inequality known as *Evection*. Fourteen centuries later, in c. 1580 Tycho Brahe, a Danish astronomer, discovered the third Inequality viz. *Variation*. In India, the ancient *Vāsiṣṭha Siddhānta* is the first known work

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1. According to the *Sid. Śir.* the *Equation of the Centre* is equal to $-301.8' \sin l$, where l = the Mean Moon minus its Apogee. The other *Siddhāntas* give equations of very nearly the same value. The correct value is $-377.3' \sin l$.

that takes into account the Equation of the Centre in the motion of the Sun, the Moon and the Star planets (*Tārā-grahas*), and gives its own peculiar formulae thereof, consisting of the summation of an Arithmetical series.¹ The next ancient *Paulīśa Siddhānta* does not improve much on this.² But the *Romaka* and the *Saura Siddhāntas* that followed, as also the later *Siddhāntas* use the usual form, $a \sin \theta$.³ Muñjāla (932 A.D.) is the first Indian astronomer⁴ — I have read since that Vateśvara (c. 904 A.D.) earlier than Muñjāla has given this — who gives the *Evection*, in his *Laghumānasa*, Prakirṇādhyāya, 1-2. What he gives is equivalent to $-65.3' \sin l + 65.3' \sin (l + 2D)$,⁴ the first term compensating to a great extent for the deficiency in the *Equation of the Centre* of Hindu astronomy, and the second term forming the *Evection* proper. (To be exact, the coefficients should be 76' each). Next to Muñjāla, Śrīpati (c. 1000) in his *Siddhāntasekhara*, ch. XI. 2-4, gives the *Evection*. Then Nityānanda (1639) in his *Siddhāntarāja* gives the *Variation*, calling it by the significant name *Pākṣika*, i.e., one having a period equal to a *pakṣa* or 15 *tithis*. If the *Bijopanaya* is Bhāskara's work, as it purports to be, then he should get the palm for being the first, (and so early as 1151 A.D.), to give the *Variation*; excepting perhaps an Arabian astronomer by name Muhammad Abul Wefa, who Prof. Godfrey in the Introduction to his *Lunar*

1. These formulae can be seen in chs. II and XVIII of Varāhamihira's *Pañcasiddhāntikā* (PS). The daily motions of the Moon and heliocentric motion of the Star-planets are supposed to increase and decrease uniformly, and the Equations of the Centre are got from this by summation. For details see the writer's paper, 'The Vāsiṣṭha Sun and Moon', *JOR. Madras* 25 (1955-56) 19-41.

2. See PS ch. III.

3. See PS chs. VIII, IX etc.

4. D is Mean Moon minus Mean Sun.

Theory says, “ observed at Baghdad in 975 A.D., and discovered a third inequality called *Muhazal* ”. But I am afraid Bhāskara is not the author of the *Bijopanaya* and the Hindus cannot have the credit for this. Before proceeding to show this, I shall explain the passage in the *Bijopanaya* giving the two corrections.

The Two Corrections in the Bijopanaya

Of the 58 ślokas in the *Bijopanaya* only 13, viz. 20-32, deal with the two corrections, the rest being devoted to various other matters like the praiseworthiness of the *Tithis* got after applying the corrections. The following are these verses :

तुंगचपदान्तस्य द्विधोर्के पदार्धतः ।
 परमं च द्रवैषम्यं ऋणत्वेन समीक्ष्यते ॥ 20 ॥
 तत्तृतीयपदान्तस्थात् पृष्ठगेऽर्के पदार्धतः ।
 परमं च द्रवैषम्यं धनत्वेन समीक्ष्यते ॥ 21 ॥
 चन्द्रतुल्यं च नीचे च शशाङ्कमयहौ यदि ।
 मन्दस्फुटगतश्चन्द्र निर्बीजस्तुल्यमीक्ष्यते ॥ 22 ॥
 अजान्तयोर्विधोस्तुल्यत्वात् शशाङ्कार्कमयहौ यदि ।
 चतुस्त्रिंशत्कलाहानं वैषम्यं तु समीक्ष्यते ॥ 23 ॥
 अयतः पृष्ठतो वापि रवेश्चन्द्रे पदार्धगे ।
 तुल्यतुल्ये चतुस्त्रिंशत्कलावैषम्यमीक्ष्यते ॥ 24 ॥
 एवं तन्नीचतुल्येऽपि वैषम्यं तावदेव 'ह ।
 एवं व्योमात् समासाच्च पीन पुन्येन वेधनात् ॥ 25 ॥
 चाबीजमदं बलमं मया सद्भिः समीक्ष्यताम् ।

रसा (6) गुणे दू. (13) शशिलोचने (1) च भूमूतकरी (27) कालगुणौ (33) नवत्रिः (39) ॥ 26 ॥

शराब्जयः (45) चन्द्रशरा (51) रसार्थाः (56) पृथ्वीरसा (61) बाणरसा (65) गजाङ्गाः (68) । शू. याद्रयो (70) बाहुगिरी (74) च वेदतुङ्गमा (14) बाणहया (76) शरागाः (76) ॥ 27 ॥

रसाब्जय (76) शराङ्गहया (76) हयाङ्गाः (77) हयाद्रयो (77) नागहया (78) गजाङ्गाः (78) । गज द्रय (78) अति फलं ऋणे ऋणं धने धनं मन्दफलेन संयुतम् ॥ 28 ॥

अर्कस्फुटाच्चन्द्रमिमं विशोध्य शिष्टे ऋणं त्वोजपदं फलं सगत् ।

अतोऽन्यथान्यत्र यथाक्रमं वै भुवे फलानामपि पिण्डकानि ॥ 29 ॥

शराश्च¹ (5) नन्दा (9) गुणतारकेशी (13) भूमदुभुवो (17) बाहुकरी (22) जिनाश्च (24) ।

साराः (27) खरामाः (30) द्विगुणाश्च (32) देवाः (33) वाराशिरामा (34) सरिदीशकालाः (34) । 30 ।

वेदाग्रयो (34) दानवशत्रवश्च (38) शशाङ्कवह्नी (31) नवबाहवश्च (29) ।

रसाश्विनी (26) वेदकरी (24) खबाहू (20) रसक्षमे (16) रुद्र (11) गङ्गा (8)

ऽनलाः (3) खम् (0) । 31 ।

एताः कला ओजपदे ऋणं स्युर्घर्णे तदन्यत्र भवन्ति भूयः ।

अनेन युक्तश्च शशी स्फुटः स्यात् कर्मार्हकालानयनेषु योग्यः । 32 ।

20. The greatest subtractive difference between the calculated² and observed positions of the Moon, occurs when the Moon is 90° forward from its Apogee and the Sun is 45° forward from the Moon.³
21. The greatest additive difference occurs when the Sun is 45° behind the Moon which is situated 270° from the Apogee.
22. There is no difference when the Sun and the Moon or either is at the Moon's Apogee or Perigee.⁴
23. When the Sun and the Moon are situated at 90° or 270° from the Apogee the difference is —34'.

1. Dr. Ghosh's edition reads 'रसाश्च' (6),

2. Calculated, i.e., by using about $-302' \sin l$, alone, as is done usually in Hindu astronomy.

3. Everywhere Dr. Ghosh has translated *Padārdha* into 180° when it really means 45°; while he himself has translated *Pada* into quadrant (i.e., 90°) everywhere correctly.

4. Dr. Ghosh adds the word 'together' here, thus unnecessarily restricting the scope of the configuration, for even if one is at Apogee and the other at Perigee, the result follows.

24. When the Moon is at Apogee and¹ the Sun is at 45° from it, either behind or before, there is a difference of $34'$.
- 25a When the Moon is at its Perigee with the Sun at 45° , before or behind it, the difference is $34'$.
- 25b-28 Thus by means of repeated observations after varying the situations, I have determined the following *periodic*² corrections; may learned people examine these: For $3^\circ 45'$, $7^\circ 30'$, $11^\circ 15'$ etc. of Anomaly there are the following quantities of correction: 6, 13, 21, 27, 33, 39, 45, 51, 56, 61, 65, 68, 70, 72, 74, 75, 75, 76, 76, 77, 77, 78, 78, 78, all in minutes of arc. These are additive or subtractive as the *Equation of the Centre* is additive or subtractive; and must be combined with it (and applied to the Mean Moon to get the True Moon).
- 29-32. This True Moon is then to be subtracted from the True Sun. If the remainder is in an odd quadrant the following corrections are subtractive, otherwise additive. For $3^\circ 15'$, $7^\circ 30'$, $11^\circ 15'$ etc. of the remainder, the following corrections are given: 5, 9, 13, 17, 22, 24, 27, 30, 32, 33, 34, 34, 34, 33, 31, 29, 26, 24, 20, 16, 11, 8, 3, 0, in minutes. These are to be applied to the already corrected Moon and we get the correct True Moon. This is to be used for getting *Nakṣatras Tithis*, etc. for ceremonial purposes.

It may be noted that only six and a half ślokas, 26b to 32, give the actual corrections with the instructions for applying them.

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1. Dr. Ghosh gives 'or' for 'and' which is evidently a slip.
 2. Periodic, as opposed to secular, corrections that go on accumulating.

It can be readily seen that the second set of quantities for correction is approximately the *Equation of Variation* given by modern astronomy, viz. $+ 39.5' \sin 2D$, where D is the Mean Moon minus the Mean Sun.

As for the first set of quantities, each of them can be resolved into two parts thus: $(5.1 + .9)$, $(10.1 + 2.9)$, $(15.2 + 5.8)$, $(20.2 + 6.8)$, $(25.1 + 7.9)$, $(29.8 + 9.2)$, $(34.4 + 10.6)$, $(39 + 12)$, $(43.3 + 12.7)$, $(47.5 + 13.5)$, $(51.4 + 13.6)$, $(55.2 + 12.8)$, $(58.7 + 11.3)$, $(61.8 + 10.2)$, $(64.9 + 9.1)$, $(67.6 + 7.4)$, $(70 + 5)$, $(72.1 + 3.9)$, $(73.9 + 2.1)$, $(75.4 + 1.6)$, $(76.5 + .5)$, $(77.3 + .7)$, $(77.8 + .2)$, $(78 + 0)$. The first parts are equivalent to $78' \sin l$, and as these have the same sign as the *Equation of the Centre* given in the *Siddhāntas*, viz. about $-302' \sin l$, both combined becomes about $-380' \sin l$, which is very nearly the correct *Equation of the Centre* according to modern astronomy. The second parts are roughly¹ given by $(30' - 30' \sin l) \sin l + 5' \sin 2l$. Of these, $(30' - 30' \sin l) \sin l$, it can be seen, is only the part of the Equation which is in excess due to the difference in the radii of the epicycle at the ends of the even and odd quadrants, given by many Hindu astronomical works.² As these corrections purport to be applied after the *Equation of the Centre* of the *Siddhānta Śiromaṇi* has been used, the addition of $(30' - 30' \sin l) \sin l$ (together with $-78' \sin l$) would mean that the *Siddhānta Śiromaṇi* wants us to take the Epicycle at even quadrants to be $42^\circ 54'$ and at odd quadrants to be $39^\circ 46'$, instead of having throughout $31^\circ 36'$ as given by it. As for the worth of this part of the correction, if it improves the values a little in two quadrants, it will make them worse in the other two quadrants. Taking the part, $(5' \sin 2l)$, it may

1. The difference in any value is less than $1.5'$.
2. The *Sid. Śiromaṇi* does not give this; as also the ancient *siddhāntas* like the *Saura*. But the *Āryabhaṭīya* and the *Sūrya Siddhānta* give it.

seem to be an improvement because it appears to be a part of the term $(+13' \sin 2l)$ of the Equation of the Centre of modern astronomy. But this correction is not an improvement, because this forms part of the first correction which has been mentioned to have the same sign as the *Equation of the Centre* (corresponding to the principal term thereof of modern astronomy); and as such this is an improvement in the second and third quadrants, but will make matters worse in the first and fourth quadrants. What is to be specially noted is that the first correction does not give the *Evection* at all, and Dr. Ghosh is wrong when he says this gives the *Evection*. It only corrects the Hindu *Equation of the Centre* which is defective by about $(-76' \sin l)$, and makes it nearly equal to the correct *Equation of the centre*. As for the $(30'-30' \sin l) \sin l + 5' \sin 2l$, this is worse than useless in certain quadrants as I have already mentioned. Bearing all this in mind we shall now discuss the authorship of the work.

Bhāskarācārya not the Author of the *Bijopanaya*

The *Bijopanaya* itself mentions Bhāskara as its author. Śloka 6 says: "I was born in 1036 Śaka. I discovered the two *Bijas* when I was 37 years of age." The first part of the śloka giving the year of his birth is the same as is found in the *Sid. Śiromaṇi*, verbatim; the second part leads us to infer that he discovered the *Bijas* the next year after he wrote his *Sid. Śiromaṇi*, for he says in the latter work that he composed it in his thirtysixth year (*Sid. Śir.*, Gola, Praśnādhyāya. 58). In ślokas 2 and 3 of the *Bijopanaya*, the author says: "I wrote the *Siddhānta Śiromaṇi* following the ancient texts alone. But this is not sufficient to give the correct positions of the planets in order to find the auspicious moments for the different rites enjoined by the *śāstras*. So I am writing the *Bijopanaya*." The idea of the first half of verse 3 is a repetition from the *Sid. Śiromaṇi*, *Ganita*,

Spaṣṭa 1. From the above statements and the colophon one would think that Bhāskara is the author of the work. But it can be shown that Bhāskara cannot be the author, and consequently all the above statements are falsehoods and the author is an impostor.

The following are the reasons why Bhāskara cannot be the author. (1) Without the first *Bija* (the second *Bija* may be left out of consideration here, because it vanishes under the circumstances we are discussing here, viz., the syzygies i.e., conjunctions and oppositions) the Moon calculated by Hindu astronomy (and, of course, by the *Sid. Śiromaṇi* also) will give the syzygies almost correctly. The application of the *Bija* will spoil this correctness, with the result that an error upto more than ± 6 *nāḍikās* will be introduced into the times of the syzygies. This error at syzygies is proportionate to the Hindu *Equation of the Centre*, $-302' \sin l$, and therefore has the periodicity of the Moon's apogee. As a result of this error the middle of the eclipses, as calculated, will be later or earlier than what they actually are, by an amount of time equal to this error; and the beginning and end of the eclipses will also be affected accordingly.¹ Now, are we to think that Bhāskara gives this *Bija* only to introduce this enormous error into his otherwise correct calculation of eclipses? And Bhāskara of all persons? It is by observations made at the times of eclipses that the ancient astronomers have discovered or corrected the mean periods of the Sun and the Moon and their *Equation of the Centre*, and got such accurate values. Even after a lapse of centuries now, the Hindu Mean Sun and Moon are fairly accurate. As for the Hindu *Equation of the Centre*, it is what the three major inequalities of modern astronomy reduce to, at syzygies. D being nearly equal to 0° or 180° , the *Variation* vanishes as already mentioned. For the same reason, the

1. See the end.

Evection, $+ 76' \sin (l-2D)$ becomes $+ 76' \sin l$, and combined with the correct *Equation of the Centre*, ($-377' \sin l$) becomes ($-301' \sin l$). This is the reason why the Hindus did not discover the *Evection* for a long time, *i.e.* until Muñjāla (Vatesvara earlier) discovered it, and after him Śrīpati. Thus the foisting of this *Bīja* upon Bhāskara means that that prince among Hindu astronomers did not make even the usual observations made by Hindu astronomers, that he did not notice the error of upto 6 *nāḍikās* in the circumstances of eclipses, and that too introduced by himself, an error which any ordinary person, not to speak of astronomers, will be forced to observe, in India eclipses being occasions of important religious rites. So Bhāskara cannot have given this *Bīja*, especially when Muñjāla and Śrīpati had shown him the correct form of the *Bīja* which would lead to no such error as above.¹

(2) There is discrepancy between the observation values given in ślokas 20-25 and the *Bījas* given in ślokas 26-32 of the work. We shall see whether Bhāskara would have allowed this to get into his work, if it is his. In 20-25 the author takes six configurations of the Sun and the Moon with respect to the Moon's Apogee, and gives the difference between the observed longitude of the Moon and the longitude got by the *Śāstra* without the *Bījas*. They are given below in a tabular form, together with certain other data that will be useful.²

1. I have since learnt that Sengupta had noticed this point. But he did not follow it up to the inevitable conclusion that Bhaskaracharya cannot be the author not being interested in this.
2. In computing the actual difference that should have been observed, I have used the correct *Equation of the Centre*, the *Evection*, the *Variation* and the *Parallactic Inequality*. The *Annual Equation* and the *Reduction to the Ecliptic* cannot be used and therefore left out; also they cannot affect the result much.

	Configurations of the Sun and the Moon with respect to the Moon's Apogee, as given by the author	Author's observed values of difference	The values that must be got according to the Bijas given	The actual difference he should have seen if he had really observed	Configurations of the Sun and the Moon with respect to the Moon's Apogee according to Ghosh's translation of <i>Padārdha</i> into 180°	Author's observed values of difference	The difference that must be got according to the Bijas	The difference that would be seen actually
i.	Moon 90° and Sun 135° from Apogee	-112'	-112'	-113'	Moon 90° and Sun 270° from Apogee	-112'	-78'	0
ii.	Moon 270° and Sun 225° from Apogee	+112'	+112'	+113'	Moon 270° and Sun 90° from Apogee	+112'	+78'	0
iii.	Moon 0° or 180° and Sun 0° or 180° from Apogee	0	0	0	Moon & Sun 0° or 180° from Apogee	0	0	0
iv.	Moon 90° or 270° and Sun 90° or 270° from Apogee	-34'	±78' ¹	0	Moon & Sun 90° or 270° from Apogee	-4'	±78'	0
v.	Moon 0° and Sun 45° or 315° from Apogee	34'	±34' ²	±39' ³	Moon 0° and Sun 180° from Apogee	±34'	0	0
vi.	Moon 180° and Sun 225° or 135° from Apogee	34'	±34' ⁴	±113' ⁵	Moon 180° and Sun 0° from Apogee	±34'	0	0

1. -78' if Moon is at 90°, +78' if at 270°.

2. -34' if Sun is at 45°, +34' if at 315°.

3. -39' if Sun is at 315°, +39' if at 45°.

4. +34' if Sun is at 135°, -34' if at 225°.

5. +113' if Sun is at 135°, -113' if at 225°.

To prove our point, we shall take configuration iv. Here the first correction is $\mp 78'$ according as the Moon is 90° forward or behind from the *Apogee*. The second correction is practically 0. So according to the *Bījas* given, the difference should be $\mp 78'$. But $-34'$ has been given as observed. If the author had observed first and based the *Bījas* on the observation, how could he construct the *Bījas* in such a way that they would disagree by $44'$ (if the Moon is 90° forward from *Apogee*) or even $112'$ (if the Moon is 90° behind the *Apogee*). This means that the author is an extremely ignorant person. Or if the author has taken the *Bījas* from elsewhere (which is very probable) and has 'cooked' the values for the different configurations, this means he cannot even 'cook' properly. One can never expect this of Bhāskara, that master among astronomers. And it is asserted that he wrote this work after writing the *Siddhānta Śīromaṇi*, his masterpiece ! Configuration vi too shows the above.

(3) I shall now justify the suggestion that the author had probably cooked the values at the configurations and had not made the observations, in spite of his assertion to have done so in śloka 25. In configuration iv above, he should have observed zero difference, but he says he has observed $-34'$ difference. There is an error of observation of $34'$. In v and vi he gives an observed difference of $34'$; he does not say whether it is plus or minus. Let us examine v. If the Sun is at 45° , the observed difference should be $+39'$, which we may take as agreeing with the $+34'$ given. But in this case the *Bījas* give $-34'$. Thus there is discrepancy amounting to $73'$ between his *Bījas* and observation. If the Sun is at 315° , the observed difference should be $-39'$ which we can take to have been observed by him as $-34'$. But in this case the *Bījas* give $+34'$ difference, and again there is the same amount of discrepancy between his observation value and *Bījas*. Thus we are driven to one of two alternatives : either the

author cannot see the disagreement between his observation and his *Bījas*, or there is a discrepancy of 73' between his observed values and the actual values he should have observed, i.e., there is an error of observation of 73'. In vi, the observed difference is $\pm 34'$ while the actual difference should be $\pm 113'$, which means an error of observation of 79'. With such large errors of observation, very nearly equal to the maximum value of the major *Bīja* and more than double the maximum value of the minor *Bīja*, how at all could it be possible for the author to fix the *Bījas*? Again, with such a large capacity for error how did he get exactly 34' in three configurations, all equal, and 112' in two? It is extremely unlikely that these observed values would be so closely correct, especially with such a large capacity for error. Then again, with such errors there is little likelihood of his discovering the constant of *Variation*, viz., 34' so nearly equal to the correct value 39', and the other constant, 78', so nearly equal to the correct 76' whether it be the constant of *Evection* or the defect in the *Equation of the Centre*. These things show that the author must have 'cooked' his observational values from *Bījas* taken from elsewhere, the 'cooking' being done wrongly in certain cases. Now can this person be Bhāskara?

Lest it should be thought that the interpretation of *Padārdha* to mean 180°, as done by Ghosh, might save the author from all the above criticism, I have given in the same table the corresponding values of the configurations according to his interpretation. It will be seen that except in iii, no observation agrees with the *Bījas*; that there is discrepancy between observation and the value that should have been observed, to the extent of 112' in i and ii, and 34' in iv, v and vi, and that the correct difference observed should have been 0 in all the configurations, because according to Ghosh's

interpretation all of them are syzygies, the Hindu values being correct at the syzygies, as I have already stated.

(4) Another reason why the *Bījas* cannot be Bhāskara's is that it is very unlikely he would have given them in the form of tabular values instead of in the form of an equation, as is generally done by astronomers, and that too without combining the first *Bija* with the *Equation of the Centre* as he could have done. He would only be following the usual practice if he had given in the *Sid. Śiromaṇi*: "The Moon's epicycle is $42^{\circ} 54'$ at the end of even quadrants and $3^{\circ} 8'$ less at the end of odd quadrants:" and then given $5' \sin 2l$ separately as also $34' \sin 2D$. Or if he had wished to give the *Equation of the Centre* with the traditional value first and then the *Bījas* in a separate chapter, as done by Muñjāla and Śrīpati, he might have given the epicycle as $34^{\circ} 44' - 31^{\circ} 36'$ in the *Spaṣṭādhikāra* and in a later chapter given $-(78' \sin l + 5' \sin 2l)$ and $-34' \sin 2D$, where D is True Sun minus True Moon. Even if he had discovered the *Bījas* later, he could have no difficulty in incorporating it in the *Sid. Śiromaṇi* written by him only one year earlier.

The author of the *Bījopanaya* seems to have anticipated this argument, for he says that Bhāskara wanted to keep the *Bījas* secret and therefore did not give them in his regular work; in śloka 58 he says: "This should not be given to one who has not served for one year." But this answer is unconvincing in the light of the following: Throughout the *Bījopanaya* the author dings into the readers the importance of the *Tithis* corrected by the *Bījas* for ceremonial purposes. Then what is the point in making a secret of this knowledge, so as not to reach the hands of astronomers and almanac-makers (for, as already mentioned, no astronomer or commentator refers to this work, excepting the author of the *Tithinirṇayakārikā* with which this work seems to be

associated ; nobody seems to have “been in tutelage for one year” and qualified himself!) ; for it cannot be kept a secret and at the same time made serviceable. A medicine can be kept a secret and at the same time used by a family of physicians to cure a disease and make money. Even in predictive astrology knowledge can be kept secret and at the same time profitable. But knowledge of the type we are talking about must be public if it is serviceable ; for *Tithis* computed by using the *Bījas* would be different from those computed otherwise, and people would want authority before following them. As for astronomical predictions like eclipses, where agreement with observation is all-important, and not authority, I have already pointed out that the *Bījas* would only serve to spoil even the existing agreement. Further, astronomers usually refer to their science as a secret one, which means that it should not be given to the layman who has no respect or fitness for it ; but secrecy of an astronomical work from astronomers themselves is unthinkable. Muñjāla or Śrīpati have not kept their *Bīja* secret, and Bhāskara must have known it. Then what is the point in suppressing a part of the *Equation of the Centre* and making a secret of it, (the first *Bīja* is only this), for anybody can do this.

(5) It has also to be noted that the style of the work is not Bhāskara’s and the spirit, not that of the *Sid. Śiromaṇi*. The *Sid. Śiromaṇi* reflects Bhāskara’s great reverence for Brahmagupta, whose authority is quoted wherever necessary ; and it is also known that Bhāskara belongs to the school of Brahmagupta. But the *Bijopanaya* is surcharged with the spirit of the *Sūryasiddhānta* to which frequent and exhaustive references are made as authority. For instance, at the very beginning of the work there is a long quotation from the *Bijopanayādhyāya* of the *Sūryasiddhānta*, a passage which Raṅganātha, commentator on the *Sūryasiddhānta*,

characterises as an interpolation. While the *Sid. Śiromaṇi* reads like a scientific work, the *Bijopanaya* reads like a *dharmaśāstra*. The matter of fact style of Bhāskara, stands in contrast with the racy style of a controversialist which characterises the *Bijopanaya*. See for instance the following passage from the *Vāsanābhāṣya* appended to the *Bijopanaya* and purporting to be by its author: एकं च बीजोपनयप्रयासभीताः तन्मालवः बीजोपनयागमस्य प्रक्षिप्ततामारोप्य अप्रामाण्य-सुररीचकः । तन्मतं निर्बीजमेवेति नाद्रियामहे, वक्ष्यमाणरीत्या तदुद्भावितप्रक्षेपानुमानानां तर्कमासत्वात् । etc.

Incidentally there seems to be a reference here to Ranganātha's allegation mentioned above. This passage also seems to reflect the controversy that raged during the seventies and eighties of the last century in South India between Karungulam Krishna Josyar as the protagonist of the *Vākya* system of almanacs and Sundaresa Srautigal, Nungambakkam Raghunathacharya etc. as the sponsors of the *Dṛk* system. One important point of controversy was whether the centre of the celestial sphere is the same as the centre of the Earth or different, and for what celestial phenomena the centre of the Earth (or of the celestial sphere if it is different) is to be taken as the point of reference, and for what others the position of the observer was to be taken as that point. Sundaresvara Srautigal and others argued that the two points are one, while Krishna Josyar argued that they are different, and that planets are given in the *Śāstras* with reference to the centre of the celestial sphere. When the former (it may be noted that they were the people who first got the *Bijopanaya* printed at Madras) quoted the following passage from it, in support of their stand : अथेदानीं खमद्ये दृश्यमानस्य भूमध्यदृक्तुल्यग्रहस्यैव पारमार्थिकत्वे हेतुमाह etc., no wonder Krishna Josyar accused them of forgery ; he also questioned the authority of the *Tithinirṇayakārikā* which

mentions the *Bijopanaya*; cf. “बीजोपनयमाध्यान्ते भास्करायैः प्रपञ्चिताः ।” “बीजोपनयमाध्यान्ते भास्करेण विपञ्चिता ।”¹

To conclude, in the above discussion we have certain pieces of conclusive evidence which have been placed first.² Those that follow are valuable as cumulative evidence, the combined effect of which is to show beyond doubt that Bhāskarācārya of 1150 A.D. cannot be the author of the *Bijopanaya*. This being so, the real author of the work remains to be identified by further investigation.

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1. Cf. Page 404. The solar eclipse of 23rd Nov. 1965 Tuesday: In the matter of calculating this these corrections will aggravate the error, as said.

Also, the statement, “If the Gaṇita of the country does not give the necessary dṛksphuṭa there is nothing wrong in taking the almanac of another country”, is tell-tale, and means the Nautical Almanac which these people were using to compute their dṛk almanacs.

2. In the *Vāsanābhāṣya* under śloka 9, an astronomer named Malla Bhaṭṭa is mentioned. If it could be discovered that he is later than Bhāskara, this will be another piece of evidence to show that Bhāskara is not the author of the *Bijopanaya*.

A HISTORICAL DEVELOPMENT OF CERTAIN HINDU ASTRONOMICAL PROCESSES

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It is interesting to study how in course of time Hindu astronomers developed more and more refined methods from cruder ones in the matter of (i) finding the true longitudes of the sun, the moon, and the star-planets, (ii) applying the equation of time, (iii) computing the eclipses, and (iv) fixing the *Mahāpātas*. The earlier *siddhāntas* that could help us in this have been lost, so far as we know at present. But Varāhamihira (V.M.) has chosen what were typical of the ancient ones, and given them in a condensed form in his *Pañca Siddhāntikā* (P.S. c. A.D 505), which we can use for our purpose. The extensive quotations of Bhaṭṭotpala from a *Paulīśa Siddhānta* and various other ancient *samhitās*, in his commentary on the *Bṛhat Samhitā* are also helpful. We shall consider these one by one.

I The True Longitudes

In the *Vedāṅga - jyautiṣa* (c. 1180 B.C.), the earliest Hindu astronomical work extant, and the less so ancient *Samhitās*, and the *Sūryaprajñapti* and *Kālalokaprakāśa*, as also the *Paitāmaha Siddhānta* condensed by V.M. in the P.S. only the mean sun and moon are given. The true *nakṣatras*, *tithis*, etc., required for religious rites and observances must have been obtained by the mean sun and moon as guides.

The *Vāsiṣṭha Siddhānta* of the P.S. (not the ones extant and available as separate works) is the first *siddhānta* giving methods for the true sun, moon and star-planets. P.S. II, i. gives the true sun, and the substance of what it says is as follows: The year begins, i.e. the sun is at the first point of the sign *Meṣa*, one and a half days before the *epoch* taken by the P.S., and it takes $31\frac{1}{4}$, $31\frac{1}{2}$, $31\frac{1}{2}$, $31\frac{1}{2}$, 31, $30\frac{1}{2}$, $29\frac{3}{4}$, $29\frac{1}{4}$, $29\frac{1}{4}$, $29\frac{1}{2}$, 30 and $30\frac{1}{2}$ days successively to traverse the twelve signs and

reach the first point of *Meṣa* again.¹ Giving the true sun thus in an empirical form, is facilitated by the absence of the knowledge of motion of the apsides, as it is very slow.

In the case of the moon, this *siddhānta* recognizes the equation of the centre (eq. cent.) superimposed on the uniform mean motion of the moon, recognizing the true motion to be zigzag, and a 'step-linear function' of time ; i.e. the rate of motion is supposed to increase uniformly @ $10\frac{1}{7}$ per *pada* (= a ninth of a day), from a minimum at apogee of $702'$, to a maximum at perigee of $879'$ and then falling at the same rate to the minimum again at the next apogee.² Thus the time between one apogee and the next (called a *gati*) divided into 248 *padas* (i.e. $27\frac{5}{9}$ days for an anomalistic revolution) forms the period of the zigzag. The mean longitude up to the last apogee is first given, by II, 2-4. We have now to add the true anomaly to get the true longitude. For that purpose the 248 *padas* are divided into two parts, the first 124 forming the first half *gati* being called plus *padas*, and the second set called minus *padas*. The true anomaly consists of two parts, the accumulated mean motion @ $1^{\circ}27'.843$ per *pada*, and the accumulated defect or excess in the motion forming the eq. cent. These two parts have to be combined and given. But the *siddhānta* takes the 1° per *pada* separately and adds it to the mean longitude already found, for the sake of convenience. The remaining $27'.843$ per *pada* is combined with the eq. cent., which here is a summation of the difference of the linear function of motion from the mean, and thus an

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1. Dr. Thibaut and M. M. Sudhakara Dvivedi (T. and S.) say that they do not understand the text, and the former even thinks it may deal with the moon.
 2. P.S. III, 4, gives this, but it is not recognized by T. and S. (Though Chap. III belongs to the *Paulīṣa*, most of the matter relating to the moon in II and III is common to both *Paulīṣa* and *Vāsiṣṭha*.)

algebraic function of the *pada* of the second degree, and therefore different from the correct eq. cent. of the form $\mp a \sin m$. The result of the summation is $\mp (665-5P) P/63$ in minutes (the upper sign for the plus *padas*, and the lower for the minus *padas*). Combining this with the residual mean motion, $27'.843 P$ separately for the plus or minus *padas*, we get the two formulae given in II, 6, $\{1094 + 5(P-1)\} \times P/63$, and $\{2414-5 (p-1)\} \times P/63$, respectively.³ When the *padas* are in the second set, the true motion for the first set of 124 *padas*, equal to 6 *rāśis* and 4 minutes, is added.

The *Vāsiṣṭha* applies the eq. cent. of the above form in the computation of Jupiter and Saturn also (Chap. XVIII, 6-20). In the case of Venus this is dispensed with, being small. As a first step for every star-planet, the number of days after its mean heliacal rising is found. The days from the true heliacal rising is found by applying the days corresponding to the eq. cent. at the point of heliacal rising. The synodic period of Jupiter, Venus and Saturn is divided into sections, for each of which the degrees of motion is given, according as the motion is 'quick', 'mean', 'slow', 'retrograde', etc., depending on the equation of conjunction (eq. conj.). The days in heliacal setting, with the corresponding degrees, is also given. From this set, the degrees corresponding to the true number of days gone is taken and added to the degrees at heliacal rising, with the eq. cent. applied. Thus the geocentric true planet is got.

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3. This subject has been exhaustively treated by me in 'The *Vāsiṣṭha* sun and moon etc.', published in the *Journal of Oriental Research*, Madras, 1957 (Vol. XXV, parts I-IV), and also in my Commentary on the *P. S.* written for the Indian Institute of Astronomical and Sanskrit Research, New Delhi. T. and S. have failed to understand this, and also some other parts of this section.

In the case of Mars and Mercury with large eq. cent. varying with the planet's situation in the twelve *rāśis*, since the planet may be in any of, or, all the 12 *rāśis* during its synodic revolution, the above method cannot work. So the varying sectional motion for each of the *rāśis* is given (Chap. XVIII, 21-56). Thus we see that the eq. cent. and the eq. conj. are mixed in the case of these two planets ; while in the case of Jupiter and Saturn they are separated, which is possible since their motion is comparatively small.

It is said that as early as the third century B.C. the astronomers of Babylonia used the zigzag or step-linear function, and also gave the star-planets according to the sections of motions in the respective synodic periods, starting from their heliacal rising, and this was used in predictive astrology. Thus there seems to be a connection between the *Vāsiṣṭha* and Babylonian astronomy, and there might have been give and take. Which borrowed which portion from which has to be determined after learning more about these. Or we may consider both as one unit belonging to a part of greater India.

At the end of Chap. XVIII of the *P.S.* the motion of the star-planets is again given in the last sixteen verses and attributed to the *Paulīśa*. But there is a lot of evidence to prove that this portion is an interpolation.¹

1. The evidence of interpolation is as follows :

- (i) It comes after V.M. has finished the work in the customary manner in XVIII, 62-63.
- (ii) It begins with a new salutation and introduction in 64-65.
- (iii) After recommending his treatment of the star-planets according to the Saura and *Vāsiṣṭha*, to people who have been vexed with the inaccuracies of other authors, V.M. is not likely to have given this approximate stuff with the praise

‘वरमिति वराहमिहिरो ददाति निर्मेत्सरः करणम्’ (65) ...contd.

In the case of every planet here the eq. cent. is neglected, so that the motion is simplified and given for sections of the synodic period, on the same model as the *Vāsiṣṭha* Venus. It is useless, being very rough on account of neglecting the eq. cent., and also of no historical value, coming as it does long after the age of the eccentric and epicyclic theories.

The next step forward is seen in the *Pauliṣa* method for the true sun (*P.S.*, III, 1-3). The instruction is as follows : Multiply the days from *epoch* by 120, subtract 33 and divide by 43,831. The revolutions etc. of the 'mean' sun is got. The 'mean' sun plus 20° is called *kendram* (Anomaly). For successive 30° of *kendram* first subtract 11, 48, 69, 70, 54, 25 minutes of arc, and then add 10, 48, 70, 71, 54, 25 minutes, to get the true sun.¹ Here again we see the zigzag, the variation from the 'mean' being given for the degrees of *kendram* instead of the time passed, and the period of variation beginning at 20° before the commencement of the so-called 'mean' year and ending at the same point. Actually the 'mean' sun has practically the maximum eq. cent. of about 139' combined with it, and so the beginning of the 'mean' year is practically that of the true. The improvement consists in relating the amplitude of the zigzag with the degrees of

(iv) In the case of Mercury and Venus, the author has interchanged *Vakra* and *aticāra*, not knowing which occurs at the superior Conjunction, and which at the inferior. This blunder could not have been committed by V.M., who has given the same in the *Saura* and *Vāsiṣṭha* correctly.

1. T. and S. have misunderstood the verses here, and failed to understand that the maximum eq. cent. is combined with the 'mean' here. By this they have not only shut out the required method of getting the anomaly, but also given a method which will give wrong values. For fuller details the reader is referred to the relevant part of my commentary on the *P.S.* written for the Institute mentioned above,

the so-called *kendram* paving the way for the appearance of the real *kendram* next, in the *Romaka Siddhānta* condensed in the *P.S.*

As regards the moon, the *Pauliṣa* has taken over the method of the *Vāsiṣṭha in toto*, and in III, 5-8 gives a correction to it, in order to yield its own values, conforming more to the ' $a \sin m$ ' formula. But owing to the corruption of the text it is not possible to interpret the verses with any certainty.

It is in the *Romaka* that we see for the first time the mean sun and moon and their respective eq. cent. recognized as separate entities, going to make up the true sun and moon. The eq. cent. is also recognized to depend on the angular distance called *kendra* (now in the true sense of the word) between the mean body and a point in its orbit (the *ucca* or apogee, but not named here.) Its increase or decrease is given for every 15° of *kendra*, with instructions when to add and when to subtract (*P.S.* VIII, 1-6). By adding the increments the sun's maximum eq. cent. can be found to be $143' 23''$ and the moon's $4^\circ 55'$. These values differ considerably and erratically from the values that will be got from the formulae, $143' 23'' \sin (0^\circ, 15^\circ, 30^\circ, \text{etc.})$ or $4^\circ 55' \sin (0^\circ, 15^\circ, 30^\circ, \text{etc.})$. Therefore, we have to conclude that the *Romaka* values were got empirically by observation, and not by using the trigonometrical form. But from better observations and better values, the next step would be the discovery of the trigonometric form, which would lead to the eccentric and epicyclic theories as an explanation of the observed result.

How the early Hindu astronomers explained the zigzag motion, now slower, now faster than the mean, now direct and now retrograde, is contained in the *Sūrya Siddhānta* (Chap. II, 1-5). Deities stationing themselves at certain points in the orbit (later to be identified with

the apogee and superior conjunction) pull the bodies backwards and forwards with invisible air-reins, which motions combining with the uniform natural motions of the bodies result in the zigzag, it says. But this theory is at best only qualitative, though satisfactory to the minds of the ancients, which could connect anything extra-normal with invisible deities.

As said before, the discovery of the trigonometrical form for the eq. cent. led to the theory of eccentrics and epicycles, as found in the *Ārdharātrika pakṣa* of Āryabhaṭa (reflected in the *Khaṇḍa Khādyaka* of Brahmagupta, and mentioned in the *Mahābhāskariya*, VII, 20 etc.), and in the *Saura* of the *P.S.*¹ According to the eccentric theory the planet (either mean or *śighra*) moves uniformly in a circle whose centre is away from the earth's centre by a distance equal to the maximum equation of the centre (as measured on the arc of the circle) in the direction of the respective *ucca*. According to the epicyclic theory the mean planet moves in a circle round the earth as centre, while the true planet moves on an epicycle of radius equal to the sine of the maximum eq. cent. or eq. conj., as the case may be, moving round the mean planet as centre. If the maximum eq. cent. is $2e$ (= sine max. eq. cent. approximately), we can get from the theory the first term of the modern eqn. cent. $2e \sin m$ (where m is the

1. It is said that these theories originated among the Greek astronomers and Ptolemy II of Alexandria (c. A.D. 140) perfected it in his *Almagest*. The Hindu astronomers adopted the theory, but used better methods like trigonometry in the computation and discovered better constants by analysis.

I have since seen the *Laghumānasa*, and found that its R , (i.e. त्रिज्या), is $8^\circ 8'$ (or 8° ?)

इन्दूच्चोनाऽर्ककोटिघ्ना गत्यंशा विभवा विधोः ।

गुणो व्यर्केन्दुदोः कोटयोः रूपपञ्चास्रयोः क्रमात् ॥ १ ॥

फले शशांकतद्गतयोः लिप्ताद्ये स्वर्णयोर्वधे ।

ऋण चन्द्रे घनं भुक्तौ स्वर्णसाम्यवधेऽन्यथा ॥ २ ॥

mean anomaly reckoned from the apogee), taking the distance between the true planet and the earth (called the true hypotenuse) as approximately equal to the radius of the circle, as many *siddhāntas* do. If the true hypotenuse also is taken into account, as logically following from the theory, which the school of Āryabhaṭa does, we get the second term also in the form, $-2 e^3 \sin 2m$, only slightly different from the modern $-5/4 e^3 \sin 2m$ (cf. the article, 'Some peculiarities of the school of Āryabhaṭa' by the author).

But these theories cannot completely take the place of the actual, for the moon or the planets really move on ellipses, with the parent body at one focus, and the motion is such that equal areas are swept by the radius vector in equal times. The necessary difference between the results of the defective theories and observation was sought to be removed in some works (like the *Āryabhaṭīyam* and the later *Sūrya Siddhānta*) by supposing the epicycles to vary; but this has not met with much success. Only in recent times, the adoption of modern methods of computation based on the correct theory could give results to any desired degree of accuracy.

In the case of the moon, the disturbance caused by the sun's pull on it is another cause of great deviation from the mean motion. This is analysed into various *equations* called the Second, Third, etc., inequalities, the eq. cent. itself being the First. Of these the Second inequality, now expressed in the form $-76'.4 \sin (m - 2D)$, (where $2D$ is the mean elongation of the moon, and m is the mean anomaly reckoned from the perigee according to modern custom), was first discovered by Ptolemy II of Egypt (second century A.D.) and called by him the *Evection*. Of the Hindu astronomers, Muñjāla was the first to give this in his *Laghumānasa* (A.D. 932) *prakīrṇādhyāya*, 1-2. It will be interesting to know how he got this. What he gives reduces to $-65'.3 \sin m + 65'.3 \sin$

($m-2D$), (m here being reckoned from apogee according to ancient practice). Of these the first term approximately compensates the deficiency in the eq. cent. of Hindu astronomy (= about 300' maximum, while actually it is 377'.3). The second term forms the Evection proper.¹ Next to Muñjāla, Śrīpati (A.D. 999) gives it in his *Siddhānta Śekhara*, XI, 2-4.²

1. Here evection and the corresponding correction to the daily motion is given. Taking the daily mean motion of the moon to be 13.2°, we have the rule for the evection:

$$-(13.177-11 \cos (\alpha'-\vartheta) \sin D \times (8\frac{8}{60})^2, \text{ the result taken as minutes),}$$

$$\begin{aligned} &= -72' \times 2 \cos (\alpha'-\vartheta) \sin D \\ &= -72' \times 2 \cos (\alpha'-\vartheta + \alpha - \alpha) \sin D \\ &= -72' \times 2 \cos (m-D) \sin D \\ &= -72' \sin m + 72' \sin (m-2D), \end{aligned}$$

where α' and α are the longitudes of the sun and the moon, ϑ is the moon's *ucca* from which its anomaly m is reckoned, and D is $\alpha - \alpha'$.

(The use of $\cos D$ will give the correction to the daily motion, when divided by 5).

It is to be noted here that Yellayāvya, commenting on the *Laghumānasa*, says that Muñjāla got this from Vaṭeśvara. If so, Vaṭeśvara (A.D. 904) must be the first.

2. त्रि भविरहितचन्द्रोच्चोन मास्वद्भुजज्या
गगननृप (160) त्रिनिघनी भवयज्याविम्वता ।
भवतिचरफलाख्यं, तत्पृथक्स्थं शर (5) घनं
द्वतमुहुपतिकर्णत्रिज्ययोरन्तरेण ॥ २ ॥
परमफलमवाप्तं तद्वनर्णं पृथक्स्थे
तुहिन किरणकर्णे त्रिज्यकोनाऽधिकेऽथ ।
स्फुटदिनकरहीनादिन्दुतो या भुजज्या
स्फुट परमफलघनी भाजिता त्रिज्ययाऽऽप्तम् ॥ ३ ॥
शशिनि चरफलाख्यं सूर्यहीनेन्दुगोलात्
तद्वनमुतधनं चेन्दूच्च हीनाऽर्कगोलम् ।
यदि भवति हि साम्यं व्यस्तमेतद्विधेयं
स्फुटगणितद्वयैक्यं कर्तुमिच्छद्भिरत्र ॥ ४ ॥

...contd.

From (2) and (3) we have the erection as follows :

$$\begin{aligned} & \sin \{\alpha' - (\vartheta - 3 \text{ राशि})\} \times 160' \times \sin D \\ &= -160' \cos (\alpha' - \vartheta) \sin D \\ &= -80' \times 2 \cos (\alpha' - \vartheta) \sin D \\ & \dots\dots\dots \\ &= -80' \sin m + 80' \sin (m - 2D), \text{ as before.} \end{aligned}$$

1. This information is taken from the *Gaṇaka Tarāṅgiṇi* by M. M. Sudhakara Dvivedi.
2. cf. Author's article, 'The Bījopanaya ; is it a work of Bhāskarācārya ?' published in the *Journal of Oriental Institute*, M. S. University of Baroda, Vol. 'III. no. 4, June 1959. The impostor author has taken great pains to make the work appear like Bhāskarācārya's. But out of several, I give hereunder two main reasons why it cannot be by that astute astronomer. (i) The evection part does not serve to give the Evection, but it only spoils the already correct Hindu sūzygies by a maximum of six nāḍikas, and phenomena like eclipses occurring at syzygies will expose this error even to a lay man, not to speak of astronomers. (ii) Observational values for certain configurations of the sun and the moon are given. These agree neither with the formulae given, nor with actualities, so that they show the author up as a 'cook', and an ignorant 'cook' at that, and certainly not Bhāskarācārya.

Nityānanda has also given the *Reduction to the Ecliptic*, which is 7' max. in the case of the moon. The earlier *siddhāntas* neglected this as being small, though it must have been known at least to Bhāskarācārya who had understood the need of the Udayāntara or *Reduction to the Equator*. The Kerala astronomers like Mādhava were using this earlier than Nityānanda.¹ Later, contact with modern Western astronomy led to the adoption of the fourth major inequality, the Annual Equation ($= +11'$ sin sun's anomaly, having a maximum value of even 14' at syzygies), as also a host of lesser inequalities.

It is interesting to consider the question why the early Hindu astronomers failed to detect even the major inequalities, excepting the eq. cent. The mean motions of the sun and the moon, as also their eq. cent., were obtained by them by the analysis of the times of eclipses, which occur only at syzygies, and so they are very accurate for the syzygies, a tribute to their powers of analysis. But since $2D$ is practically zero at these times, the variation becomes zero, and the Evection is reduced to the form $+76'.4 \sin m$, which combined with the eq. cent. $-376'.4 \sin m$, becomes $-300' \sin m$, very nearly the same as the Hindu eq. cent. Similarly $+11' \sin$ (sun's anomaly) merges in the sun's eq. cent., having the same form with its sign reversed, since the sun is subtracted from the moon to get the times of syzygies, so that the sun's eq. cent., about $-119' \sin$ sun's anomaly (this was its value about A.D. 500), becomes about $-133' \sin$ anomaly, as given by the *siddhāntas*. If instead of analysing the times of eclipses accurate observation of

1. cf. Rāśi gola sphuṭānīti (verse 47) by the Kerala astronomer Acyuta :

पातो नस्य विधोस्तु कोटिभुजयोर्जिवे मिथस्ताडये-
दन्त्यक्षेपश्राहतं वधममुं विक्षेपकोटया हरेत् ।
लब्धं व्यास दलोद्धृतं हिमकरे स्वर्णं, विपाते विधौ
युग्मायुग्मपदोपगे, विधुरयं स्पष्टो भगोले भवेत् ॥

the moon's longitude had been used, at least the major inequalities could have been detected early, one after another, as the accuracy of observation improved.

II The Equation of Time

The Equation of time (eq. time) is the difference between the mean and true moons or mean and true midnights. This is required in Hindu astronomy to take the sun, moon or star - planet, computed by using the mean days, and therefore got for the end of the mean day, to the true day, which is the practical unit of time. (This is enough for systems that begin the day at midnight like the *Ardharātrika* of the *Saura Siddhānta* of the *P.S.* or the *Sūrya Siddhānta* school. But if the day begins at sunrise as in most *siddhāntas*, the variation in daytime also will have to be corrected for. If the place is east or west of the prime meridian like Ujjain, a correction for this also will have to be made.)

The eq. time consists of two parts: (a) the part caused by the true sun generating the true day being behind the mean sun or in advance, owing to the eq. cent., and called *Bhujāntara* ($= -22 \sin \text{sun's anomaly in } \text{vinadis}$); (b) the part caused by the true sun moving along the ecliptic, so that its projection on the celestial equator is behind or forward of the mean sun assumed to move along the celestial equator that forms the *nāḍi maṇḍala* and called *Udayāntara* (i.e. reduction to the equator $= -25 \sin 2\odot$ in *vinadis*, \odot being the *sāyana* true sun).

Even at the stage of the *Romaka Siddhānta* of the *P.S.* the need for correcting the sun etc. for the eq. time was not recognized by Hindu astronomers. In the *Saura* of the *P.S.* the *Bhujāntara* correction appears for the

first time, followed by all later astronomers.¹ The *Āryabhaṭīyam* does not mention this, but Āryabhaṭa's followers have argued that this is intended in *gītika* 2 by the expression, *ajārkodayācca laṅkāyām*, as done by Govindaswamy under Mahābhāskariyam, IV, 7. Its appearance in the *Khaṇḍakhādyaka* shows this probable. At any rate, Bhāskara I gives it in his work (IV, 7, 24, 29-30).

Vaṭesvara sees the need to refine the minutes of *bhujāntara* by projecting it on the celestial equator, by instructing that the minutes of the eq. cent. should be multiplied by the *prāṇas* of ascensional difference of the sign occupied by the sun and divided by 1800.² Śrīpati gives the *bhujāntara* without this refinement since it will be taken care of by the Udayāntara, which he gives for the first time in Hindu astronomy (*Siddhāntaśekhara* II, 46; XI, 1).³ After this Bhāskarācārya and others continue to give this.

1. P.S. IX.9 though given here for the sun and the moon, it must be taken as intended for the star-planets also in XVII, for a similar reason. Āryabhaṭa's *ārdharātrika pakṣa* must have given this at least for the moon (as the correction may be as large as 5' in this case) as seen from the *Khaṇḍa Khādyaka*:

मांशो (27) ऽर्कफलस्येन्दोः

षड्दशयूनाऽधिके केन्द्रे (I. १६)

2. वटेम्बरसिद्धान्त स्पष्टाधिकार I—

...सूर्यफलकलाऽभिहताः । ६३ ।

राश्युदयारचरेहोरात्रासुभाजितास्तेन संगुणिताः ।

गतयो ग्रहस्य शून्याभनगमही (1६00) भाजिताः फलं रविवत् । ६४ ।

3. रविफलगतिघातात् चकलिप्ताभिराप्तं

स्वमृगमिहविदध्यात् अर्कवत् खेचरेषु ।

सवित्फलषट्शो योविलिप्तासुभानोः

दुधिनमदसि कुपात्तस्य वा विषय (27) भागम् । II ४३ ।

...contd.

III The Computation of Eclipses

Eclipses are mentioned even in the Vedas, the earliest works of the Hindus. Even in the early Fifth *Maṇḍala* of the *R̥gveda*, in the 40th *sūkta*, the solar eclipse is mentioned as caused by *Svarbhānu* (otherwise called *Rāhu*), and well understood by the *Atri* family of priests. In the *Chāndogya Upaniṣad* the lunar eclipse is mentioned as caused by *Rāhu*.¹ Therefore the Saros or something like it might have been known then. But computation as such could not have been possible before the stage of the *Vāsisṭha* of the *P.S.*, which is the first to give the true sun and moon forming the basis.

P.S. Ch. VI gives the lunar eclipse according to the *Vāsisṭha*. (It does not attempt the solar eclipse). The instruction is as follows :

(a) Find the time of true opposition, and the true moon then. (b) Subtract $1^{\circ}36'$ from *Rāhu* (Head or Tail, whichever node is near the moon), and find the distance between this and the moon in degrees. If this is within 13° , there is an eclipse. If within 15° , there is only a slight darkening. (*Rāhu* is not given by this *siddhānta*.)

अन्त्य भ्रमेणगुणिता रविबाहुजीवा-

ऽभीष्टभ्रमेणविहता फलकार्मुकेन ।

बाहोः कलासु रहितास्ववशेषं ते

यातासवो युगयुजोः पदयोर्धनर्णम् ॥ XI १ ॥

(अत्र भ्रम इति दृज्या II = स्पष्टाधिकारः, XI = ग्रहयुद्धाधिकारः)

In the introduction to the *Vaṣeṣvara Siddhānta*, page 23, the editor has looked for the *udayāntara* of the *Siddhānta Sekhara* in the wrong place, and naturally has not found it there.

1. यत्वा सूर्यं स्वर्मानुस्तमसाऽविध्यदासुरं ।

... ..

गुळहं सूर्यं तमसाऽगदुरतेन तुरीयेण ब्रह्मणाऽविन्ददन्निः ॥

यं वै सूर्यं स्वर्मानुः तमसाऽविध्यदासुरः ।

अत्रयस्तमन्वविन्दन् न ह्यन्ये अश्वनुवन् ॥ (ऋग्वेदः V.40.)

छान्दोग्योपनिषदि अन्ते, “चन्द्र इव राहोमुखात्प्रमुच्य”

So we are expected to use the Pauliśa's. Subtraction of $1^{\circ}36'$ may be an empirical correction.) (c) The minutes of arc of half-duration = $\sqrt{55^2 - L^2}$, where L is the latitude of the moon in minutes. (This latitude is Pauliśa's.) (d) $60 \times$ minutes got in (c) $\div D$ (i.e. difference of the true daily motions of sun and moon) = time of half duration in *nāḍikas*. Subtract this from and add to the time of opposition to get the first and last contacts. (A correction is given next for having used the *pauliśa* L ., whose significance is not understood by T and S.) (e) $21 \sqrt{25 - d^2}$ = minutes of arc of half total phase, where d is moon~Rāhu in degrees. (f) $60 \times$ minutes got in (e) $\div D$ = time of half total phase, from which the beginning and the end of total phase is got.

From (e), the critical latitude for total obscuration is $21'$, from (c), the same for the eclipse is $55'$. Therefore the angular semidiameters of shadow + moon = $55'$ and shadow-moon = $21'$, from which we get the radius of shadow = $38'$ and radius of moon = $17'$, both considered constant. Add to the error caused by these, the roughness of taking the opposition as the middle of the eclipse, and not taking the variation of the latitude during the eclipse into consideration.

In *P.S.* VII, 6, the *pauliśa* gives the duration of the lunar eclipse directly in *nāḍikas*, equal to $3 \sqrt{169 - d^2}/4$. Since D is not used here, the computation is rougher still. It does not give the total phase.

The instruction, common to both *siddhāntas*, to find the points of first and last contacts, is as follows : Divide the moon's half orb on the side opposite to the direction of its latitude into 13 strips of equal width, by lines parallel to its east-west (this with reference to the ecliptic) diameter. The eastern and western points of the $(d + 1)$ parallel line beginning with the diameter are the points of first and last contacts. These have to

be reduced to the observer's east-west by the two *valanas*, *ākṣa* and *āyana*. The latter is omitted here. The former is given roughly by multiplying a quarter of the moon's rim by the degrees of the hour-angle and the degrees of latitude of the observer, and dividing by 8100. Thus, instead of their sines, the degrees themselves are used for the proportion (VI, 7-8).

The *Pauliṣa* is the first *siddhānta* to attempt the difficult solar eclipse (P.S. VII). Here the parallax in longitude is transferred to the time of conjunction, as done by all Hindu *siddhāntas*. The correction to the conjunction is taken roughly as $4 \times \sin h$ (= Hour. angle) in *nāḍis*. The manner of giving the parallax in latitude (l) is peculiar, in that it is done as three corrections to *Rāhu*, so that the (moon ~ corrected-*Rāhu*) will give the parallax-corrected-] direct. (T. and S. have failed to understand the instruction : see commentary under VII, 2-4).

The first correction to *Rāhu* is : multiply the degrees of latitude of the place (ϕ) by 5 and divide by 27. Add to *Rāhu* if Head (i.e. ascending node), subtract if Tail (descending node). If we resolve the total parallax in latitude ($= \pi \sin$ zenith distance of the nonagesimal) into three parts involving the latitude of the place, the declination of the moon or sun, and the hour-angle, this approximately does duty for— $\pi \cos \omega \cdot \sin \phi$, where π is the difference of the horizontal parallaxes of the sun and the moon (about 49' according to Hindu astronomy), and ω is the obliquity of the ecliptic (taken as 24° in *do*). Latitudes in India being low, $\sin \phi$ is taken as $\phi \div 57$ approximately. Since the *Pauliṣa* gives 55' of moon's latitude (l) for 13° of moon ~ *Rāhu*(d), the correction to *Rāhu* = $-49 \times .92 \times 13^\circ \times \phi \div (55 \times 57) = -5^\circ \times \phi \div 27$. This being negative decreases north l and increases south. So this is to be added to the head and subtracted from the tail as given.

The second correction is : Add 3 *rāsis* to the sun. Take the declination (ϕ) of this point in degrees. Multiply these degrees by the number of *nāḍikas* of correction to conjunction, and divide by 22. If the time of eclipse is *Uttarāyana* and forenoon, add the resulting degrees to *Rāhu*, if Head. Subtract instead of adding if any one of *Uttarāyana* 'forenoon' and 'Head' is changed, or if all three.¹ This rule does duty for— $\pi \sin \omega \cdot \cos \phi \cdot \sec \delta \cdot \cos \odot \cdot \sin h$, \odot being the sun, and δ being the sun's declination. This *siddhānta* has taken $\cos \phi$ as .9 approx. for North India, and $\cos \delta$ is taken as unity. $\sin h = \text{nāḍis of correction} \div 4$. $\sin \omega \cdot \cos \odot = \sin \omega \cdot \sin (\odot \text{ plus } 3 \text{ rāsis}) = \sin f = f$, approx. Thus we get $49 \times .9 \times 13 \times f \times \text{nāḍis of parallax got in } (1) \div (55 \times 57 \times 4) = f \times \text{nāḍis of } (1) \div 22$. This has the same sign as $-\cos \odot \cdot \sin h$. Since $\cos \odot$ is positive for *Uttarāyana* and $\sin h$ for morning, the degrees resulting from the rule are additive to Head. It follows that change of any one or all three must make it subtractive while the change of any two must keep it additive.

The third correction is : find the *nāḍis* elapsed from sunrise to corrected conjunction if forenoon, or the *nāḍis* to elapse till sunset if afternoon (i.e. find the *unnata nāḍis* of corrected conj.). Multiply this by the degrees of declination (δ) of the moon, and divide by 80. If the moon's longitude is within 6 *rāsis* the resulting degrees are to be subtracted from the Head or added to the Tail. If more than 6 *rāsis*, add to Head and subtract from Tail. This serves for the one remaining correction, $+\pi \cdot \cos \omega \cdot \cos \phi \cdot \tan \delta \cdot \cos h$. Here, $\cos \phi$ is roughly taken as unity, and $\tan \delta$ equal to δ approx. As for $\cos h$, this is very roughly taken as *unnata nāḍis* divided by 15. So the correction takes the form, $+ 49 \times 13 \times 0.92 \times \delta \times \text{unnata nāḍis} \div (55 \times 57 \times 15) = \delta \times \text{unnata nāḍis} \div 80$. Since h is

1. I have emended 'द्विविभक्तः' into 'द्विद्वि भक्तः' which supplies also the one *mātrā* wanting.

roughly taken to be within $\pm 90^\circ$, and therefore $\cos h$ always positive, the sign of the correction depends only on the moon's declination, which is positive for the moon in the first 6 *rāsis* and negative otherwise. So the correction is subtractive and additive to the Head respectively, and vice versa to the Tail. These three corrections must have been obtained empirically.

An empirical correction of $-1^\circ 36'$ is made on this corrected *Rāhu*, as done for lunar eclipse. The duration of the eclipse is given in *nādis* by $3\sqrt{64-d^2}/4$. The corrected conjunction \mp half this, gives the first and last contacts. The total phase is not attempted. As in the lunar eclipse, here too the variation in the moon's latitude is not taken into account, as also the true *D*. From the formula we get, $d=8^\circ$ as the limits, i.e. when the corrected latitude is $55' \times 8 \div 13 = 33' \cdot 8$. From this, since the angular radius of the moon has already been found to be $17'$, that of the sun is $16' \cdot 8$ according to the *Pauliṣa*, both constant.

The *Romaka Siddhānta* gives a very much improved method for the solar eclipse. (Its lunar eclipse is not given in the *P.S.*) It is the first to use the *Tribhonalagnam* (i.e. the nonagesimal) in the computation (exhibiting thereby knowledge of the trigonometrical problem involved), though the method given is rough. The conjunction corrected for parallax is to be found first, in the same way as in the *Pauliṣa*. For this time the *lagna* (i.e. Orient Ecliptic point) is to be found. This minus 3 *rāsis* is the nonagesimal. The sine of the zenith distance of the nonagesimal (*zdn*) has to be multiplied by the relative horizontal parallax π to get the parallax in latitude. But the *Romaka* takes the *zdn* as equal to the declination of the nonagesimal $\pm \phi$, the upper sign being used for south declination and the lower for north. (This is given as an approximate method by Brahmagupta.) π is taken as roughly proportionate to the moon's

true daily motion, as generally done in Hindu astronomy. Thus the parallax in latitude is given by the formula, tabular sine zdn ($=120 \sin zdn$) \times moon's true daily motion $\div 1800$. (This will give $53'$ for mean π . Other *siddhāntas* make this $49'$ by subtracting $4'$ for the sun's parallax from the moon's $53'$. But as the sun's is really $0'15$, this $53'$ of the *Romaka* is far near the correct $57'$).

Since the moon is not on the ecliptic when its latitude is other than zero, a correction has to be made for this in the parallax in latitude. But this is too small (maximum half a minute) and neglected by some *siddhāntas*. The *Romaka* and some others intend to give it, but by the following method which is wrong (and gives rise to an error of $4'$ and more): Treat the nonagesimal as the moon and find its latitude. Add this to the declination of the nonagesimal, and use this corrected declination to find the zdn above (P.S. VIII, II). Bhāskarācārya (II) sees the error in this and criticizes it in his *Bhāṣya*.¹

The moon's latitude is to be got by multiplying the tabular sine of moon $\sim Rāhu$, by 21, and dividing by 9. (This will make the obliquity of the moon's orbit $280'$, better than the $270'$ given by other *siddhāntas*).²

The mean angular diameters of the sun and the moon are given as $30'$ and $34'$ respectively, and they are made true by multiplying by their true daily motions and dividing by the mean, as done by most *siddhāntas*. It must be noted that the sun's diameter here given is too small by $2'$ and the moon's too large by $3'$. The half duration is given in *nādis* by $60 \times \sqrt{\Delta^2 - P^2} \div D$, where Δ

1. सिद्धा शिरो-गणित-सूर्यग्रहणाधिकार-उपसंहारभाष्यम्—
रश्मिदक्षेपार्थं यद्विन्निमलनेषुणाऽत्र संस्करणम् ।
जिष्णुजमसं तदुक्तं न मन्मते वच्मि युक्तिमिह ।

2. T. and S., by a misinterpretation, make it 4° .

is the sum of the semidiameters, l is the parallax corrected latitude at corrected conjunction, and D is, as already said, the difference of the true daily motions. This is subtracted from or added to the corrected conjunction (as in the *Pauliṣa*) to find the first and last contacts. This is defective in so far as the l of the first and last contacts are not found by successive approximation, as done by the later *siddhāntas*, and used to find the respective time. The total phase is not given by this *siddhānta*.

The method for the solar eclipse given by the early *Saura Siddhānta* condensed by V.M. in the *P.S.* (Chap. IX) is more refined and typical of the later *siddhāntas*, and uses far better constants. Like the *Romaka* the *Saura* also uses the true motions of the moon and the sun to get their respective true parallaxes and angular diameters. But in the case of the moon the true motion at the time of the eclipse is used (which is proper) instead of the true daily motion (IX, 14). Also these are not got directly from their mean values as in the *Romaka*, but from their orbits and diameters in *yojanas* as in the regular *siddhāntas*, though these are not the actual numbers of *yojanas* but those reduced by a factor 270 in the case of the orbits, and other appropriate factors in the case of the diameters and parallaxes. By examination we can find that the mean diameters of the sun and the moon are 32'.1 and 32'.2 respectively, and their mean horizontal parallaxes 3'.8 and 51'.4 (IX. 15, 16, 22).¹

To find the sine of the zdn also a better method than the *Romaka's* is used: The sine of the zenith distance of the meridian ecliptic point is first found. Using the amplitude of the orient ecliptic point, which is equal to the azimuth of the nonagesimal, the sine of the angular

1. T. and S., have made several mistakes here, which have been discussed by me in my commentary written for the above-mentioned institute.

distance between the meridian ecliptic point and the nonagesimal is found next. With these two sines treated as hypotenuse and one side, the sine of the zdn forming the other side of the right-angled plane triangle is found, approximately, instead of getting it by solving the spherical triangle. After this the parallaxes in latitude and longitude are got by accurate steps.

The parallax corrected conjunction is got from the parallax corrected longitudes, and made accurate by successive approximation. From the parallax corrected latitude of this time, the approximate half duration and first and last contacts are found. Using the parallax corrected latitude of the first or last contact a more accurate time of the respective contact is got, and thus by successive approximation the correct times are got. In this too the *Saura* is an improvement on the *Romaka*. It also gives the total phase, together with that of the lunar eclipse, in X, 7.

For the shadow also in computing the lunar eclipse, the reduced orbits and diameters are used and the result given in a more reduced form. If we reduce it still more and express the angular diameter in radians, we get $36 \div \text{moon's orbit} - 572 \div (5 \times \text{sun's orbit})$, the first term representing double the moon's horizontal parallax of the modern formula, and the second representing double the difference between the sun's angular radius and horizontal parallax. The mean diameter of the shadow can be found to be $78'.4$. As already said the total phase also is computed. Successive approximation is used. The method to find the fraction of the moon eclipsed at any given moment is given, as in later works.

A separate chapter (XI) is devoted by the *Saura* to the graphical representation of eclipses. In order to make the figure appear as big as the real, a scale of one *angula* or digit for $2'$ is instructed to be used near the

horizon, and for 3' at the zenith, and proportionately in intermediate positions (XI, 6). But the difference in size according to its position in the sky is an optical illusion depending on the atmospheric conditions, and various scales are given by various *siddhāntas*, the *Saura* being the first to give it.¹

In order to mark the points of first and last contacts at the exact positions as seen by the observer, the lay of the segment of the ecliptic where the moon is situated has to be fixed with references to the east-west of the observer. Two corrections, one due to the latitude of the place called *ākṣavalana* and the other due to the moon's *ayana* called *āyanavalana*, have to be applied to the east-west points. (We saw the *ākṣavalana* given by the *Vāsiṣṭha* itself, but very roughly using degrees instead of their sines.) But the versine of the hour-angle is used instead of its sine, a mistake of some ancient astronomers, pointed out by Bhāskarācārya (II). While the *Vāsiṣṭha* dispenses with the *āyanavalana*, the *Saura* gives it fairly correctly as $\sin^{-1} (\cos \text{moon's longitude} \times \sin \omega)$, instead of $\tan^{-1} (\cos \text{moon's longitude} \times \tan \omega)$ (XI, 2-3). It is noteworthy that the *Saura* does not use the versine here, a mistaken practice of the ancients condemned by Bhāskarācārya (II).² (The *Romaka* is silent about the directions).

1. e.g., i. मौरिकाऽर्धाङ्गुला ज्ञेया यथावा लक्षते दिवि । (महाभास्करीयं V 63)
with Parameśvara's interpretation of the second part as
'क्षितिजाऽऽसन्ने ग्रहे एकैकालिप्ता एकाङ्गुलप्रमाणा भवतीत्यर्थः ।
- ii. सोन्नतं दिनमध्यमं दिनार्घाप्तं फलेन तु ।
छिन्द्याद्विक्षेपमानादि तान्येषामङ्गुलानि तु । (सूर्य. सि. IV 26.)
- iii. त्रिज्योद्धृतस्तत्समयोत्थशंकुः सार्धद्वियुक्तोऽङ्गुललिप्तिकस्स्युः । स्थूलास्सुखार्थं
द्युदलेन भवतं समुन्नतं सार्धयमान्वितं वा । (सि. शिरो. गणित. चन्द्रग्रहणाधि. (24)
2. दृष्टिकर्मवलनं च केनचिद्भ्रान्तितः कथितमुत्क्रमज्यया ।
तत्कृतं तदनुगेस्ततोऽपरैरेवपूरुषपरम्परोपमैः ।

...Contd.

The *Āryabhaṭīyam*, and the *Mahābhāskariyam* following it, give the same method as the *Saura* for parallaxes, angular diameters and *valana*. But they find the first separately for the sun and the moon, using in the moon's case its own orbit instead of the ecliptic, to find the parallaxes. We have already mentioned that this trouble is not worth taking in view of other errors. The mean angular diameters and the horizontal parallaxes of the sun and moon according to these are 33', 31'·5, 3'·9 and 52'·5.¹ It is remarkable that the *Mahābhāskariyam* uses parallax in the lunar eclipse, of course without purpose (Chap. V, 68-70). The mean angular diameter of the shadow can be found to be 79'·8. The later *Sūrya Siddhānta*, representing a school to which the *Soma*, *Brahma* and *Vṛddha Vāsiṣṭha Siddhāntas* belong, also uses the same method as the *Saura* of the *P.S.* with one difference. It gets the *nāḍis* of parallax in conjunction direct by a formula that reduces to $4 \times \cos zdn \times \sin$ (sun ~ nonagesimal). We have seen that the *Pauliṣa* and the *Romaka* also give it in *nāḍis* with the same maximum of 4 *nāḍis*. The parallax in latitude is given by $\sin zdn \times$ difference of mean daily motions $\div 15$ (V, 7-8). We see that here the mean horizontal parallax is taken to be the mean daily motion divided by 15, which gives 3'·9 for the sun and 52'·7 for the moon. Using the true daily motions

ब्रह्मगुप्तकृतिरत्र सुन्दरी साऽन्यथा तदनुगैर्विचार्यते ।

नोद्धता कृतेरथोद्धतास्तु वा मामिका सुगुणकैर्विचार्यताम् ।

(सि. शिरो. गोल. दृक्कर्मवासना 16-17)

यैरुत्क्रमज्या विधिनैतदुक्तं सम्पृङ्गते गोलगतिं विदन्ति ।

(सि. शिरो. गणि. चन्द्रग्रहणा—23. Also see 38-39).

In the *Āryabhaṭīyam* (गोलपाद—45 Dr. Kern's edition) 'नतक्रमज्या' seems to be a corrected reading for 'नतोत्क्रमज्या' because Bhāskara gives only 'उत्क्रमज्या' in the *Mahābhāskariyam*, and Paramēśvara's commentary there also gives the same.

1: cf. i. आर्यभटीयम्-गोल—39-40, ii. महामास्करीयं V 1-7, iii. महामास्करीयं V 12-23.

as done by the *Romaka* and the *Saura* or using the true hypotenuse like the school of Āryabhaṭa would have made the parallax true, which is desirable. But in the case of the *nāḍis* of parallax in conjunction the constant 4 is justified, since the multiplication by the true motions to get the true parallaxes is cancelled out by the division by the true motions to get the *nāḍis*. The mean diameters of the sun and the moon according to this school are 32'·4 and 32'·0 and the shadow 82'·6. This school uses the correct formula for the two *valanas* (Chap. IV, 24-25).

Bhāskarācārya, following Brahma Gupta, gives the same formula for the parallax¹ as the *Sūrya Siddhānta*, except that he gets $\sin zdn$ and $\cos zdn$ direct. So the remarks about the *Sūrya Siddhānta* hold in his case also. The mean diameters according to him are 32'·5, 32'·0 and 80'·8. About the correct formulae for *valana* used by him, we have already written, as also about his criticism of Brahmagupta (and certain others) who have given wrong correction in parallax for the moon's latitude.

IV. The Mahāpātas and Yogas

The *mahāpātas* are *Vyatipāta* and *vaidhyti*, from which the 27 *yogas*, *Viṣkambha* etc. were later derived. *Vyatipāta* is the situation when the sun and the moon having different *ayanas* have the same north declination or south. *Vaidhyti* is the situation when they have the same *ayana* and equal and opposite declinations. The 27 *yogas* are computed like the *nakṣatra*, using the sum of the longitudes of the sun and the moon for the purpose. The seventeenth in this series called *Sārpa Mastaka* (and also *Vyatipāta*) together with the two *mahāpātas* are said to be extremely inauspicious for auspicious rites like marriage, but very efficacious for *dāna*, *homa*, *japa*, *tapas*

1. सिद्धान्तशिरोमणि. गणित. सूर्यग्रहणाधिकार. 3-14. This includes several alternative approximate methods.

and offering to the manes. The seventeenth *yoga Vyatipāta*, and the twenty - seventh also called *Vaidhṛti*, are included among the 96 *śrāddha days*.

Neither in the *Vedas*, nor in the *Vedāṅga Jyautiṣa*, are these mentioned.¹ The *Vyatipāta* (*Mahāpāta*) first occurs in the *Paitāmaha Siddhānta*, as seen from its condensation in the *P.S.* (Chap. XII). The rule is to multiply the days from the beginning of any five-year *yuga* by 12 and divide by 305 and the mid-*vyatipāta* falls when there is no remainder. This is equal to saying that when the sum of the longitudes of the sun and the moon reckoned from the Winter Solstice is equal to 12 *rāśis* it is *Vyatipāta*, which is the same as saying, if the sum is equal to 6 *rāśis* reckoned from the Spring Equinox, as defined in later times, it is *Vyatipāta*. It must be noted that this can give only the approximate time, the exact time being got in earlier times from observation, and in later times (beginning with the *Pauliṣa Siddhānta*) by computing the declinations (cf. *P.S.*, Chap. III. 22).²

The *Vyatipāta* and *Vaidhṛti* forming the seventeenth and twenty-seventh of the *yogas*, first appear in the *Pauliṣa* (*P.S.* III 20). The *Vyatipāta* here can be shown to be a relic of the one of the *Paitāmaha Siddhānta*. This *Siddhānta* placed the winter solstice at *Sraviṣṭhā*, from where longitudes were reckoned by it (as by the *V.J.*). The sum of the longitudes thus reckoned, when equal to 12 *rāśis*, would give the *Vyatipāta*, satisfying (though approximately) the condition of equal declination. But in course of time Winter Solstice would be occurring earlier and earlier than *Śraviṣṭhā*, so that the criterion of

1. In his edition of the *Vedāṅga Jyautiṣa*, Shyama Shastri, Curator, Mysore Government Oriental Manuscripts Library, fancies seeing the *Vyatipāta* in verse 19.

2. विपरीतायनयातो यदाऽर्ककाष्ठं शशी सविशेषः ।
भवति तदा व्यतिपातः दिनकृच्छशियोगचकार्थे ॥

equal declination would not be satisfied, and still people would be observing *Vyatipāta* at the time calculated from the sum of longitudes, much as we are now observing the *ayanas* at *Makara* beginning and *Karkaṭaka* beginning though these have precessed more than 20 degrees. At the period when the Spring Equinox was observed to be near *Aśvinī*, the astronomers shifted the first point from *Śraviṣṭhā* to *Aśvinī*, as done in the *Pauliṣa* by V. M. If longitudes are reckoned from *Aśvinī*, which is 5 *nakṣatra* segments in advance of *Śraviṣṭhā*, the traditional *vyatipāta* would happen when the sum is 12 *rāśis* minus 10 *nakṣatra* segments (i.e. 17 segments on the whole) and it is this that is given by (*P.S.* III, 20). Thus was formed this seventeenth *yoga*, called also *Sārpamastaka*. Its sanctity, guaranteed by tradition, would still be there. At least from the time when the new position of the W.S. could be observed, the *Mahāpāta Vyatipāta* also should have been computed and used as satisfying the condition *par excellence* of equal declination. But reckoning longitudes from the Spring Equinox, which is 3 *rāśis* forward from W.S., the sum should be 6 *rāśis* to satisfy the condition, as we have already said, instead of the 12 *rāśis* reckoned from W.S. Meanwhile the other *Mahāpāta (Vaidhyti)* arose, whose characteristic (already defined) would be approximately given by the sum reckoned from the Spring Equinox being equal to 12 *rāśis*. Viewed as formed by the sum of longitudes alone, without the reckoning from the Spring Equinox, this became the twenty-seventh *yoga vaidhyti*. It is noteworthy that V.M. does not give the *Mahāpāta, Vaidhyti*. He also does not mention any of the other 25 *yogas* (not even in his *Bṛhatsaṃhitā*), which means that the series of 27 *yogas* had not been formed in his time.

The *Saura* of the *P.S.* also does not give the series, from which we can surmise its absence from the

ārdharātrika system also. Its figuring in the *Khaṇḍakhādyaka* (which follows the *ārdharātrika*) is likely to be an interpolation, since the verse giving it is the same as in the *Brāhmasphuṭa Siddhānta* (*Spaṣṭa* 63), where it is considered by critics to be an interpolation, since it is not commented upon by Pṛthūdakasvāmi and does not figure in the question and answer (Chap. XIV-6, 31).¹ The *Āryabhaṭīyam* gives the two *Mahāpātas*, *Vyatipāta* and *Vaidhyti*, under the one name *Vyatipāta*.² Bhāskara I mentions these two separately, and also the seventeenth *yoga*, *Vyatipāta*, calling it *Sārpamastaka*.³ (It is these three that are classified sometimes under the one name *Vyatipāta* and considered very inauspicious etc.)⁴ Āryabhaṭa's direct pupil, Prabhākara (sixth century A.D.), has mentioned seven inauspicious *yogas* (six of which can be identified in the series of 27 *yogas*) as seen from a quotation by Śaṅkaranārāyaṇa in his commentary on the *Laghubhāskarīyam*, under II, 29. It can also be inferred from the quotation that he does not know the other *yogas*, and the practice of giving the *yogas* *Viṣkambha* etc. as an item of the *Pañcāṅga* had

1. रविचन्द्रयोगलिप्ताः खल्वसुभिर्माजिताः फलं योगाः ।

2. रविशशिनक्षत्रगणास्संमिश्राश्च व्यतीपाताः ॥ काल. ३ ।

Here 'रविशशिनक्षत्रगणाः' is interpreted to mean वैद्युति, 'संमिश्राश्च' to mean व्यतीपात. The *Vaṣeṣvara Siddhānta* *Madhyamādhikara* II 5, gives the same:

द्विगुणितपर्ययसंयुतिरुक्ता दिनकरचन्द्रमसोर्व्यतीपाताः ।

3. सूर्येन्दुयोगे चक्रार्धे व्यतीपातोऽथ वैद्युतः ।

चक्रे च, मैत्रपर्यन्ते विज्ञेयस्सार्धमस्तकः ॥

4. व्यतीपातत्रयं घोरं गण्डान्तत्रितयं तथा ।

एतद्भस्त्रितयं सर्वकर्मसु वर्जयेत् । (सू. सि. XI 22)

स्नानदानजपश्राद्धव्रतहोमादिकर्मभिः ।

प्राप्यते सुमहच्छ्रेयः तत्कालज्ञानतस्तथा । („ „ 18)

not yet come into vogue.¹ But in the *Siddhāntas* later than Brahmagupta (seventh century), like those of the *Sūrya Siddhānta* school, *Vaṭeśvara Siddhānta* etc., the series of 27 *yogas* is seen to be systematically computed.

As for the two *Mahāpātas*, the early works even up to the time of Bhāskara I give only the rules for their approximate time of occurrence as mentioned already, leaving the computation of the exact time and duration to astronomers, from the criterion of equality of declinations.² The *Brāhmasphuṭa Siddhānta* is the first, as it itself claims,³ to deal with them in detail (Chap. XIV, 33-34). First comes the test for possibility⁴ and

1. सूर्येन्दुयोगनक्षत्रे रूप (? मनु = 14) वस्वलर्क 12 पञ्चभिः ।
अत्यष्ट्या 17 चैव धृत्या 18 च पङ्क्त्या 10 युक्त्वा पृथक् पृथक् ।
निरोधं परिधं वज्रं दण्डं खड्गं (गण्डं) च चू (शू) लक्षम् ।
व्यतीपातं च सप्तैतान् महादोषान्प्रचक्षते ।

The instruction to get these individually (पृथक् पृथक्) shows that the series of 27 *Yogas* was not computed in Prabhākara's time.

2. आर्यमटीयभाष्यं कालक्रिया. 3.

“एवं स्थूलव्यतीपातभुक्तिः यथा सूक्ष्मो भवति

तथा प्रदर्श्यते । उक्तम् —

नानाऽयने व्यतीपातः तुल्यापक्रमयोस्तयोः ।

उद्देशस्तस्य चकार्यं विक्षेपस्त्वधिकोनकम् । इति ।

सूर्याचन्द्रमसौ नानायेने तुल्यापक्रमौ यदा भवतः तदा व्यतीपातः ।

चन्द्रस्य, विक्षेपसहितो रहितो वा अपक्रमः । ”

3. व्यतीपातवैधृत्यानयनन्यतन्त्रेषु न ब्राह्मात् । (XIV. 143)

4. This is faulty in certain extreme cases as pointed out by Bhāskarācārya in his *Vāsanā Bhāṣya*, under (सि. शि. गणित पाताधि. 11-12). Here, the *śiṣya* of Lalla, who prides himself on the excellence of his treatment of the *mahāpātas*, as also सिद्धान्तशेखर, and another work सिद्धान्तचूडामणि by one माधव, are also shown to be faulty.

next is found out whether the mid-event has gone or is to come. Then taking a point of time arbitrarily before or after, the increase or decrease in the difference between the declinations during the interval is found. Using this, the time when the difference is zero, and the duration for a change of difference equal to the sum of the angular semidiameters, are found. The time of zero difference gives the mid-event, which minus or plus the half duration gives the beginning and end. The (later) *Sūrya Siddhānta* devotes chap. XI to this topic. This school finds the zero - difference - point by successive approximation, using the longitudes corresponding to the declinations. It gives the half duration in *nāḍikas* by multiplying the sum of the semidiameters into 60 and dividing by the difference of the daily motions, which is very rough indeed. Lalla, who immediately followed Brahmagupta, attacked the problem with zest in his *Śiṣyadhivṛddhida*, as also Śrīpati and others, with more or less correctness (cf. note 4. P. 73). Bhāskarācārya in a special section devoted to this subject (*Sid - Śiro - Gaṇita*, *Pātādhikāra*) gives the most systematic and correct treatment. Using the obliquities of the ecliptic and the moon's orbit, and the *sāyana* longitude of *Rāhu*, the obliquity of the moon's orbit to the celestial equator, with the distance of *Rāhu* from the point of intersection, is got. This is fairly fixed, since *Rāhu* moves very slowly (about 3' per day). The correct declination of the moon and its variation is now as easy to compute as those of the sun. Thus the times of zero difference and difference equal to the sum of the semidiameters are got easily. The possibility or impossibility can be visualized clearly and correctly when the matter is thus simplified. It must be said in conclusion that the *Mahāpātas*, made so much of by the astronomers, are not even heard of nowadays in connection with religious rites or fixing auspicious moments.

This article is intended to help historians of Hindu astronomy, but is not exhaustive. Fear of making it too long has prevented considering the development of other concepts like the heliacal setting and rising of planets, etc. The nonavailability of most Hindu astronomical works in the libraries in Madras has limited the matter of even the topics chosen. I expect others to follow up and supply the need.

THE SYSTEM OF THE VAṬEŚVARA SIDDHĀNTA

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The *Vaṭeśvara Siddhānta* (A.D. 904) is one of the most famous of Hindu Astronomical works, and cited frequently by writers on *Dharmaśāstra*. Albiruni mentions the author with another work of his. But later on the study of the work became so rare that only recently it appeared in print from an only manuscript in the Lahore University Library. The new commentary with which it is printed has masked its peculiarities as a work belonging to the school of Āryabhaṭa, as also many of the fundamental constants like the different numbers of cycles, etc., and given wrong ideas and numbers instead. This has led to further mistakes in interpretation. Hence, a good deal of research has to be done to salvage these and make them available to scholars for further study.

The findings are as follows: The *yuga* (i.e. *mahāyuga*) is divided into four *equal* quarters. 72 *yugas* make a *manvantara*; fourteen *manvantaras*, without any *sandhi* (1008 *yugas*), form the *kalpa* or half-day of Brahma; 720 *kalpas* form his year, and his life-span is 100 such years. At present, $8\frac{1}{4}$ years and 15 days of Brahma's lifetime has gone. On the next day, 6 *manvantaras* and $27\frac{1}{2}$ *yugas* have gone, up to the beginning of this *Kali*. The constants denoting the number of civil days, revolutions of the Sun, Moon, Jupiter, Saturn, Mercury, Venus as well as of Moon's apogee, nodes, etc., are presented according to the *Vaṭeśvara Siddhānta* and discussed in relation to wrong numbers given by the commentary. Several other wrong interpretations presented by the commentary are also dealt with and the correct interpretations given. It is pointed out that in the *Spaṣṭādhikāra* of the commentary also there are a number of mistakes to be rectified.

The *Vaṭeśvara Siddhānta* is one of the most famous of Hindu Astronomical works, and cited by writers on *Dharmaśāstra* like Kamalākara Bhaṭṭa. The author is Vaṭeśvara, son of Mahadatta, of Anandapura, and the work was composed in Śaka 826 (A.D. 904), when the author was 24 years old (*Madhyamādhikāra* I, 21). With his other work *Karaṇa Sāra*, he is mentioned by Albiruni. But now manuscripts of the work have become rare, so much so that the late M. M. Sudhakara Dvivedi regrets,

in his history of Hindu Astronomy, that he is not able to procure a transcript. The Indian Institute of Astronomical and Sanskrit Research, Delhi, got the work printed and published for the first time in 1962, from a manuscript in the Lahore University Library, adding a new commentary in Sanskrit with a Hindi translation of the same. Now, this commentary has masked the system of this *siddhānta* by superimposing on it its own ideas and constants, unwarranted by the text, so as not to be available to readers without doing research. This has led to further mistakes in interpretation. Hence, this attempt of mine here. For the present I shall confine myself to the *Madhyamādhikāra* mainly.

I give the system first, for the sake of scholars who may not be interested in the research. In the *Mahāyuga* (usually called *yuga*), there are 43,20,000 solar years. The *yuga* is divided into four *equal quarters*, *Kṛta*, *Treta*, *Dvāpara* and *Kali*, of 10,80,000 years each, a peculiarity which it shares with the *Āryabhaṭṭīyam*. *Seventy-two yugas* make a *manvantara*, and 14 *manvantaras* or 1,008 *yugas* make a *kalpa*, which ideas also are peculiar to this system and shared by the *Āryabhaṭṭīyam*.¹

1. (i) *Vaṣeṣvara Siddhānta Madhyamādhikāra* (Chap. I) :

दन्ताऽब्धयो (482) ऽयुतहता युगमर्कमाना-

दस्त्राऽब्धयो (72) युगगुणा मनुरेक उक्तः ।

कल्पश्चतुर्विंशतिमनुयुजिंशं चोद्दिष्टः

कस्य त्ववर्षशतमत्र सदाऽऽयुर्वक्तम् । 9 ।

...सदृशं ह्ययस्त्रयः... । 10 ।

(ii) *Āryabhaṭṭīya Gītikāpāda* :

काऽहोमनवो ढ (14) मनुयुगरख (72)

गतास्ते च (6) मनुयुगच्छा (47) च ।

कल्पादेर्युगादा ग (3) च

गुरुदिवसाच्च भारतात्पूर्वम् । 8 ।

(iii) *Āryabhaṭṭīya Kālakriyāpāda* :

अष्टोत्तरं सहस्रं ब्राह्मो दिवसो ग्रहयुगानाम् । 8 ।

(Other *siddhāntas* assert that the duration of *Kṛta*, *Treta*, *Dvāpara* and *Kali* are in the ratio, 4 : 3 : 2 : 1, that 71 *yugas* make a *manvantara* and that 14 *manvantaras* with 15 *sandhis* make up the *kalpa* consisting of 1,000 *yugas*.)

To continue, two *kalpas* make a day of Brahma, 360 such days form his year, and his life - span is 100 such years. Now, the commentary has missed every one of the peculiarities of the system mentioned above, though, strangely enough, the introduction seems to accept some of them, at the same time stating that there are 10,000 *yugas* in a *kalpa*.

As for the constants, according to this *siddhānta* there are 157,79,17,560 civil days in the *yuga* as derived from the statement that there are 158,22,37,560 sidereal days in it. The revolutions of the sun are 43,20,000 and of the moon 5,77,53,336. The revolutions (mean) of Mars are 22,96,828, of Jupiter 3,64,220, of Saturn 1,46,568, and those (*Śighra*) of Mercury are 1,79,37,056 and of Venus 70,22,376, and those of the moon's apogee (*ucca*), 4,88,203, and of its nodes (*pāta*) 2,32,238. *The revolutions of the Seven Sages are 1,692 in the yuga.*

In the life - span of Brahma the revolutions of the apogee of the sun are 1,65,801, of Mars 81,165, of Mercury 4,77,291, of Jupiter 13,948, of Venus 1,52,842 and of Saturn 72,974, and those of the nodes of Mars 20,684, of Mercury 988271456418719, of Jupiter 39,202, of Venus 19,61,27,48,06,36,835 and of Saturn 1,542. The great circle on the celestial sphere is in *yojanas*, 12474720576000 (*Madh.* VII, 3). Of these constants, the commentary gives wrong numbers for the italicized digits, and obliterates the revolution of the *Sages*.

This *siddhānta* says that time with its indicators, the sun, moon and star-planets were created together with Brahma and coexistent with him, the moment of their

creation being mean sunrise at Lankā, and the day Saturday.⁹ (The other *siddhāntas* say the sun, moon, etc., are created anew in every day of Brahma, getting into dissolution at night - fall; while the *Āryabhaṭīyam* is silent on this point.) At the beginning of the present *kalpa* eight and a half years *together with fifteen days* have elapsed since Brahma's creation. The period of 15 days has to be established by reasoning, since the words giving it in I. 10, have been spoiled by the scribes. (*Āryabhaṭa* leaves out the 15 days, which may be a variation or an approximate statement. According to the other *siddhāntas* 50 years of Brahma's life have passed). Six *manvantaras* and 27 *yugas* in the seventh have gone in the current *kalpa*, and three quarters (not nine - tenths) of the twenty - eighth *yuga*, up to the beginning of the present *Kaliyuga*, which falls on a Friday according to all. To link this with the present time it may be stated in passing, that this, as also all other *siddhāntas* (except the *Surya Siddhānta* school, not mentioning any era), say that the *Śaka* era began 3,179 years after *Kali* set in. (The *Āryabhaṭīyam* gives the *Kali* year direct).

Now for the *kṣepas* for the beginning of *Kali*. All but two of the *yuga* revolutions being divisible by four, the *kṣepa* is naturally zero for them. Of the two exceptions, *for the moon's node it is six rāśis and for its*

2. *Vaṭeśvara Siddhānta Madhyamādhikāra*:

- (i) त्र्युद्धादिपद्मोद्भवजीवितान्तः
क.लम्समं तेन क्षणान्तमन्धी ।
लङ्काकुजस्थद्युचरैः प्रवृत्तः
शनेर्दिने चैत्रसिताऽऽदितेऽयम् । 11-9 ॥
- (ii) मन्दतुल्लभगणोऽब्जजीविते...॥ I 16 ॥
- (iii) जगदुत्पत्तिप्रलयौ
कमलजानत उवाच यत्तदसत् ।
वेदानां नित्यत्वात्
श्रुतिवाक्यानां गतिर्भवति । X. 8 ।

apogee it is three rāsis, as mentioned in IV, 55. But the commentary here makes the absurd statement that these *kṣepas* are for the beginning of the *kalpa*. The *kṣepas* of the longer period revolutions can be found by multiplying the number of revolutions by 24798639, viz. the quarter *yugas* since creation, and dividing the product by the quarter *yugas* in Brahma's life-span, viz. $100 \times 360 \times 2 \times 1008 \times 4$. They are :

Sun	apogee	2r	18°	51'	37''
Mars	..	4	8°	50'	50''
Mercury	..	7	16°	42'	54''
Jupiter	..	5	22°	48'	31''
Venus	..	2	20°	3'	26''
Saturn	..	7	26°	55'	4''
Mars	node	10	20°	10'	12''
Mercury	..	11	10°	19'	54''
Jupiter	..	9	0°	54'	2*''
Venus	..	5	24°	1'	56*''
Saturn	..	8	20°	1'	0*''

The number of civil days from creation to the beginning of *Kali* is 9782551985550210. (Here too the commentary gives wrong numbers for the italicized digits, or even omits or adds digits. The verse giving the *kṣepas* of the nodes of Jupiter and Venus is missing in the text.)

Now, proceeding from *what is undisputed*, I shall build up the whole structure given by me as the correct one. Let me begin with showing that the number of civil days in the *yuga* is 1577917560, and not what it will be according to the commentary, four days more. Using III 26, the civil days in a *yuga* = $4320000 \times 13149313 \div 36000 = 1577917560$, as I have given above. Again, using III 23-25, the solar months in the *yuga* = 4320000×12 . The *adhimāsas* = $66389 \times 4320000 \times 12 \div 2160000 = 1593336$. The synodic months = solar months + *adhimāsas*

= 53433336. The *tithis* = synodic months $\times 30$ = 1603000080. The *avamās* = *tithis* $\times 209021 \div$ (total *tithis* in the *yuga* $\div 120$) = 25082520. Therefore the civil days = *tithis* — *avamās* = 1577917560. Thus we see it is not 1577917564, as given by the commentary. III 12, 13, 14, 15, each one can also *independently* give the same number of civil days. But everywhere the commentary assumes a year of days 365-15-31-15, (this is Āryabhaṭa's), and proceeds with the proof, unaware that this will make the *yuga* days equal to Āryabhaṭa's 1577917500, 64 days short of what the number will be even according to it. Adding the sun's revolutions in the *yuga*, the sidereal days = 1582237560, and not 1582237564 as given by the commentary under II, 1. The mistake lies in its interpreting *jaladhara* as 4, while it actually means zero (*Jaladhara* = *abhram* = 0).

That *Kṛta*, etc., are equal quarter *yugas* is got by the statement *Sadṛśāṅghrayastyaḥ* in I, 10. Therefore they are not in the usually given proportion, 4:3:2:1, as mistaken by the commentary. This is also confirmed by X, 7, where the author defends Āryabhaṭa against Brahmagupta, on this point.³ Therefore, the civil days in each quarter *yuga* are a fourth of the total, i.e. 394479390.

I shall now show that according to this *siddhānta* 8½ years plus 15 days, etc., have gone since creation, and there are 72 *yugas* in a *manvantara*, etc. III, 18 states that the number of days since creation up to *Kali* is the product of 24798639 and days in a quarter *yuga* (found above to be 394479390) and has given it, viz., 978255198 5550210. (It must be noted that the commentary omits the two digits 5, 0, and thus makes the number a hund-

3. *Vaṣeṣvara Siddhānta Madhyamādhikāra* :

चरणरचतुरंशकस्मृतो

वतलोके न दशंशकः क्वचित् । X-7 ।

redth of the actual. Strangely enough, it asks us to use for getting the product, the days in a quarter *kalpa* instead of *yuga*, which will make the number more than one thousand times the actual! The disputed 15 days or the extra duration of the *kalpa* and *manvantara* in this system are too small to help removing the discrepancy; they will only mar the complete agreement in the digits.) Therefore, it is obvious that 24798639 given is the number of quarter *yugas* since creation. We have it therefore that the *yugas* gone must be $24798639 \div 4 = 6199659\frac{3}{4}$. I shall show this can be so only with the 15 days more. In $8\frac{1}{2}$ years there are $8\frac{1}{2} \times 360 \times 2$ *kalpas*. In the next 15 days there are 30 *kalpas*. Thus there are 6150 in all, in which there are $6150 \times 1008 = 6199200$ *yugas*. The *yugas* gone in the present *kalpa* = $6 \times 72 + 27 + \frac{3}{4} = 459\frac{3}{4}$. Therefore, the total *yugas* gone = $6199200 + 459\frac{3}{4} = 6199659\frac{3}{4}$, as required. Omitting the 15 days or making the *kalpa* equal to 1000 *yugas*, etc., as stated by the commentary, will give $8\frac{1}{2} \times 360 \times 2 \times 1000 + 6 \times 71 + 7 \times 4 + 27 \cdot 9 = 6120456 \cdot 7$ *yugas* only. Note the difference. Our conclusion is also reinforced by the agreement in the *kṣepas* of the apogees and nodes, as we shall see later. It may be mentioned in passing that the relevant portions of III, 18, and I, 9-10, must have been wrongly read by the commentary or tampered with by somebody; as for instance, *ccandrādrayo* in I, 9, must be *ddusrādrayo*; *tathā* in I, 10 must be *tithi* (= 15), *bhujābhra* in II, 7 (peculiarly interpreted by the commentary as zero zero), must be *gajābhra* and *śara śara* in III, 18 must be *khaśaraśara śara*. (This last one will also make the metre correct.)

We shall now show that the first day of creation according to this *siddhānta* (as also of every *kalpa*) is Saturday. All *siddhāntas* agree that the first day of *Kali* is Friday. Dividing out the days from creation given above by 7, the remainder is 6, which means that the week - day of creation is 6 days previous to Friday, i.e.

the day immediately following Friday, viz., Saturday. This is also mentioned in III, 19, by *mandasitādyo vyastagaṇanayā*. Also logical reasoning, on the pattern of the position taken by the author in X, 9-11, points to Saturday as the first day. From this it follows that the first day of every *kalpa* is also a Saturday, because the days in the *kalpa*, since they have a factor, 14, must be divisible by seven. In the light of our finding, *vyomacarādhika* in III, 1, should be taken to mean *grahanāyaka* or Saturn,⁴ or it must be read as *vyomacarādhika* and interpreted as the planet having the longest period or the greatest orbit, i.e. Saturn. In II, 9, *raverdine* must be a misreading for *śanerline*. Let it not be thought that the acceptance of the 71 year *manvantara*, etc. attributed to the system by the commentary, will give Sunday either for the *kalpa* or creation. It will not, as can be tested easily. (Under X, 11, by the way, in trying to show that the first day of the current *kalpa* is Thursday according to Āryabhaṭa, the commentary makes several mistakes, a patent one being the omission from reckoning of the 27 *yugas* in the current *manvantara*.)

We shall now pass on to the revolution constants. We have stated that at the commencement of *Kali* the *kṣepa* of the moon's node is six *rāśis* and of its apogee, three *rāśis*. The revolutions of these being whole numbers and given for a *yuga*, and three - fourths of the *yuga* having gone at the beginning of *Kali*, three - fourths of the number of the respective revolutions clearly give half and a fourth of a revolution, i.e. six and three *rāśis*. At the commencement of the *yugas* the *kṣepas* must be zero, and therefore at the commencement of the *kalpas* too. But the commentary makes them six *rāśis* and three *rāśis* at the beginning of the *kalpa*, stating *kalpādaḥ* =

4. Cf. *Śanaṭscara Dvādaśanāma Stotra* :

कोणशशैश्चरो मन्दः छायाहृदयनन्दनः ।

मार्ताण्डजस्तथा सौरिः पातकी ग्रहनायकः ।

srṣṭyādaḥ, exhibiting also ignorance of the fact that according to this *siddhānta* there is no creation of the planets at the beginning of every day *kalpa*. Evidently *kalyādaḥ* has been misread as *kalpādaḥ*.

I have given the *yuga* revolutions of Mercury as 17937056, while the commentary gives 80 for 56. Let us verify : by V, 16, in a *yuga*, i.e. for 43,20,000 solar years gone, the cycles of Mercury = $4 \times 4320000 + 4320000 \times 20533 \div 135000 = 17280000 + 657056 = 17937056$, agreeing with what I have given. We can reinforce this confirmation by VII, 9, giving Mercury's orbit as 695472 + 11424,560533 *yojanas*. For, using the correct number of cycles got above in VII, 5, we get Mercury's orbit = $4320000 \times 57753336 \div (20 \times 17937056) = 695472 + 11424 / 560533$, the same as given by VII, 9. This will also show that the correct reading of VII, 9 is, 'नेत्राऽगवेदसायक न तु भिन्निमदुश्च शशि चन्द्रैः। सुरशस्त्राऽङ्गक्षलत्रैः.....' Therefore, in I, 13 giving Mercury's cycles *kharasairhi* should be read as *rasāgni*. This will also correct the error in prosody.

The cycles of the Seven Sages are given in I, 15, as 1692 (*bhujagoṣṭayaḥ*) per *yuga*. But the commentary misses it, interpreting the verse with the next one, and making the remark, the number 1692 seems to be useless: 'भुजगोऽस्तयः इति निरर्थकं प्रतिभाति' (There is another comment to match, under II, 5, 'अर्थनिपातः, अर्थनिपातसंज्ञकाः...अथात रविचन्द्रयोः द्विगुणतमगणयोगस्य नाम अर्थनिपात इति ।' Apart from the unheard of nature of the name, the commentary is unaware that the *sandhi* in the text cannot give the word. The word is 'व्यतिपाताः' and the 'r' is due to *sandhi*. *Vyatiṣṭā* here means the two types, *vyatiṣṭā* and *vaidhṛti*, hence *dviguṇita*, vide the same instruction given by the *Āryabhaṭīyam*, *Kāla* 3.)⁶

5. *Aryabhaṭīya Kalakriyāpāda* :

रविशशिनक्षत्रगणा-

स्संमिश्राश्च व्यतीपाताः । 3 ।

We now pass on to the revolutions given for Brahma's life-span and the *kṣepas* relating to them. Let us start with the cycles of the apogee of Mercury, 477291, and of the nodes of Mars, 20684, about which there is no dispute. The *kṣepas* for *Kali* can be found conveniently in two steps, first up to the beginning of the current *kalpa*, and from then up to the beginning of the current *Kali*. (Those who do not want to use this easy method may multiply the cycles by the *yugas* gone and divide by 72000×1008 the *yugas* in Brahma's life-span to get the required *kṣepas*.) To get the first part, the fraction to be used for multiplication is the *kalpas* gone since creation, by the total *kalpas* in Brahma's life-span, i.e. $(8\frac{1}{2} \times 360 \times 2 + 30) \div (100 \times 360 \times 2) = 6150 \div 72000 = 41 \div 480$. If the number of cycles be *c* and the result given in *rāśis*, this first part of the *kṣepa* = $12 \times c \times 41 \div 480 = 41c \div 40 = c + c/40$. (This is given by the first half of IV, 53, *khaka*=40, being misread as *svakha*, and *grhādayo* as *grahādayo* and the meaning given as, 'वशू-य मन्तलब्धयुत-भगणाः' with the comment, 'अत्र स्व खदतलब्धयुतभगणाः इत्ययुक्तं प्रतिभाति !!')

The multiplier to get the second part is the *yugas* gone in the present *kalpa* amounting to $459\frac{3}{4}$ as already found, and the divisor is the same as before. If this is given in minutes of arc, again *c* being the cycles, the second part = $c \times 12 \times 30 \times 60 \times 459\frac{3}{4} \div (72000 \times 1008) = c \times 613 \div 4480$. (The fraction $613 \div 4480$, or its equivalent, must have been given by the last word of the first half, and the second half of IV, 53, spoiled beyond recognition.) Now, using the two fractions, the *kṣepa* of the apogee of Mercury is the *rāśis*, etc., 7-8-15-0+0-8-27-54=7-16-42-54, exactly as given by the text in IV, 57. Applying to the nodes of Mars, the *kṣepa* is, 9-3-0-0+1-17-10-12=10-20-10-12, again exactly as in IV, 59. (Incidentally, this confirms again that up to this *kalpa* $8\frac{1}{2}$ years and 15 days have gone, and in this *kalpa* six man-

vāntaras of 72 *yugas* each have gone, and 27½ *yugas* in the seventh *manvantara*, up to *Kali*. Otherwise we cannot get the agreement in the numbers.)

The above procedure gives us the means of verifying the cycle numbers on the one hand, and the *kṣepas* on the other, when in doubt about one of them. We shall use it. The revolutions of the apogee of Mars are 81165. From this the *kṣepa* is $10-3-45-0+6-5-5-50=4-8-50-50$. The commentary gives 5 instead of 8 here, interpreting *dhiyah* (wrong'y read *dhayah*) as 5. We see it is 8 according to *Vaṭeśvara*. This is confirmed by *spaṣṭa* IV, 11, giving the synodic period of Venus as 584 (पयोजि धी पवनाः, where धी must be taken as 8, but the commentary even here taking it as 5). Therefore *dhīkṛtāṅka dahanendavo*, giving Jupiter's apogee-cycles must be 13948, not 13945, as given by the commentary. Further, only the former will give the *kṣepa*, 5-22-48-31, and not the latter, confirming our conclusion. (The latter will lessen the *kṣepa* by more than 3 *rāśis*.) In the case of Mercury's nodes, to agree with the *kṣepa*, 11-10-19-54, *śara* (in I, 18a) must be read as *rasa*, of course *dhī* as also *matī* (meaning the same), each being interpreted as 8; and we get 988271456418719. The number made out by the commentary 955271455418719, cannot give the *kṣepa*. In the case of Venus's nodes in I, 18b, 3 for *śikhi* (meaning fire), is most probably intended (though it may also mean arrow or 5), for in I, 14, it is used for 3, and no author will use one word for two different numbers, which will be confusing (though the word may have two meanings). *dhī* is of course 8. We cannot verify the cycle number here because the verse giving the *kṣepa* is missing, together with that of Jupiter's nodes.

From Venus's number of cycles of apogee, 152842, the *kṣepa* must be 2-20-3-26, and not the commentary's 2-20-13-2; and therefore the reading should be 'यमली नखास्त्रयो रसयमाः' From Saturn's number of cycles of

apogee, 72974, its *kṣepa* must be 7-26-55-4, and the reading should be corrected as 'सुनयोऽङ्गदशोऽक्षरा वेदाः'. Using Saturn's cycles of nodes 1542, the *kṣepa* got is 8-20-1-0, and the reading is to be corrected as 'अष्टौ नखारव भूःखे'. From the *kṣepa* for the sun's apogee, the number of cycles should be 165801, and not 16511 as given by the commentary. The latter does not yield the given *kṣepa* but 5-0-54-13. Therefore in I, 16, 'पंकज' is a misreading for 'खं गज'.

The first six significant figures of the node cycles of Mercury and Venus can also be verified by using I, 20: The *śighra* cycles of Mercury and Venus, for Brahma's lifetime, calculated from their *yuga* values, are respectively 988271746560000 and 196127640576000, and we find that their node cycles given by us agree to six significant figures. The difference in the other places is of course expected and necessary.

The *yojana* measure of the great circle on the celestial sphere, given in VII, 3, can be verified by using VII, 5, and therefrom it is $4320000 \times 57753336 \div 20 = 12474720576000$ as given by me, there being no doubt about the cycles used and the method.⁶ Therefore, the commentary's 1222514920576000 making the number about a hundred times what it is, is wrong. So we see that in VII, 3, *graha* should be interpreted as 7, and not 9, as the commentary does (in II, 7, the commentary itself interprets *graha* as 7), and that 'भूतस्व' (=2251) is a misreading for 'भूम-कृत' (=47). The commentary's reading also in against prosody and uses 'स्व' for 2, not generally used.

6. See also, (i) *Aryabhaṭīya Gītikāpāda* :

शशिराशयष्टु (12) चक्रं

तैऽशकलायोजनानि य (30) व (60) अ 10 गुणाः । 4 ।

(ii) *Mahābhāskariyam* :

हन्दोर्गुणाः खखविषदसद्वन्द (216000) निघनाः

व्योम्नो भवेयुरिहवृत्तसमानसंख्याः । VII-20 ।

There are scores of other places also where the commentary gives wrong constants, as for example in V, 1. These are fundamental, and are used in a number of other places to derive and give other multipliers, divisors and *kṣepas* for shortening the work. It is funny to see how the commentary misinterprets many of them and adduces proofs, of course wrong, for them too! I have mainly taken the *Madhymādhikāra* for study here. In the *Spaṣṭādhikāra* too there is a lot to be corrected.

THE SCHOOL OF ĀRYABHAṬA AND THE PECULIARITIES THEREOF

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It is possible to classify early Hindu astronomical works into specific schools, on the strength of certain peculiarities of each. One such school is that of Āryabhaṭa, revealed in his *Āryabhaṭīyam*. Prabbākara, Bhāskara I and the Kerala astronomers, Govindasvāmin, Haridatta, etc... belong to this school, as also Vāṭṣeṣvara in the north.

Āryabhaṭa and Vāṭṣeṣvara give peculiar lengths for the different eons. According to them the *yuga*, though consisting of the usual 4,320,000 years, is divided into four equal quarters, *kṛta*, etc., instead of in the ratio 4 : 3 : 2 : 1. Seventy-two *yugas* make a *manvantara* and 14 *manvantaras* or 1,008 *yugas* constitute the *kalpa*, with no *manvantara-sandhi*. Time and its indicators—the sun, the moon and the star-planets—were created together with Brahma and considered to last as long as Brahma lasts, instead of being dissolved and recreated in each *kalpa* forming the day-time of Brahma. The moment of this creation was mean sunrise at Lanka and the day, Saturday. At the beginning of the current *kalpa*, eight and a half years and 15 days of Brahma's life had elapsed. At the beginning of the present *kali yuga*, six *manvantaras* and $27\frac{1}{2}$ *yugas* had gone in the *kalpa*. Vāṭṣeṣvara gives the revolutions of the apses and nodes (excepting those of the moon) as so many cycles in the lifetime of Brahma, and therefore all this is not of mere academic interest.

We also find variations from other schools in the number of cycles of the mean or *śighra* motions, as also in the degrees of epicycles. An important peculiarity is the use of the true hypotenuse in computing the equation of the centre (condemned by Bhāskarācārya II). Another peculiarity is dispensing with the first operation (i.e. the application of half-equation of conjunction to the mean) in the case of Mercury and Venus.

It is possible to classify early Hindu astronomers and astronomical works into specific schools on the strength of certain peculiarities of each. One such school is that of Āryabhaṭa, as revealed in his *Āryabhaṭīyam* (A.D. 499). Āryabhaṭa has written another work known as his *Ārdharātri* system which has been

adopted by Brahmagupta in his *Khaṇḍakhādyaka*¹ (A.D. 665), which he claims to give the same result as the *Ārdharātri*, whose constants and other peculiarities are given by Bhāskara I (c. A.D. 628) in his *Karmanibandha* (better known as *Mahābhāskarīyam*), chap. VII,² and which, the followers of Āryabhaṭa think, has been intended only to be examined and refuted, as Govindasvāmin claims in his *Bhāskarīya Bhāṣya* under VII, 35. This *Ārdharātri* system is representative of another school to which belong the *Saura Siddhānta* condensed by Varāhamihira (V.M.) in his *Pañca Siddhāntikā* (P.S., c. A.D. 505) and the *Paulīśa* often quoted by Bhaṭṭotpala in his commentary on the *Bṛhatsaṃhitā*. The school of Āryabhaṭa is represented by his direct pupils like Prabhākara, Bhāskara I who has written the *Āryabhaṭīya Bhāṣya* and whose *Karmanibandha* is claimed to be a *vārtika* of the *Āryabhaṭīyam*, Govindasvāmin its commentator³ Haridatta, the author of the *Grahacāranibandhana*, and a long line of illustrious Kerala astronomers, as also the famous Vateśvara of Ānandapura. Besides the peculiarities we are going to talk about, the fact that these astronomers belong to the west coast from Valabhī in the north to the Cape in the south, and that they were conscious of belonging to this school, as gathered from statements made in their works,³ point to the existence of the school.

1. Cf. *Khaṇḍakhādyakam* :

वक्ष्यामि खण्डखाद्यकमाचार्यभिमततुल्यफलम् ॥ (I.1)

2. Cf. *Mahābhāskarīyam* :

निबन्धः कर्मणां प्रोक्तो योऽसावौदयिको विधिः ।

अर्धरे त्रे त्वयं सर्वः यो विशेषस्त बध्यते (VII.21)

3. (a) *Vide*, e.g. Bhāskara I's statements in the *Mahābhāskarīyam* :

(i) चिरं च जीव्यासुरपेत कल्मषाः

भट्टस्य शिष्या.....जितरागशत्रवः । (I.3)

(ii) अद्भुतमन्यैरिदमाश्मकीयैः--- (I.21)

.....contd.

One important peculiarity of the school is that its system of eons differs from that of other schools and the Purāṇas, as can be gathered from the *Āryabhaṭīyam* and the commentary thereon, and from the *Siddhānta* of Vateśvara, by *guṇopasaṃhāranyāya*. According to this, in the *mahāyuga* (also called the *yuga*), there are 4,320,000 years; but this is divided into four *equal quarters* (*kṛta*, *tretā*, *dvāpara* and *kali*) of 1,080,000 years each, as seen from *Vaṭ. Sid.*, I, 10: 'four equal quarters', *Āryabhaṭīya Bhāṣya* under *gītikā* 3, 'for us the four quarters of the *yuga* are equal', and as inferred from *Vaṭ. Sid.*, *madhya*: X, 7, and *gītikā* 3.¹ Seventy-two *yugas* make a *manvantara*, and 14 *manvantaras* make a *kalpa* (half-day) of

(iii) व्यावर्णयन्ति गणका भटशास्त्रचित्ताः (I 40)

(iv) अध्वानं गणितविदो भटस्य शिष्याः ॥ (II.5)

(b) And vide his *Āryabhaṭīya Bhāṣyam*:

अस्माकं पुनर्युगपादास्सर्वे एव तुल्याः (*gītikā* 3)

(c) Not only does Vateśvara declare that his source is the same as that of the *Āryabhaṭīyam*, but he also salutes the same deities as Ārpabhaṭa does and professes to follow his work and school: cf. *Vaṭ. Sid.* *Siddhānta*:

(i) ब्रह्माऽवनीन्दुबुधशुक्रदिवाकराऽर-

जीवार्कसूनुभगुरुन् पितरौ च नत्वा ।

ब्राह्मं यदक्षगणितं महदत्तसूनु-

र्वक्ष्येऽखिलं स्फुटमतीव वडेस्वरोऽहम् । (*madh.*, I.1)

(ii) कालक्रिया गणितगोलमहाऽऽगमार्थ-

ज्ञानप्रपञ्चविमलीकृतचारुधीभिः ।

दिव्यैः प्रदर्शितमिदं मुनिभिर्यदज्ञाः

कुर्मो वयं तदवलोक्य युगस्स तेषाम् । (*madh.*, I.2)

The defence of Āryabhaṭa which he puts up at the end of the first section also points to this.

1 (i) कल्पादेर्युगपादा ग (3) च गुरुदिवसाच्च...(*gītikā* 3)

(ii) युगत्रिवृन्दं सटशङ्खयस्त्रयः (*Vat. Sid.*, *madh.*, I.10)

(iii) चरणश्चतुरंशकस्मृतोऽबतलोक न दशंशकः ववच्चित्

(*Vaṭ. Sid.*, *madh.*, X.7)

Brahma, consisting of 1,008 *yugas*. These are not only mentioned in various places, but also can be inferred from various *kṣepas* involving these periods.¹ (On the other hand, it is well known that other schools, and the Purāṇas, say that the duration of *kyta*, etc., are in the ratio 4 : 3 : 2 : 1, that 71 *yugas* make a *manvantara*, and that 14 such *manvantaras*, plus 15 *sandhis*, each equal to $\frac{2}{3}$ *yuga*, make up the *kalpa* of 1,000 *yugas*.)

To continue, two *kalpas* make a day of Brahma, 360 such days form his year, and his life span is 100 such years, as commonly held. But this school asserts that time with its indicators—the sun, the moon and the other heavenly bodies—was created with Brahma (the moment of their creation being mean sunrise at Ujjain) and co-existent with him, unlike the statement of others that these bodies are created afresh in every day-time *kalpa* and get into dissolution during the night-time. At the beginning of the present *kalpa* $8\frac{1}{2}$ years with 15 days have elapsed since Brahma's creation according to Vaṇeśvara. (According to others 50 years have just passed and we are in the next day.) Āryabhata is silent on this point, since he has no use for it as he does not give the slow-moving cycles of apogees and nodes. But Bhāskarācārya II, in his *Siddhānta Śiromaṇi* (*Gaṇita : madh. : kālamāna* : 26), gives it as the opinion of some that Brahma's age is $8\frac{1}{2}$ years. We do not know whether this is made as an

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- 1 (i) काऽहोमनवोद (14) मनुयुग शूख (72)
गतास्ते च (6) यनुयुगच्छना (27) च । (*Ārya., gitikā* 3)
- (ii) अष्टोत्तरं सहस्रं ब्रह्मो दिवसं महयुगानाम् । (*Ārya., kāla* 8)
- (iii) दन्ताऽब्धयो (432) ऽयुतहता युगमर्कमाना-
दक्षादयो (72) युगगुणा मनुरेक उक्तः (*Vaṇ. Sid., madh., I.9*)

In the edition brought out by the Indian Astronomical Institute, New Delhi, 'चक्षादयो' is a wrong emendation for 'दक्षादयो' as shown in the author's thesis, 'The System and Constants of the *Vaṇeśvara Siddhānta*.'

approximate statement, or as the opinion of a variant of the Āryabhata school, and held by Āryabhata himself. But we can be sure that in *Vat. Sid., madh.* : I, 10, this extra period of 15 days has been given (but masked by scriptory errors) as a study of *madh.* : III, 17-18, and IV, 56-60, will show.¹ (Before proceeding I wish to state that the commentary of the only printed edition available of the *Vaṭeśvara Siddhānta* exhibits spoiled constants in most places, so that they contradict one another, and the correct constants have to be determined by investigation and research, which I have done in a separate paper with the title, 'The System of the *Vaṭeśvara Siddhānta*'. What I am using here is part of that, and for fuller knowledge or in doubt that should be consulted.)

In III, 17-18,² the number of quarter *yugas* from creation up to the beginning of *kali* is given as 24,798,639, and this multiplied by the days in one quarter *yuga* (=394,479,390) gives the days from creation up to *kali*

- 1 कजन्मनोऽष्टौसदलास्समा ययुः
तथासमाप्ता (तिथिर्दिनानां ?) मनवो दिनस्य वा । (षट् ?)
(*Vat. Sid., madh.*, I.10)

In the above-mentioned edition, the reading in the second foot gives nothing. The author's emendation is only to give the idea that should be there, which alone can be done in the absence of the original manuscript.

- 2 (i) तद्योगः कल्यादौ द्युगणः,
कोत्पत्तितोऽथवा निम्न ।
नवगुणरसाष्टनवनग-
वेदमुजैः (24798639) कुदिन वेदां (4) शैः ।
(*Vat. Sid., madh.*, III.17)
- (ii) खेकाक्षिस्त्रशरशर-
वसुनवरूपाक्षतस्ववस्वगाङ्गाः (9782551985550210) ।
कल्यादौ द्युगणोऽयं
कलिगतद्युगणेन संयुतस्त्विष्टः (*Vat. Sid., madh.*, III.18)

For details, see the article, 'The System and Constants of the *Vaṭeśvara Siddhānta*'.

as 9,782,551,985,550,210. Dividing the quarter *yugas* by 4, the *yugas* from creation is 6,199,659 $\frac{3}{4}$, and this must be got if we compute the number. With the 15 extra days, the (Brāhma) days from creation are $8\frac{1}{2} \times 360 + 15 = 3,075$. The number of *kalpas* are $3,075 \times 2 = 6,150$. In this period there are $6,150 \times 1,008 = 6,199,200$ *yugas*. The *yugas* gone in the present *kalpa* are $6 \times 72 + 27 + \frac{3}{4} = 459\frac{3}{4}$. Adding, we get the total number of *yugas* to be 6,199,659 $\frac{3}{4}$, the same as we want. Thus we see it is 15 days more than $8\frac{1}{2}$ years. Our conclusion is reinforced by the fact that only with the 15 days more can the *kṣepas*, given in IV, 56-60, be got. The cycles of the apses and nodes of the sun and the star-planets have been given as so many in Brahma's life span of 72,000 *kalpas* (or 72,576,000 *yugas*) in *Madh*, I, 16-19. Using these it can be seen that for the 6,199,659 $\frac{3}{4}$ *yugas* gone up to the present *kali*, with the 15 days included, we get these *kṣepas* to the second.¹

1. For those who care to work these out and verify, the details are given below. The *kṣepas* marked * have not been given by the printed text mentioned, perhaps this portion of the manuscript is missing. They have been worked out and given here. (For further details, see the article, 'The System and constants of the *Vaṇeśvara Siddhānta*'.)

Body		Number of cycles in the life-span of Brahma	<i>Kṣepa</i> in <i>rāśis</i> , etc.
Sun :	apogee	165801	2-18-51-37
Mars :	..	81165	4- 8-50-50
Jupiter :	..	13948	5-22-48-31
Saturn :	..	72974	7-26-55- 4
Mercury :	..	477291	7-16-42-54
Venus :	..	152842	2-20- 3-26
Mars :	node	20684	16-20-10-12
Jupiter :	..	39202	9- 0-54- 2*
Saturn :	..	1542	8-20- 1- 0*
Mercury :	..	988271456418719	11-10-19-54
Venus :	..	196127480636835	5-24- 1-56*

But though Āryabhaṭa has not given the number of the above cycles of the apsides and nodes, Bhāskara I gives evidence of his knowledge of their motion when he says in his commentary under *gītikā* 9 that Āryabhaṭa has given these in loose verses.¹ But as the only two manuscript texts of the *Bhāṣya* available are much vitiated, it is not possible to connect these with the cycles given by Vaṭeśvara.

Another point of distinction is that this school asserts that a great circle on the celestial sphere, measured in *yojanas*, is 12,474,720,576,000. This is the distance moved by the sun, the moon or the star-planets in a *yuga*, Hindu astronomy asserting that all have the same rate of motion. In the moon's orbit it is 10 *yojanas* per minute of arc according to this school, as given by Āryabhaṭa in *gītikā* 4, by Bhāskara I in *Mahābhāskarīyam*, VII, 20, and by Vaṭeśvara in his *Siddhānta, Madh.*, VII, 3-7.² They all give the same manner of

1. Cf. *Āryabhaṭīya Bhāṣya* of Bhāskara I, under *gītikā* 9, about the node :

(i) 'मुक्तककथितं किलाचार्येण-
एकद्वित्रिचतुरिषून् क्रमशो मगणान्प्रयान्ति सर्वेषाम् ।
कल्पादेर्गतकालात् गणनीयमतोगतिस्तेषाम् ।'

and about the apogees :

- (ii) 'अथ किमिति मन्दोच्चगतिर्नाऽभिहिता ?...मुक्तकेनैव आचार्येण कथितमिति संप्रदायाऽविच्छेदादवधार्यते अत्यन्तसूक्ष्मत्वात् वर्षगणेनैव आचार्येण यदाऽऽख्यातं तदेवाऽविच्छिन्नसंप्रदाय प्रतिपत्त्याऽभिधीयते । तद्यथा—

अष्टिकृताऽत्यष्टिनवैरुच्चयुगं तिग्मदीधितेरुत्तम ।

दशगणगुणितैरब्दैः विरवान्भुङ्क्ते क्रमाद्भगणान् ।

...यथा पितुः शास्त्रसंप्रदायाऽविच्छित्तिरुच्यते महापातेषुक्तं तदत्राऽप्यवधारणीयमिति ।'

- 2 (i) शशिराशयः (12) चक्रं

तेऽशकलायोजनानि य (30) व (60) अ (10) गुणाः (*Āry.*, *gītikā* 4)

- (ii) हन्दोगणाः खखवियस्रस्रन्द (216000) निघनाः

व्योम्नो मवेयुरिह वृत्तसमानं संख्या । (*Mahābhāsk.*, VII, 20)

.....cond.

getting it, $10 \times$ the minutes in a circle ($= 21,600$) \times the moon's cycles per *yuga* ($= 57,753,336$). As all schools hold that the absolute motion is the same, the ratios of the measure of the celestial sphere, according to them, will give the inverse ratios of the absolute values of the *yojanas* used by them. For example, the celestial sphere is $10 \times 32,400 \times 57,753,336$ according to the *Ardharātrika* system; and the *Sūrya Siddhānta* school. Therefore the *yojana* measure they use is $\frac{2}{3}$ that of the *Āryabhaṭa* school.

Another important peculiarity of this school is the use of the true hypotenuse in the computation of the equation of the centre. The use of the hypotenuse in the equation of conjunction is common and accepted by all schools, as justified by the eccentric or epicyclic theory of the motion of the planets, which can be readily seen from a geometrical representation of the motion. By the same logic, the hypotenuse should be used for the equation of the centre also, the theory being essentially the same. That is why this school uses it, as a geometrical consequence of this theory set forth by *Āryabhaṭa* in *Kāla-kriyā*: 17-21, combined with the theory of uniform motion given in *Kāla*: 12-14. Thus, in the *Mahābhāsk.*, IV, 8-12, the manner of getting the true hypotenuse as based on the theory of epicycles is given, and in 19-20 the same as based on the eccentric theory. In 21, the approximate sine equation of the centre is asked to be multiplied by the radius and divided

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- (iii) कोशस्तु तैर्वन्धु (4) समैहिं योजनं
 तैर्व्योमवृत्तं कथयन्ति सन्तः
 स्वव्योमपूर्णतुनगेषुखाऽक्षि
 महाऽब्धिभूमृत्कृतपक्षचन्द्रेः (12474720576000)
 (Vat. Sid., madh., VII.3)
- (iv) रविशशियुगघातः खाऽक्षि (20) भक्तः सकक्ष्या
 शशिमगणहता वा दिगू (10) धनचक्रस्य लिप्ताः ॥
 (Vat. Sid., madh., VII.5)

by the true hypotenuse to get the correct sine equation of the centre. *Vat. Sid. Spasṭādhikāra*, II, 3-4, gives the method of getting the true hypotenuse, and III, 11 instructs its use to divide the approximate equation of the centre to get the correct one.¹

The use of the hypotenuse is not only a logical result of the theory, but it will also give a better result. It supplies part of the second term of the modern correct equation of the centre. Neglecting powers of e (eccentricity) higher than the square, the first two terms are $2e \sin m - 5/4 e^2 \sin 2m$, where m is the mean anomaly reckoned from the higher apsis, as in Hindu astronomy. The distance between the centres of the original and eccentric circles is equal to $2e$. It is also the radius of the epicycle. According to the theory, correct sine equation of centre = $2e \sin m \div h$ (=hypotenuse). But $h = \sin m \div \sin (m - \text{eq. cent.})$, if the radius of the eccentric circle is taken as unity. Therefore $\sin \text{eq. cent.} = 2e \sin m \times \sin (m - \text{eq. cent.}) \div \sin m = 2e \sin$

1. (i) Cf. *Mahābhāskariyam* :

कोटयाः पदवशाद्धित्वा युक्त्वा वाऽन्त्यफलं पुनः ।
तद्वर्गबाहुवर्गस्य योगात्कर्णः पदं भवेत् ।
कर्णेनान्त्यफलं हत्वा विष्कम्भाधेन लभ्यते ।
पूर्वकोट्यां घनर्णं स्यात् यावत्कर्णस्समो भवेत् ।
भुजज्यया हतां त्रिज्यां कर्णेनाप्तधनुः क्रमात् ।
केन्द्रात्पदविभागेन धनं स्वोच्चे प्रकल्पयेत् ॥ (IV.19-21)

The same is said in (IV. 8-12) also.

(ii) Also cf. *Vaṣeṣvara Siddhānta* :

लब्धस्य चापमिह शीघ्रफलं प्रदिष्टं
एवं मृदु श्रवणको द्युचरस्य साध्यः ।
बाह्ययोस्स गुणकस्त्रिगुणश्च हारः
ताभ्यामसावसकुदेवमनिश्चलत्वे ॥ (*spasṭ.*, II-4)
त्रिज्या हताभुजज्या
कर्णहता तस्य कार्मुकं तु फलम् ।
देयं मध्ये शोध्यं
शीघ्रोच्चे स्यात्स्फुटोद्युचरः ॥ (*spasṭ.*, III-11)

$(m - \text{eq. cent.}) = 2e \sin (m - 2e \sin m)$ (since the eq. cent. is small) $= 2e \sin m - 4e^3 \sin m \cos m = 2e \sin m - 2e^3 \sin 2m$. Though we get $2e^3$ as the coefficient of the second term, instead of the correct $5/4 e^3$, it will not make much difference, being the second power of e . Also, the point is that we get the term instead of neglecting it. Using the moon's epicycle of $31\frac{1}{2}$ degrees, which gives $7/80$ as the value of $2e$, we get for the second term $-13' \sin 2m$, the same as the modern correct one. (The apparent complete agreement is due to the Hindu coefficient of the first term being defective by about a fifth.)

Bhāskarācārya II discusses the point, why other schools do not use the hypotenuse for the equation of centre. He says that some do not use it thinking that the difference is small. This depends upon what we consider small and negligible and may be accepted. But the other argument he gives, quoting his master Brahmagupta, that the theory itself is that the epicycle instead of being uniform, is proportionate to the true hypotenuse and has to be multiplied by it and divided by the radius, and therefore the division by the true hypotenuse is cancelled out, is untenable, for this kind of argument helps only to shut out a tolerably good theory already existing and nothing more, and is just a way of escape, as pointed out by Caturvedācārya in his commentary on the *Brāhmasphuṭa Siddhānta* (cf. *Sid. Śiromaṇi: Gola: Chedyaka*; and commentary thereon).¹

1. (i) Cf. *Brāhmasphuṭa Siddhānta* :

त्रिज्या भक्तः परिधिः कर्णगुणोबाहुकोटिगुणकारः ।

असकृन्मन्दे तत्फलमाद्यसमं नाऽत्र कर्णोऽस्मात् ।

- (ii) And also cf. *Siddhānta Śiromaṇi, gola* :

स्वरूपान्तरत्वान्मृदुकर्मणीह

कर्णः कृतो नेति वदन्ति केचित् ।

त्रिज्योद्धृतः कर्णगुणः कृतेऽपि

कर्णेस्फुटस्स्यात्परिभिर्यतोऽत्र ॥

.....contb.

Another peculiarity is dispensing with the first of the four operations (according to this school) of applying half the equation of the centre to the mean, in the case of the Mercury and the Venus. This is according to Āryabhaṭa, Bhāskara I and Haridatta.¹ But Vaṭeśvara seems to vary even from this (if the printed text is correct), dispensing with the first two operations, and changing the order of the third and fourth, though he himself as also other astronomers all give the application of the equation of the centre as the third, and the application of the equation of conjunction as the fourth.² The question naturally arises whether this is an improvement in the method, so as to give a better result. In the first place, even the original method, with its four steps, is far from satisfactory. The third and fourth steps alone are capable of yielding correct results (modern astronomy uses only these), provided (i) the true sun is used for the *śighra* of Mars, Jupiter and Saturn and the *madhya* of Mercury and Venus, (ii) the true hypotenuse of the sun

तेनाऽऽद्यतुल्यं फलमेति तस्मात्

कर्णः कृतो नेति च केचिदूचुः ।

नाऽऽशंकनीयं न चले किमित्थं

यतो विचित्रा फलवासनाऽत्र । (*sphuṭagativās.*, 36-37)

1. Cf.

(i) बुधभृग्वोः पुनस्साध्यं मान्दमेव स्वकर्मणा ।

तेन सिद्धौ चलाद्भूयः स्फुटावेतौ प्रकीर्तितौ ॥ (*Mahābhāsk.*, IV.53)

(ii) विनाद्यमन्दसंस्कारं बुधशुक्रौ स्फुटौ मतौ ।

(*Grahacāranibandh.*, III.10)

(iii) शीघ्रोच्चादर्धेन

कर्णव्यमूणं धनं स्वमन्दोच्चे ।

स्फुटमध्यौ तु भृगुबुधौ

सिद्धान्मन्दात्स्फुटौ भवतः ॥ (*Āryabh. kāla.*, 24)

2. Cf. *Vaṭeśvara Siddhānta, spaṣṭādhikāra* :

ग्रहोनात्स्वचलात्कृत्स्नं फलं शोध्यं हाशुकयोः ।

मान्दं चैव स्वमन्दोच्चात् सकलं मध्यमाद् ग्रहात् ॥ (II.2)

is introduced in the proper place and (iii) the equation of the centre is applied to the *śighra* of Mercury and Venus instead of their *madhya*. It is noteworthy that in the case of Venus, Hindu astronomy actually does (i) though purporting to apply the equation of centre to Venus. Venus's being less, is masked by the sun's. That is why its apogee given is really that of the sun. But otherwise dispensing with the *half-manda* application is not going either to make or mar the correctness of the result. As for the very peculiar instruction of the *Vaṭeśvara Siddhānta*, to apply the equation of conjunction and then apply the equation of centre is quite wrong. But Vaṭeśvara is too astute to make this mistake, and the text in the printed edition (got from an only manuscript) must be wrong.

Another peculiarity is that the *yuga* is divided into two parts, *utsarpiṇī* and *avasarpiṇī* on the one hand, and *suṣamā* and *duṣṣamā* on the other, as given by the *Āryabhaṭīyam* (*kāla* 9) and the *Vat. Sid. (madh., II, 6)*.¹ Even from a time as early as Bhāskara I, who is later than Āryabhaṭa only by a little more than a century and who comes in a direct line of pupils, commentators are not sure of the import of these divisions (*vide* commentary under *kāla* 9 in the *Āryabhaṭīyabhāṣya* of Bhāskara I and *Bhaṭṭadīpikā* of Parameśvara). A general interpretation

1. (i) Cf. *Āryabhaṭīyam*:

उत्सर्पिणी युगार्धं

पश्चादवसर्पिणी युगार्धं च ।

मध्ये युगस्य सुषमा-

ऽऽदावन्ते च दुष्षमेन्दूच्चात् । (*kālakri., 9*)

(ii) And also cf. *Vaṭeśvara Siddhānta, madhyamādhikāra*:

उत्सर्पिणी प्रथममेव युगार्धमुक्ता

शेषा द्वितीयमवसर्पिणिकाभिधाना ।

मध्ये युगस्य सुषमा खलु दुष्षमा स्या-

दाद्यन्तयोः कुमुदिनीवनबन्धयोगात् । (II.6)

is that during the first half of the *yuga*, the longevity, strength, ability, etc., of creatures increase and, in the second half, these decrease, an idea current among the Jains.

Only three sections of the *Vaṇeśvara Siddhānta* are available to us in print now. When the rest is also published, we can come across more items peculiar to this school.

SOME MIS-INTERPRETATIONS AND OMISSIONS
BY THIBAUT AND SUDHAKARA DVIVEDI
IN THE *PAÑCASIDDHĀNTIKĀ* OF
VARĀHAMIHIRA*

The *Pañcasiddhāntikā* (PS) of Varāhamihira (VM) is very valuable as a source of information on the state of Hindu astronomy before the 6th century A.D. because in this work VM has condensed the basic texts of five astronomical schools in vogue before him. VM has depicted herein, very faithfully, the constants and methods of the said five schools. The work was edited in 1889 by Thibaut (T) and Sudhakara Dvivedi (S) on the basis of two badly vitiated manuscripts available to them. Since no ancient commentary was to be had, S added to the edition a short Sanskrit commentary, and T supplied it with an English translation and explanatory notes. Unfortunately, certain important verses of the text have not been touched by T-S, as being obscure, and in many places their interpretation is wrong, some being seriously so. These mistaken ideas have been repeated and made the basis of further research by later scholars, who perhaps, did not have the time or ability to go into the relevant verses and draw conclusions for themselves. During the more than eight decades now, since the work was issued, nobody has re-edited the work, supplying the omissions or rectifying the mistakes. There have just been two reprints (1930, 1968), perpetuating the errors in the earlier edition. An attempt is made in the following pages to correct some of the more important errors,

* A paper presented to the International Sanskrit Conference, New Delhi, March, 1972.

1. PS III. 1-3 gives the sun according to the Pauliśa (P). The meaning is: "Multiplying the days from epoch by 120, subtracting 33, and dividing by 43831, the revolutions etc. of the 'mean' sun is got. 20° added to this is called *kendram*, ('anomaly'). For each Rāśi of *kendram* subtract one for one, $11'$, $48'$, $69'$, $70'$, $54'$, $25'$, and then add $10'$, $48'$, $70'$, $71'$, $54'$, and $25'$. The 'mean' sun becomes true."

This straight interpretation supplies all the information required to get the true sun. For e.g., let the days from epoch be 620 $(120 \times 620 - 33) \div 43831 = 1$ Rev., 8 Rāśis, $10^\circ 52' = 8$ Rasis, $10^\circ 52' =$ 'mean' sun. This plus 20° , i.e., 9 Rāśis $0^\circ 52'$, is the *kendram*. So, for the 9 full Rāśis have to be applied, $-11'$, $-48'$, $-69'$, $-70'$, $-54'$, $-25'$, $+10'$, $+48'$, $+70'$, and $+2'$ for the $52'$ left over, i.e., in all, $-147'$ ($= -2^\circ 27'$). Therefore the true sun $= 8R 10^\circ 52' - 2^\circ 27' = 8R 8^\circ 25'$.

It can be seen that by *madhyama-sūrya* the Siddhānta actually means the mean sun plus a large part of the equation of the centre at epoch, and that by *kendram* it means not the anomaly as we understand it now, but only the argument to get the correction to the so called mean to become true. (It must be noted that the *Vākyakaraṇa* gives a similar method for the sun, in the shape of what are called *Bhūpādivākyas*, to be applied to the sun from the beginning of the true year).¹

But the translation of T-S says, "Add 20° to the *kendram*," and raises to questions: (1) How are we to get the *kendram* without the *ucca* being given, and (2) why should 20° be added to the *kendram* (which is not even used), while to use the equation of the centre for the Rāśis would be easier and better? If the real anomaly plus 20° be called *kendram*, why not call the so called *mean*

1. *Vākyakaraṇa*, Cr. Ed. by T. S. K. Sastry and K. V. Sarma, (Madras, 1962), p. 15.

plus 20° so? Then, against all canons of interpretation, they add together the quantities instructed to be subtracted, given in one sentence, and the quantities instructed to be added, given in the next sentence, and say that these are the equation of the centre values, and thereby face the further difficulty that, according to their interpretation, the instruction where to add and where to subtract has not been given. Why at all should each value be split into two almost equal parts and the two series given in two different places? Further, they do not see that their made up values are so erroneous as to represent anomalies 20° off even.

2. In *PS* III 15, the true sun or moon, computed for the mean sunset at Yavanapura (Alexandria), is reduced to the true sunset of a desired place. The meaning is: "From the *nāḍikās* of *deśāntara* in longitude (given by 14) subtract half the *cara* (ascensional difference given in 10) if the sun is in the six signs, Meṣa to Kanyā, and add the same if the sun is in the next six signs. Subtract the motion (of the sun or moon) calculated for this period (from the sun or moon) already found. We get them for the sunset of the place."

T-S have missed the import of the verse and have translated the last part as: "Reject the remaining ascensional difference." If it means this, need it be said? From the instruction it can be inferred that the work is for places in India.

3. In *PS* III. 10, the computation of two *yogas*, viz., *Vaidhyā* and *Vyātīpātā*, (also called *Sārpamastaka*), that are days of *śrāddha*, are given. (Note that the subsequent verses also give the times for the propitiation of the manes.) The meaning is: "When the sum of the true sun and moon is equal to 12 signs, then is the *yoga*, *Vaidhyā*. When the sum plus 10 *nakṣatra* segments (ie., 4R 13° 20') is equal to 12 signs, then is the *yoga*, *Vyātīpātā*."

The exact time of day (of their occurrence) should be sought by using the degrees remaining over (the 12 signs)” (The technical *Vyatipāta* is given in 22). Not realising the purpose of the verse, T-S have unnecessarily changed *cakre* into *ṣaṭke*, and given a meaning not intended here.

4. PS III. 28-29 give the Pauliśa Rāhu (moon’s nodes). To know the position of the node at any time it is necessary that (i) its longitude at epoch should be known, and (ii) its motion during the time from epoch, which should be subtracted from (i) to get the present position.

Verse 28 gives (ii): “Multiply the days from epoch by 8, and divide by 151, and add minutes equal to the revolutions, to get the degrees of motion during the period.”

Then 29 gives (i) and how to use (ii), thus: 7R 25° 59’ are Rāhu’s (longitude at epoch). The first, *i.e.* the motion given in 28, being subtracted from this is Rāhu’s Head, (*i.e.*, ascending node). This plus six signs is called Rāhu’s Tail, (*i.e.*, descending node). ”

Note that from this we can see the ascending node at epoch, (Śaka elapsed 427, Sunday, 37-20 nāḍikā from sunrise at Ujjain, which is 509432 days, 22-40 nāḍikās before mean sunrise at Ujjain on Jan. 1, 1900) to be 7R 25° 59”, *i.e.*, 235° 59”. Compare: The ascending node computed for the time by modern astronomy is 236° 11’, *Siddhāntaśiromaṇi* 237° 6’, *Romaka* 235° 49’ and *Saura* 236° 6’. See the close agreement, which will show the need for the *kṣepa* at the epoch.

But T-S have mistranslated (29) and obliterated the *kṣepa* as they have done in the case of the Vāsiṣṭha moon.¹ They have missed to note that if left with (28),

1. See the present writer’s paper, ‘The Vāsiṣṭha Sun and Moon, *Jl. of Or. Res.* 25 (1955-56) 19-41. (pp. 1-28 in this book).

and the *kṣepa* was assumed zero at epoch, there should at least be the instruction to subtract the result of (28) from 12 signs to get Rāhu, the motion being retrograde. The relevant section of S's commentary is nonsensical and T confesses ignorance of the expression *vṛścikā-bhāgā rāhoḥ*. S says: "There are already 25', being the length of the scorpion-like limb of Rāhu. *Subtracting this 25' from the Rāhu* found in (28), Rāhu's Head is obtained. For the fact that Rāhu has this scorpion-like limb, we have only to depend on the statement of the ancients, no other reason can be given." The fact is that T-S do not realise that *vṛścikā-bhāgā*...means 7R 25°59'. This method of stating positions is common, as for instance, *sārdhāḥ pañcālino bhogāḥ* (XVIII. 1), *kanyāṁśān ṣaḍviṁśatim* (XVIII. 2), *navasārdhāḥ kanyāṁśāḥ* (XVIII. 11), *ṣoḍaśa vṛṣabhasyāṁśāḥ* (XVIII. 18), *mithunadalāś śodhyate* (VIII. 2); and these expressions have mostly been interpreted by T-S in the way intended by VM.

5. III. 31. gives the Pauliśa moon's latitude. Here the maximum given is 280', by both manuscripts. But T-S, for no apparent reason, make it 270', by introducing an unlikely emendation. (The Romaka too gives 280', as can be seen from VIII. 11 and 14, though, here too, T-S introduce unnecessary emendations making them 240' in one place, and 270' in the other.) There are however, very strong reasons to believe that the maximum latitude given is actually 380', that this Siddhānta got this by extrapolating the latitude 55' for the ecliptic limit 13° by the proportion 13° : 90° :: 55' : max-lat., since the proportion mentioned here is, as elsewhere in this Siddhānta, according to the degrees and not their sines, and that VM's "*liptāśatatrāye*..." has been mis-corrected by some scribe into "*liptāśatadvāye*..."

6. T-S have not interpreted III. 32-37, as being too obscure. Here VM criticises certain astronomers of his

time like Bhadraviṣṇu whose works gave wrong tithis and nakṣatras, and the people who follow them blindly. He criticises the Romaka stating that the tropical year given by it will, in course of time, overthrow the luni-solar year of the Hindus based on the fixed nakṣatra system and play havoc with the Dharmaśāstra rites and national festivals like Rāma-Navamī. By giving this criticism at the end of the Pauliśa Siddhānta, VM exhibits his intention that the Pauliśa should be followed by the people, alternatively with the Saura, for which purpose he gives it so very fully, indicating the days of Śrāddha etc. It may be remembered that Pauliśa includes the Vāsiṣṭha *mutatis mutandis*, as in the case of its moon and star-planets.

7. In the second half of IV. 1, VM says, "Here, assuming *that* (meaning by '*that*' the diameter of the circle mentioned immediately before,) to be 4° , the R sines of the eighth parts of signs (*i.e.*, $3\frac{3}{4}^\circ$, $7\frac{1}{2}^\circ$, etc.) are given, (with the method of computing them)". From this it is clear that VM has taken R to be $120'$, and the sine of 3 signs. But T-S have missed this meaning, and have taken the word '*that*' to indicate the circumference. They say: "Dividing the circumference into 4 parts, the R Sines of the eighths of the signs are given." So, according to them, the radius is not given as being equal to 120, exactly, but what it would make up by adding the last sines given, to the sine of 2 signs got, and by an error arrive at $120' 1''$ for the radius. This seemingly innocent $1''$ has perplexed scholars, and some have asked me what VM intends by this tail of $1''$. In comparative studies they quote this extra $1''$ with meticulous care. Why, T-S themselves have taken it as sacrosanct, and in filling up the gaps in 13-14, with sine-intervals, according to the context, choose the numbers in such a way that they all add up to exactly $120' 1''$.

The fact is that all this is the result of an apparently wilful mistake T-S have committed. The text gives the

sine of 60° as $43' 55''$ over the $60'$ of the first sign, to make up the correct $103' 55''$. By some freak of thought, T-S have emended *hīnā manubhir viṣayaiḥ* in IV. 9. into *hīnā manusāgaraiḥ*, and, given $43' 56''$, instead of $43' 55''$. As a result, all the R sines in the third sign are in excess by $1''$, and the last is got as $120' 1''$. It passes one's comprehension why T-S emended the correct *viṣayaiḥ* into *sāgaraiḥ*, and created all this trouble for themselves and for others.

8. T-S have left IV. 16-18 untouched, with the remark that these probably give the moon's latitude. Actually they give a rule to find the sun's declination, and then the worked out declination intervals for every $7\frac{1}{2}^\circ$ of the sun's longitude. The meaning is: "The R sine of the sun's longitude $\times 61 \div 150$, is the R sine of the sun's declination, (the maximum R sine declination being, $120' \times 61 \div 150 = 48' 48''$, giving 24° , as the maximum declination). The moon's declination is this, plus the moon's latitude. The declination intervals for quarter signs are $183'$, $180'$, $175'$, $166'$, $156'$, $144'$, $128'$, $104'$, $90'$, $63'$, $40'$, $11'$, (total $1440'$)."

9. In IV. 41-44, the sun's shadow at a given time is computed. Here, in (42) the sentence, *aviśodhanena jīvāśaḍghnīnām eva kartavyā* means, "If the degrees of ascensional difference cannot be subtracted from the degrees of the given time, being larger, take, simply the sine of the degrees of the taken time." This instruction is necessary, in the case when the ascensional difference, which is asked to be subtracted when the sun is in the six signs Meṣa etc., happens to be larger. But T-S do not see the need for this instruction, which is the correct meaning, but say that this gives as a general rule: "Of the *nāḍikās* multiplied by 6, take the sine without any correction, (to the said *nāḍikās*)."

In fact the question of correction does not at all arise here. Then what can this mean?

10. In IV. 50, the moon's shadow is sought to be calculated, as that of the sun done earlier. Here the instruction is: "Take the time after sunset for which the moon's shadow is wanted. Add to it the time elapsed from moonrise to sunset, if the moon has risen in the daytime. If the moon rises in the night time, subtract the time of moonrise after sunset. Take this as the given time, and work out as for the sun. The moon's shadow is got." T-S interpret the verse in such a way that the moon's shadow would be got in the day-time, when the sun is shining!

11. In IV. 52-54, and then in 55-56, VM uses the word *arkāgrā* or *sūryāgrā* in the sense of *śaṅkvaḡram* of Bhāskara I or *śaṅkutalam* of Bhāskara II. T-S have understood this to mean what is usually called *agrā* or 'sine amplitude of the rising or setting sun'. This has resulted in the wrong interpretation of the verses, though with great difficulty they struggle out, with the correct ideas intact.

12. In V. 1-3, giving the time for the first visibility of the moon, the correct expression *ayanānukūlavikṣipte*, given by the manuscript text, meaning, 'if the latitude has the same direction as the northward or southward course of the moon', has been emended by T-S into '*apamānukūlavikṣipte* and the meaning given as, 'if the moon's latitude has the same *direction as the difference of the declinations*', which is nonsensical.

13. In VI. 2, VM instructs that $1^{\circ} 36'$ should be subtracted from the calculated Rāhu (nodes) and then the computation of eclipse proceeded with. (This must be an empirical correction by VM himself.) But T-S interpret *saśaṅkṣṭikalām hitvāṁśam* to mean 'subtracting 26', neglecting the word *amśam*, and think that this is the same as given by them in III. 29, as correction for "Rāhu's scorpion-like limb", (which we have discussed

in 4 above). But there it was given as 25'. How has the limb grown into 26' here? If this is meant here, it has been given according to T-S as an item in getting Rāhu already, and that Rāhu is taken in this chapter as ready computed. Why again this instruction?

14. The meaning of VI. 4 is: "Multiply (moon ~ node), which must be less than 13° for eclipse to occur, by 5. These are the *vinādis* that should be added to the total duration if the node is greater than the moon, and subtracted if less." The purpose of this correction is to take into account the empirical correction of $1^\circ 36'$ mentioned before. In the computation of the duration in verse (3), $55'$ latitude is taken as the limit. This latitude has been computed according to III. 30-31 from the un-corrected Rāhu, and is not what it would be if the empirical correction is made. Hence this correction, though rough.

T-S give a wrong interpretation; T says that he does not understand the rationale, while S says that it is a correction for the changing velocity of the moon, not understanding how. In truth, the small change in the moon's rate of motion during an eclipse will have practically no effect on the duration, for it has to be noted that it is the lunar eclipse that is being dealt with here, and that there is no question of change due to parallax.

15. In VI. 5. T-S propound two extraordinary things in their explanation: (i) The maximum latitude of the moon is $240'$, which is against what they themselves have said in III. 30, as $270'$. (ii) If moon ~ node is 10° there is total eclipse, while actually it is 5° , giving $21'$ limit in latitude.

16. In VI. 7, the points of first and last contacts on the moon's rim, with respect to the ecliptic east-west is given. If (moon ~ node) is zero, these are due East and

West points. If (moon ~ node is 13° it is due North or South, as the latitude is South or North. If (moon ~ node) is from 0° to 13° , the quarter-rim from East or West points to the North or South point is divided into 13 parts, one for a degree, and the number of parts from East or West points equal to the difference in degrees, is taken. This is a rough rule and considered satisfactory as far as this Siddhānta is concerned. But T-S do not understand the instruction, and bring in *Valana* here, which is treated later. But T actually says: "We do not know the reason for direction given in stanza 7 to divide each quarter circumference into 13 parts." It can be easily guessed from the 13° ecliptic limit.

17. VII. 2-4 contain the parallax in latitude required in the computation of the solar eclipse to correct the moon's latitude, and given in the form of a correction to Rāhu. The obscurity has been cleared by the present writer elsewhere.¹

18. In VIII. 11; the Romaka gives a correction to the declination of the nonagesimal, since the corresponding point on the moon's orbit should be taken instead of the nonagesimal, which is on the ecliptic. The instruction is: "Twice the tabular sine of (nonagesimal ~ Rāhu) plus $\frac{1}{8}$ of itself, is to be applied as correction to the declination of the nonagesimal." This will agree with the $280'$ max-latitude of the moon given in verse 14, since $(2 \times 120') + 2 \times 120' \times \frac{1}{8} = 280'$. But T-S make it $240'$ unnecessarily, by carrying out unwarranted emendations.

In (14) the latitude of the moon is given by the rule "Multiply the tabular sine of (moon ~ node) by 21 and

1. See T. S. K. Sastri, 'A historical development of certain Hindu astronomical processes', *Indian Jl. of History of Sciences*, 4 ii (1969) 114-15. (pp. 46-75 in this book).

divide by 9". T-S also give this rule. Still they do not see that this would make the max-latitude, $120' \times \frac{2}{3} = 280'$, but assume 270', and arrive at the strange conclusion that the multiplication by $\frac{2}{3}$ is approximate! They do not realise that in verse 11 too it must be 280', and their 240', got by their emendation of the text is wrong.

19. In VIII. 4 giving the Romaka mean moon, the *kṣepa* must be 10984, deductive, given by *kṛtāṣṭanava-khaikavarjitāt*, and not the one given by T-S as 1984. My reading will keep the mean moon at epoch in its correct place $356^\circ 12'$, while T-S make it $359^\circ 19'$. Their value will take the Tithi earlier by a quarter of a day, and spoil the agreement with all other Siddhāntas, and modern astronomy also. In addition, this will disagree with the *kṣepa* for *avamā*, 514, given in I.10, in getting the days from epoch according to the Romaka.

20. IX. 15, the Saura sun and moon's 'orbits' required in eclipse work, are given. The meaning is, "The true hypotenuse of the sun (given in 14) multiplied by 5347 and divided by 40, is the sun's 'orbit'. The moon's true hypotenuse (given in 14) multiplied by 10, is the moon's 'orbit'." T-S give the divisor 120 in the place of 40, and the multiplier 3 in the place of 10. According to all Hindu astronomical works, as also the original Saura, the mean ratio of the sun's orbit to the moon's must be the Moon's *yuga* revolutions divided by the sun's, $= 2406389 \div 180000$ (see I. 14) $= 13.37$. Here according to my meaning, the ratio got is, $(120 \times 5347 \div 40) \div (120 \times 10) = 13.37$ exactly. But T-S's corrections will make it, $(120 \times 5347 \div 120) \div (120 \times 3) = 14.86$ which is absurd.

21. IX. 16 gives the true angular diameters of the sun and the moon according to the Saura. The meaning is: "Divide 514787 by the true 'orbit' of the sun (found in 15) to get the sun's true angular diameter. For the

moon, divide 38640 by its true 'orbit'." To agree with (15), there should be only six digits in the dividend for the sun, and five digits in the moon's. But T-S make them seven and six, respectively, and use the numbers 5147080 and 333640, by wrong emendations, and get thirty times what the angular diameters would be, and are dismayed by the result, saying, "But 962.6 can represent that quantity, (i.e., the angular diameter) only if it be divided by 30.....For some reason or other, the text, provided it be correct, does not mention the divisor 30. The rule for finding the true diameter of the moon is analogous, and we there also miss the mention of the divisor 30."

22. In IX. 22, Saura's Parallax in longitude is given. The work given here is: "Square the R sine of altitude given in (21) and subtract from 120^a . From the remainder subtract the square of the sun's $dṛkṣepa$ (got in 20). Take the root, etc." Here T-S say that 120^a minus $R^a \text{ sine}^a$ altitude should be deducted from the square of the $dṛkṣepa$ and the root taken, not realising that the square of the $dṛkṣepa$ will always be less than 120^a minus $R^a \text{ sine}^a$ altitude.

23. In IX. 27, a very important correction to the duration of the solar eclipse found in (26), and thereby to the first and last contacts, is given. T-S have missed it, and give something unrelated to the point. Now, other things being the same, the nearer the solar eclipse is to noon, the longer is the duration. It is this that is taken into account in this stanza. The meaning is: "Find the parallax in longitude for the moment of first contact (found in 26). Deduct from this the bending of the moment of new-moon (i.e., the correction of the new-moon for parallax) already found, or *vice versa*. The remainder should be added to the duration. In case the moment of new-moon and the first contact are, one in the forenoon and the other afternoon, the above two

parallaxes should be added, and the sun added to the duration. The same should be done for the last contact, (i.e. in the above, substitute last contact for first contact, and do the work)."

24. In XVIII. 1-2, the preliminaries for Venus according to the Vāsiṣṭha are given. The meaning is: "Subtract 147 - Really it is 167 and 147 is due to a scribal error - from the days from epoch. Divide by 584. The quotient are the risings of Venus. The motion of Venus for each rising is 7 Rāśi 5° 30' 20", Venus moving on to 26° of Kanyā, i.e., 5 Rāśi 26° goes to its first rising (after 30 days). To the remainder above, add days equal to $\frac{1}{11}$ of the number of risings gone. The motions during each cycle are given, (for the days remaining)."

T-S here change *guṇāptaiḥ* into *guṇāmśaiḥ*, and thus introduce into the correct motion an error of $10\frac{1}{2}$ minutes of arc per cycle, when even the difference between one Siddhānta and another does not exceed one minute of arc. They have also changed *kanyāmśān* into *kālāmśān* and shut out the *kṣepa* for the beginning of the first cycle, as they have in the case of the Pauliśa Rāhu, and the Vāsiṣṭha moon, already referred to. It is to be noted that *kālāmśa* is meaningless when applied to motion. It is the interval between the rising or setting sun and the planet, (here Venus), in time-degrees and for Venus it is 8° to 10°, and never 26°.

25. In the computation of the Vāsiṣṭha, Jupiter and Saturn following, VM gives a method similar to that given for the Vāsiṣṭha moon, using the technical terms *ghana*, *gati*, and *pada*. As in the case of the moon, here too, T-S have not understood the nature of the work, and have given wrong interpretations.

26. The last eighteen *āryās* of the work, purporting to deal with the Pauliśa star-planets, are spurious, and not VM's, for which there are strong internal evidences:

Stanzas 61, 62, 63, patently close, not only the chapter, but the whole work, which can be seen from the words, *āvāntyakāḥ Varāhamihiraḥ.....tārāgrahakārikātantram cakre*, and after criticising certain astronomers, occurs, “*dyṣṭam Varāhamihireṇa sukhaprabodham*”, in which the changed metre itself shows that the work has come to a close. In *dyṣṭam* etc., the next three feet are missing, and I feel that it is a purposely done blackout, to obliterate sure signs of the closing of the work, and it must have been done by somebody at a later time, with a view to append the next eighteen *āryās* and pass them off as VM’s. It is to be noted that if the eighteen *āryās* also belong to the work, then the work would be without the customary completion.

These eighteen *āryās* form a second set of rules, of which the first two stanzas form a new salutation and introduction. It avers that VM considered this to be a superior work, and that with a liberal heart he gives this to the world. And these stanzas are just worthless stuff. The rules completely neglect the equation of the apses, and therefore what are given, are only the *mean* geocentric positions of the planets. That explains the boasted simplicity of these rules. But what is it worth? How on earth can it be styled superior, on which VM is said to take pride and pass them on to posterity, a thing which even a novice can do? Fancy VM, who says in 62, “Let people, who have been discouraged by the inaccurate Mars of Pradyumna, Jupiter of Saura and Mercury of Vijayanandi, have recourse to this part of the manual.” Fancy VM giving this second set, which gives mean, *i.e.*, incorrect, results instead of true, *i.e.*, correct results. Further, in Mercury and Venus given in this set, there are mistakes which one cannot presume to have been committed by VM.

T-S do not say anything about the possibility of this being an interpolation, because they themselves say,

(vide Introduction pp. xlvi-viii), that they do not understand it. They are perplexed at the strange and apparently incorrect constants appearing, and T is very sore that this is attributed to the Pauliśa, which he cherishes. He cannot emend the constants into his desired values, because they are "checked and found correct" in the work itself. But actually they are no more strange than familiar friends in different dress. Thus: instead of using the days as unit, the author uses the time taken by the mean sun to move one degree as the unit of time, and asks us to call this 'day'. This is nothing strange because Hindu astronomers define what they call a 'solar day' thus, and this is distinguished from the *sāvana* or civil day, equivalent to the mean solar day of our daily parlance. For certain given periods the planets move stated amounts of degrees forwards or backwards relative to the sun. Therefore, if the sun is known, we can get the planet by adding or subtracting the degrees. Thus, knowing the sun is a desideratum. So, the author gives the synodic periods in his new units of 'solar days' instead of ordinary days, because that will be conducive to uniformity and convenience, and, for the same reason, the period of the motions are also given in the same units. We shall now convert the author's synodic periods into civil days: For 360 solar days there are 365-15-30, (this is the Pauliśa year), ordinary days. So we have:

<i>Planet</i>	<i>Solar days given</i>	<i>Civil days</i>	<i>The period according to Vāsiṣṭha</i>
Mars	768 $\frac{3}{4}$	779-58-43-26	779-57-19-0
Mercury	114 $\frac{6}{9}$	115-52-30-12	115-52-45-0
Jupiter	393 $\frac{1}{7}$	398-53-7-3	398-53-20-0
Venus	575 $\frac{1}{2}$	583-54-21-43	583-54-32-44
Saturn	372 $\frac{2}{3}$	378-6-36-4	378-6-0-0

It is seen that the agreement is close, and the difference is not greater than that among the Siddhāntas themselves.

It may be pointed out here that only the more important misinterpretations have been dealt with in this paper. The omissions and misinterpretations in chapter II, have been dealt with exhaustively in the author's paper on 'Vāsiṣṭha sun and moon'.¹ A more detailed explanation of the items treated above, besides many more points less important but still deserving notice, has been given by the present writer in his exhaustive commentary in Sanskrit and translation, of *PS* prepared by him for the Institute of Astronomical and Sanskrit Research, New Delhi.

1. See *Journal of Oriental Research*. 25 (1955-56) 19-41. (pp. 1-28 in this book).

THE MAIN CHARACTERISTICS OF HINDU ASTRONOMY IN THE PERIOD CORRESPONDING TO PRE-COPERNICAN EUROPEAN ASTRONOMY

1. The System of Measuring Longitudes

One characteristic of Hindu Astronomy is that from time immemorial it has been following the sidereal system instead of the tropical system. From the earliest R̥gvedic period of their history, the Hindus have been following a lunisolar calendar. The lunar month, which is the period between two conjunctions (or oppositions) of the sun and moon, was observed as a natural unit of time, and used to measure time. It was also discovered that another natural unit, the year, in which the seasons recur, contains twelve of these months and about eleven days more. Continued observation showed that nearly once in twenty-seven days, the moon came practically to the same point in its path, marked by certain stars and star-groups, twenty-seven in number, *Citrā*, *Svātī*, *Viśākhā* etc., each one roughly corresponding to one day. The twelve lunar months were named after certain of these asterisms, selected for their suitability, at or near which the full moons occurred, like *Caitra*, *Vaiśākha* etc. When the full moons of the lunar months receded too far away from the asterisms to which they were assigned, by the eleven day defect of the lunar year accumulating, an extra month was given to the year, in order to bring them back to their assigned asterisms. This automatically tied the lunar year to the solar sidereal year, and has ever been keeping the names of the months *Caitra* etc., meaningful. Why sidereal and not tropical, it may be asked. The asterisms mark fixed points on the zodiac. (We neglect here their proper motions), and the opposi-

tions, (and thereby also the conjunctions) of the moon with the sun, being associated with these fixed points, make the periods of the moon or the sun once round the zodiac, sidereal.

It is true that the solar year, conceived through the coming round of the seasons, is tropical, i.e. equal to the time taken by the sun to move once round the movable zodiac from equinox to equinox, made shorter than the sidereal year by the precession of the equinoxes. But the difference is so small that hundreds of years would have to pass before it would be observed. Indeed, during the long Vedic period itself the difference was observed at long intervals, and since the seasons occur earlier and earlier in the sidereal year in the course of time, earlier and earlier new moons or full moons were enjoined by the vedic seers for the rituals depending on the seasons, like “*caitrasya amāvāsyā*”, “*caitrasya pūrṇimā*”, “*phālguna amāvāsyā*” “*phālguna pūrṇimā*” etc. This itself shows that the sidereal year was taken for granted.

This is confirmed by the fact that in the period immediately following, the *Vedāṅga Jyotiṣa* (V.J.) (c. 1200 B.C.) was based on the sidereal system. It divides the zodiac into twenty-seven equal asterismal segments, with the twenty-seven asterisms roughly corresponding to each, and says that in one year the sun traverses these twenty-seven asterismal segments, and in five such years, (called a *yuga*), the moon makes 67 such revolutions, with the result that there are 62 lunar months in it, two of them being intercalary. The V. J. also says that the zero-point of this sidereal system was at the beginning of the *Śraviṣṭhā* segment, where the winter solstice was declared to be situated at that time. Earlier, during the vedic period, we have evidence that the zero-point was taken to be situated near the Orion, and then near *Rohiṇī* and then again at *Kṛttikā* at successive periods, where the spring-equinox was situated at those periods.

The *V. J.* was intended to provide a scheme for a religious and civil calendar, which purpose it could serve excellently, provided an intercalation was dropped now and then on actual observation. We are sure that this was done, so that for more than a thousand years afterwards, even during the period of the astronomical *saṃhitās*, when the winter solstice had receded to the first point of *Śravaṇa* the zero-point was still taken to be *Śraviṣṭhā*, and predictions made. Only the Jain and Buddhistic astronomical works took the reality of the situation into consideration, and shifted the zero-point of their sidereal system to *Śravaṇa*.

About 570 A.D. the vernal equinox was very near the first point of *Aśvinī*, near *Zeta-piscium*, and the *siddhāntic* astronomers took this as the zero-point of their sidereal system, in which the position of the sun, moon and star-planets among the 27 asterismal segments was sought to be found, and their periods were intended to be sidereal. Excepting the *Vāsiṣṭhasiddhānta* of the *Pañcasiddhāntikā* which used a rough year of 365½ days, and the *Romaka* of the same, which followed the tropical system accepted to be foreign, all other *siddhāntas* give a sidereal period for the sun in the neighbourhood of 365-15-31, (while the actual sidereal period is 365-15-23), so far removed from the tropical that it borders on the anomalistic, though intended to be sidereal. The result is that the zero-point has *processed* at the rate of about a degree per 420 years, and is now forward by more than 3°, creating all sorts of confusion in fixing the *ayanāṃśa* (i.e. the distance between the Hindu zero-point, and the Vernal equinox forming the zero-point of modern astronomy). Consistent with this processing of the first point, all Hindu astronomers have given in general longer sidereal periods for the moon and the planets and the moon's apogee, and a shorter period for the moon's nodes, and larger rate of precession. In comparing the Hindu sidereal periods and rates of precession with the modern ones, this should be borne in mind,

Thus it will be seen that throughout in Hindu astronomical history, the sidereal system has been in use, implicitly in the vedic period, and explicitly from the time of the *V. J.* onwards. I have made this discussion a little elaborate, because for various reasons, (like astrology and modern convenience) some people have wished that the Hindu system be tropical, and so have asserted that it was tropical during the vedic period, (and according to some even in the later classical period) and tried to distort history instead of looking facts squarely in the face.

Another thing must be mentioned here. Recently some people have expressed the novel idea that the Hindu longitude is polar, i.e. the great circle arc joining the celestial pole, (as distinguished from the pole of the ecliptic), and the moon or planet is projected on the ecliptic, and the longitude of this point measured from the first point of *Aśvinī* is given as its longitude by the Hindus. But no *siddhānta* or *karāṇa* says so. They give the polar longitude for a specific purpose: For the sake of astrological predictions the moon, sun and planet's conjunction in polar longitude is given. This is also given to check their correctness in position by comparing them with the polar longitudes of the stars. For this purpose most *siddhāntas* give the co-ordinates of the stars in polar longitude readymade, and this has misled these people. Commentators have all made this point clear, not to speak of the texts themselves.

In contrast to the Hindu system, western astronomers, like the Egyptians, Greeks and Romans, very early adopted the tropical or seasonal year, on which essential civil activities like agriculture depend. When would Father Nile bless them with his yearly floods, was the problem of the Egyptians, and seasonal effects would be more marked in regions of higher latitudes like Greece

and Rome. On the other hand the asterisms, and the days and months occurring with these at full or new moon, became sacred to the Hindus even in the vedic period, and important religious rites like *darśapūrṇamāsa* on these, and so the sidereal calendar, originating in the way we indicated, has persisted, requiring the sidereal system of longitude reckoning, so much so that even the rituals depending clearly on the tropical year, later came to be fixed according to the sidereal calendar by the *Dharma Śāstras*.

2. Cycles in Yugas and Kalpas

In order to give the revolutions of the sun, moon etc., and of their apogees and nodes in whole numbers, and at the same time secure sufficient accuracy, the *siddhāntas* have used long periods called the *yuga* and even vastly longer ones called the *kalpa*. The *yugas* and *kalpas* are not equal in duration for all. There are four main schools of *siddhāntins* in this respect (1) The Old *Sūrya Siddhānta* condensed by Varāhamihira used perhaps a *yuga* of 1577917800 days. Āryabhaṭa's *ārdharātri* system clearly had this number, and following it, the *Khaṇḍa khādyaka*. A *Paulīśasiddhānta*, quoted by Bhaṭṭotpala in his commentary on the *Bṛhatsaṃhitā* belongs to this school. We do not now whether this school had a *kalpa*, nor how long it was.

(2) The School of Āryabhaṭa, represented by his *Āryabhaṭīyam*, has a *yuga* of 1577917500 days. This divided into four equal sub-*yugas* *Kṛta*, *Tretā*, *Dvāpara* and *Kali*. 1008 *yugas* make a *kalpa*. (3) The *Viṣṇudharmottara* School has 1577916450 days for the *yuga*. The *Brāhmasphuṭasiddhānta* and the *Siddhānta Śiromaṇi* belong to this school. The length of the sub-*yugas* are in the ratio 4:3:2:1. One thousand *yugas* make the *kalpa*. (4) The later or new *Sūrya siddhānta* has 1577917828 days for the *yuga*. A number of new

siddhāntas like the *Soma*, the *Brāhma*, the *Vṛddha Vāsiṣṭha*, the (new) *Romaka* etc. belong to this school. The sub-*yugas* are in the ratio 4:3:2:1, and the *kalpa* equal to 1000 *yugas*. But the revolutions begin, not with the beginning of the *kalpa*, but 3.95 *yugas* later, when according to these the creation of the planets was completed.

The same number of revolutions per *yuga* for the sun, is given by all schools. Excepting school no(3) all other three give the same number of *yuga* revolutions for the moon also viz. 57753336. No. (3) gives 57753300. As for the revolution of the other planets, the four schools agree or disagree with one another in various ways. It must be noted that even if two schools agree in the number of revolutions of a particular body, the body cannot have the same period, because the *yuga* days are different. This is natural, and must be expected, or else what is the distinction between the schools?

In spite of the above mentioned various differences, all schools take almost the same point of time as the beginning of the *Kali* sub-*yuga*. Schools (1) and (4) take the mean midnight at Ujjain, February 17/18, 3102 B.C. beginning Friday, as the beginning of *Kaliyuga*, (the *Kali* epoch), while the other two schools take the time of mean sun-rise following six hours later, as the point at which *Kaliyuga* begins. (This would mean that the *yugas* and *kalpas* according to each may vary with regard to the times of their beginning by several years, and even *yugas*, inspite of the popular feeling that they do not differ, but occur at the same point of time.)

Further all schools agree that the mean sun and moon is at the zero point of *Aśvinī* at the above mentioned *Kali* epoch. Excepting school (3) the others are also agreed that the mean planets too are at that point at the epoch, and that the moon's apogee is 90°, and the node

180° from that. What are we to understand from this? Are we to think that at such an ancient date as 17/18, February, 3102 B.C. the Hindu astronomers gave this result as got from their observations? Or was this point of time fixed by some later astronomers as a convenient epoch for starting their calculations, and followed by all the later *siddhāntins*? The former alternative cannot be accepted, because the mean sun, moon and planets were not the same but differed widely from one another, nor were they at zero *Āśvinī* as calculated by modern astronomy for that epoch. Scholars like Bentley first conceived this idea of verification by calculation. Bentley showed that starting from the epoch and working by each *siddhānta* the error gradually became less and less, until at the time of the *siddhāntas* the error became a minimum, as must be expected. Thus he proved that the second alternative was the correct one, and that the *Kali* era starting from this epoch, was an extrapolated era, founded by the astronomical *siddhāntins*.

Before this was done by the astronomers, the concept of the four sub-yugaṣ *Kṛta* etc., was a vague one, without any definite number of years attached to them. The concept of the *yuga* itself had arisen in the vedic times, and was perfected in the *Vedāṅga Jyautiṣa* period, the idea being that at the beginning of every *yuga*, consisting of five solar years, the sun and moon came together at *Śraviṣṭhā* where the Winter Solstice was situated at that period. At the time of the *jyautiṣa saṃhitās* a larger sixty year *yuga* was conceived, in which not only the sun and the moon, but also Jupiter met together at a given point in the zodiac. (The *Romaka-siddhānta* of the *Pañcasiddhāntikā* gives a lunisolar *yuga* of 2850 years of which the famous Metonic cycle is the 150th part. (P. S. I, 15). The *Sūryasiddhānta* condensed in it gives a lunisolar cycle of 180,000 years (P. S. I, 14). We do not know whether these were of the original *siddhāntas* themselves, or *Varāhamihira* gave them as

fractions of the original, for convenience). Then the idea of a larger *yuga* was evolved, at the end of which the other planets also came together with them, and repeated their motions. Along with this concept of a very long *yuga* arose the idea that the first part of the *yuga* was a golden age, the second not so very good, the third passable, and the fourth bad, an idea naturally occurring in the minds of ancient peoples. The names *Kṛta*, *Tretā*, *Dvāpara* and *Kali*, used for very good, good, passable, and bad throws of dice, current from vedic times were borrowed and given to these sub-*yugas*. But no definite periods were attached to them, until the astronomers used the concepts for their own purpose, giving a definite number of years to them, as we have seen, and fixing the beginning of *Kali* as an important epoch. The vagueness of the original concept can be inferred from various incidents of different times being taken as occurring at the beginning of *Kali*, by different ancient authorities. For example, the astronomical *Kali* is accepted by most *purāṇas* and given as the date when Kṛṣṇa left off his mortal body, and ascended to heaven. But certain other traditions give the first coronation of Yudhiṣṭhira at Indraprastha, others the Mahābhārata war, and yet others the abdication of Yudhiṣṭhira and starting on the *mahāprasthāna*, as the starting point of *Kali*. The Jain and Buddhistic writers take a period 468 years after the astronomical *Kali* as the time when Yudhiṣṭhira lived and established his Yudhiṣṭhira era (corresponding to 2634 B.C.) Varāhamihira and following him Kalhaṇa, state that the Yudhiṣṭhira era began in 2449 B.C. While the *Śākalya Saṃhitā* and the *Matsyapurāṇa* say that the *Saptarṣis* were at *Kṛttikā* at the beginning of *Kali*, the *Garga Saṃhitā* and other *purāṇas* say that they were at *Maghā* at the beginning of *Kali*. Thus there is a difference of about 700 years in the times given by them for the *Kali* epoch. Thus we see that the early astronomers fixed the *Kali* epoch arbitrarily, as a

convenient starting point for their calculations, and others accepted and followed it.

3. The Concept of Precession of the Equinoxes

In most Hindu *siddhāntas* the precession of the equinoxes has been given as oscillatory about the fixed first point of *Aśvinī* and not as a continuously regressing phenomenon, which it is. The later *Sūryasiddhānta* says that there are 600 such to and fro full oscillations in the *yuga*, i.e. the period of oscillation is 7200 years. According to this the equinox coincided with the First Point of *Aśvinī* at the *Kali* epoch. During the first 1800 years of *Kali* it uniformly moved forward by 27°, i.e. to a point 20' beyond *Bharaṇī* and then in the next 1800 years, i.e. at 3600 *Kali*, (corresponding to 499 A.D.), it regressed to the First Point, and from that time on is continuing the regression till in 2299 A.D. it will stop at 27° behind *Aśvinī* and begin moving forward again (*Sū. S.* III. 9-10). From this we can also see that the rate of motion is 3° per 200 years. Some works give the amplitude of oscillation as 24° instead of 27°. Some like *Muñjāla* quoted by Bhāskara II, give the phenomenon as continuously regressing and making 199669 revolutions in a *kalpa*, i.e. about 22000 years per revolution, giving a precession of about 1' per annum. The earlier *Siddhāntins* like Āryabhaṭa I and Brahmagupta do not mention any motion of the equinoxes. But Bhāskara I, a senior contemporary of Brahmagupta and follower of Āryabhaṭa, discusses it in his *Āryabhaṭīya Bhāṣya*, (pp. 174-76 of R 14850, Oriental Mss. library, Madras), mentions the *Romakas* who assert there is precession, characterises them as people who do not know the truth, (*aviditaparamāṛthāḥ*) and dismisses their view saying it is a temporary and unnatural phenomenon of the nature of a portend. But Varāhamihira who came before him and just after Āryabhaṭa, is more positive and says, (*P.S.* III. 21, 23 *Br. S.* III 1-2) "Certainly in the ancient times the

Winter Solstice was at *Śraviṣṭhā* and the Summer Solstice at the mid-point of *Āśleṣā* as mentioned in the earlier *Śāstras*, (like the *Vedāṅga Jyotiṣa*). Now they are at the beginning of *Makara rāśi* and *Karkaṭaka rāśi* respectively. If they have moved away even from these points at a later period, the amount of this unnatural change can be found out by observation of the sun's shadow and examination." Not knowing the rate of precession, he does not mention calculation for this purpose. For the matter of that even Bhāskarācārya II, advocates finding the amount of precession by observation, since there is difference of opinion among the *siddhāntins* about the rate of motion. Earlier than Varāhamihira, excepting the *Romaka* which is clearly of foreign origin, no Hindu astronomical work mentions precession, though its effect on the seasons was observed even in the vedic times, as I have already mentioned. Even the shifting of the Vernal Equinox from *Mṛgaśiras* to *Rohiṇī*, and from *Rohiṇī* to *Kṛttikā* had been observed in the Vedic period. But this had not resulted in the idea of the tropical year as such, as distinguished from the sidereal year, so strong was the hold of the sidereal year upon the astronomical and calendric system of the age. This must be the reason why, even after knowing the continuous motion of the equinoxes, they considered the phenomenon oscillatory, i.e. something temporary which would rectify itself by an equal motion in the opposite direction. Even if a *Muñjāla* gives it as continuously regressing, it can have no value unless he had adequate reason to know it for certain (and he had no reason, it could only be a guess), for the real cause of the phenomenon, viz. the behaviour of the earth rotating like a spinning top in resisting the pull of the sun, moon and planets on the extra matter on its equatorial bulge, this was not known at that period.

Another fact, also indicated earlier must be mentioned here. We have seen that the Hindu sidereal

year, being more than 8 *vinādis* longer than the correct sidereal year, the point of *Aśvinī* itself has a progressive motion of more than 8" per annum. Since the correct precession is about fifty and a quarter Seconds, the rate of precession with respect to the Hindu First Point must be more than $58\frac{1}{4}$ ", since the *siddhāntas* advocate getting the precession by observation of the sun's shadow. Accordingly, *Muñjāla* and the later works give a rate of precession nearly equal to 1' per annum, which is quite proper. It would be a mistake to suppose as far as ancient Hindu works are concerned that the nearer their rate of precession is to $50\frac{1}{4}$ " the more correct it is, e.g. it would not be proper to commend the *Sūryasiddhānta* for its rate of precession of 54" per annum, on the ground that it is so near $50\frac{1}{4}$ ".

4. The Eccentric and Epicyclic Theories of Planetary Motion

From the time of the Old *Sūrya Siddhānta*, i.e. the one condensed by Varāhamihira in his *Pañcasiddhāntikā*, the eccentric and epicyclic theories have come to be used in Hindu astronomy in computing and geometrically explaining planetary motion. The ancient *Vedāṅga Jyotiṣa* gives only the mean sun and moon, and problems connected therewith, like the ending moments of *tithis* etc. Even these are rough, keeping in view the convenience of a civil calendar. But the religious rites like *darśapūrṇamāsa* require the correct computation of things, like the day when the moon sets or rises heliacally. Certainly by long observations the ancient priests must have arrived at rules to rectify the positions of the sun and the moon got by the rough rules, so that at least for a few synodic revolutions forward, the heliacal setting and rising of the moon etc. were computed tolerably correctly. But even in the later *Jyotiṣa saṃhitās* like the *Garga saṃhitā* we do not see rules for the computation of the true sun and moon, not

to speak of the planets. It is so in the *Paitāmaha siddhānta* also (the one condensed by *Varāhamihira*). It is in the *Vāsiṣṭha siddhānta* of the *Pañcasiddhāntikā* that we see for the first time distinction made between mean and true sun and moon. A rough and empirical rule is given to get the true sun. The variation in the moon's motion is recognised, but this is supposed to increase and decrease uniformly over its mean motion, during its anomalistic revolution, and the true moon is computed by arithmetically summing the varying motions, the excess or defect over the mean answering for the equation of the centre. Of course this cannot be very correct, since the variation is really with the cosine of the anomaly, and the equation of the centre is a function of sine anomaly. This was recognised and used in the later *Pauliṣa* and *Romaka siddhāntas*, as seen from the empirical values given being roughly proportionate to the sine of the anomaly. This must have suggested that the equation of the centre is truly proportionate to the sine of the anomaly, and was accepted as such.

A natural tendency would be to picture the motion by a graphical representation. Circular motion, the synodic revolution of planets, and the representation of the equation of conjunction of planets by circles, with which the astronomer's mind was already familiar, readily suggested the epicycle, with its centre moving on a deferent, as a means of representing the equation of the centre, and this picture was adopted, as first seen in the Old *Sūrya siddhānta*. Very soon an exactly equivalent method of representation was discovered, viz. making the body move uniformly on a circle, called the eccentric circle, the centre of which is off the centre of the earth in the direction of the point of slowest motion, by a length equal to the radius of the epicycle. The *Aryabhaṭīyam* gives both representations.

While both can give the same locus of the moving body forming the orbit of the body, the eccentric would

seem more real, because in the former an imaginary non-physical mean body, moving on the deferent would not be acceptable to the mind, and would appear only as a devise to represent the effect of the equation of the centre graphically. This would mean that the distance of the actual planet from the earth's centre called the *manda karṇa* (hypotenuse of Eq. cent) need not be taken seriously. This explains why most *siddhāntas* like the *Siddhānta Śiromaṇi* do not use the *karṇa* in finding the equation of the centre. On the other hand the School of Āryabhaṭa, on the plausibility of the motion of the actual body to be on the eccentric, has taken the representation more seriously, and used the *karṇa* in the epicyclic representation also, as being more logical. Incidentally, this would accord better with the actual orbit of the body, which is an ellipse. The equation of the centre of a body moving in an elliptic orbit is of the form $-2e \times \text{sine anomaly} + 5/4e^3 \times \text{sine } 2 \text{ anomaly}$ —etc. where e is the eccentricity of the ellipse, and the anomaly is reckoned from the apogee (or aphelion). Taking the *karṇa* into consideration would give part of the second term also, while neglecting it would give only the first term. (This has been shown by me in the paper, 'A Historical development of certain Hindu Astronomical processes,' presented at the seminar of the Indian National Science Academy, held in 1968, and published in the *Indian Journal of History of Science* Vol. IV 1-2, 1969) (pp. 46-75 in this volume).

A question may now be asked: as early as c. 225 B.C. in Greece, Apollonius of Perga developed the theories of epicycles and eccentrics. The Old *Sūrya siddhānta*, using the epicycle, must be later and the *Āryabhaṭīyam* mentioning both theories, is certainly later. There is no doubt that there was contact between India and Greece from earlier times. Did the Hindu astronomers borrow these ideas from the Greeks, or did these occur to the Hindus naturally, as it had occurred to

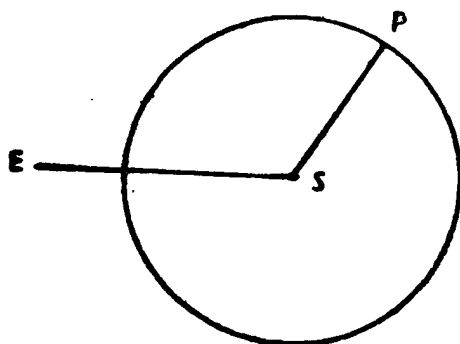
the Greeks? We cannot answer this question with certainty. But an independent origin in India seems more probable, when we see that the clearly western and earlier *Pauliṣa* and *Romaka* did not have either of these theories, that the Hindu constants were different and better in general, that there was already the analogy of representation of the equation of conjunction, which must occur to the astronomer naturally to explain the motion of the star-planets as seen from the earth, and that the Hindus, having already a theory of the variant motion in the form of the pull or repulsion of the apogee on the planet, required the epicyclic representation as a geometrical model, for which there would certainly be an urge.

Before proceeding we must understand in terms of modern astronomy the meaning of certain Hindu terms used in the context of planetary motion. *Madhya graha* is the mean planet in its own orbit round the earth, when applied to the moon. When applied to the sun it means the mean longitude of the earth round the sun, plus 180° . In the case of Mars, Jupiter and Saturn, it means their mean heliocentric longitude, and the longitude of the superior conjunction in the case of Mercury and Venus, the mean sun being that point. *Śīghra graha* means the mean planet in its own orbit round the sun in the case of Mercury and Venus, and the superior conjunction in the case of Mars, Jupiter and Saturn, the mean sun being that point. According to modern astronomy, and correctly too, the equation of the centre is to be applied to the mean body in its own orbit round its central body. This will give the true longitude in the case of the sun and the moon, and the true heliocentric longitude in the case of the five planets Mercury etc. The equation of conjunction is then applied to these five planets, using the true sun to convert their heliocentric longitudes into geocentric longitudes. Any instruction given in Hindu

astronomy must in essence follow this procedure, or else the instruction would be wrong.

Now, Hindu *siddhāntas* instruct what amounts to applying the equation of the centre, in the case of Mercury and Venus, to the mean sun instead of their own mean longitude, and this is wrong. Further instead of taking the true sun as the superior conjunction the mean sun is so taken. On account of these and the error in the equation of the centre itself, the computed true planets did not agree with observation, and various attempts were made to correct these errors, with very little success, since they rarely touched the real cause. For example, the *Āryabhaṭa* and New *Sūrya Siddhānta* schools made the radius of the epicycles vary with the anomaly. Before applying the equation of the centre and the equation of conjunction, the Old and New *Sūrya siddhānta* schools made a preliminary operation, and applied a half-equation of conjunction and half-equation of the centre respectively, to the *madhya* of the planet. The *Āryabhaṭīya* varied this by instructing the half equation of the centre to be done first, and dispensing with it altogether in the case of Venus and Mercury. The *Siddhānta Śiromaṇi* instructed full equation of the centre, and full equation of conjunction, for the preliminary operation too, except in the case of Mars, where repetition of the whole set of operations is enjoined using half of both equations in the preliminary work. This disagreement among the *siddhāntas* itself shows that the preliminary operation is an unessential hotchpotch.

We have said that the application of the equation of conjunction converts the heliocentric longitude into the geocentric. This can be readily seen in the case of Mercury and Venus (see fig). If the line joining the earth and sun be taken as unity, the sun being the planet's mean longitude in these two cases, the epicycle of conjunction round the sun is the planet's orbit and the radius of the epicycle is the distance of the planet from



the sun, taking the distance of the sun from the earth as unity. Since the ratio of the radius to the circumference in any circle is constant, the degrees of epicycle $\div 360^\circ$ will give the ratio of the distance of the planet to unity. (If the epicycle varies we can take the mean.) In the case of the superior planets, viz. Mars, Jupiter and Saturn also, since the application of the equation of conjunction converts the heliocentric planet into the geocentric, the geometrical representation must be true, which can be shown thus: Interchange *P* and *E* in the figure. This is a representation of the earth circling the sun, as seen from the planet, and the earth will be viewed always near the sun, even as Mercury and Venus are seen always near the sun as viewed from the earth. By comparison, we can now see that if we take the distance of the earth from the sun as before, the degrees of epicycle $\div 360^\circ$, gives the reciprocal of the ratio of the distance of the superior planet from the sun to the distance of the earth from the sun. In the table below, these ratios are given. They can be compared with the correct ratios of modern astronomy given.

Planet	Modern	<i>Siddhānta</i> <i>Sīromaṇi</i>	Old <i>Sūrya</i> <i>Siddhānta</i>	New <i>Sūrya</i> <i>Siddhānta</i>	<i>Ārya-</i> <i>bhaṭṭayam</i>
Earth	1.00	1.00	1.00	1.00	1.00
Mercury	.39	.37	.37	.37	.38
Venus	.72	.72	.72	.73	.73
Mars	1.52	1.48	1.54	1.54	1.54
Jupiter	5.20	5.29	5.00	5.07	5.16
Saturn	9.55	9.00	9.00	9.11	9.41

We see the agreement is fairly close, and the *Ārya—bhaṭṭiyam* comes closest. If the Hindu *siddhāntins* had only taken the geometrical representation more seriously, and if they had not been wedded to the geocentric hypothesis so strongly, they could have noticed the correct ratios of the distances of the planets from the sun, and even postulated the heliocentric hypothesis, as Aristactūs of Samos had done before, in c. 280 B.C. They would have also seen that their postulate of equal linear motion for all planets as well as the sun and the moon, is wrong. (The Hindus estimated the distance of the moon to be about 65 times the earth's radius, fairly correctly by using the moon's horizontal parallax. But they calculated the sun's distance to be about 13.4 times the moon's distance, since their postulate of equal linear motion is equal to postulating that the distances are proportionate to the periods of revolution. Really the sun's distance is about 390 times the moon's. This enormous real distance was not realised even by Copernicus, who gives it as about 25 times that of the moon. The Greek Aristactus of Samos had estimated it to be about 20 times, using a method ingenious but useless, on account of the impossibility of making correct measurements. According to the Hindu theory, Jupiter will be 12 times and Saturn 29 times distant as the sun. Quite arbitrarily, they gave a distance to the stars, i.e. the stellar sphere also, as 60 times that of the sun, and for the celestial sphere, 4320000000 times, practically infinity. The distance they gave to the stars is one reason why they clung to the geocentric hypothesis, for they thought that if the earth moved round the sun there would be a large stellar parallax that could be observed, but no such parallax was observed. But in fact there are stellar parallaxes. On account of the stars' enormous distances these are so small that only modern instruments can measure them. It was left for Copernicus to propound the heliocentric hypothesis anew. Whether it is the epicyclic theory or

the eccentric theory, whether it is the geocentric theory or the heliocentric theory, one is as good as the other as far as the result is concerned, and this was known to the ancients, both Greeks and Hindus. As long as the real reason why the sun should be considered the central body, viz. the strong pull it exerts over the other bodies, is not known, the only recommendation for the heliocentric hypothesis is its simplicity. But simplicity can masquerade as truth, and the simpler hypothesis can appear to the human mind as more real, for nature's laws are characteristically simple. It should be noted that Copernicus, in propounding the heliocentric hypothesis in his '*Revolutionibus Orbium Clestium*', published in 1543 A.D., very carefully explained that he was adopting the theory only for its simplicity, for if he had not done so he would have lost his valuable influence with the Church, and might have had even to face the inquisition like Galileo later.

5. The Concept of The Sidereal Day

Hindu astronomy from the time of the *Vedāṅga Jyotiṣa* has conceived the sidereal day as being caused by the rotation of the stellar sphere round the earth from east to west, the time of one rotation being one sidereal day. Classical Hindu astronomy supplies a reason for the rotation, by assuming a wind called *Pravaha* in the upper regions, blowing the stellar sphere round. But Āryabhaṭa seems to have felt that the same result can be obtained by assuming the earth to rotate on its axis, west to east. This can be seen from his *Āryabhaṭīyam-Gīṭikā* (1), where together with the number of eastward revolutions of the sun, moon etc. in a *yuga*, he mentions the earth and the number of sidereal days in the *yuga*, (1582237500), as the number of its eastward revolutions. This is confirmed by his statement in *Golapāda* (9): "Just as a man moving along in a ship sees the non-moving mountains moving in the opposite direction, so also the non-moving stars

seem to move directly westward in the sky above the equator.” But he seems to contradict himself when he says in the very next verse, “Being blown by the *Pravaha* wind for the sake of rising and setting, the stellar circle moves westward above the equator at the rate of 1' per *prāṇa* (i.e. 4 seconds of time). In *Gītikā* (4) too he says, “The star moves 1' per *Prāṇa*.” How are we to resolve the contradiction? His successors, even from Bhāskara I onwards have taken the latter as his own view, and interpreted the former so as to agree with the latter. The stock argument of the generality of astronomers against a rotating earth, (like the impossibility of birds leaving their nests reaching them again) seem to have made them interpret Āryabhaṭa thus. But it seems that the former is Āryabhaṭa's conviction and the latter is what he gives as a *pūrvapakṣa*, i.e. the opinion of others, in the exceptional *sūtra* style. For Brahmagupta, following only a century after, in the *dūṣaṇādhyāya* of his *siddhānta* condemns Āryabhaṭa for holding this view of a rotating earth. He even reads *Gītikā* (4) as *Praṇenaitikalāmbhūḥ* meaning, “The earth moves 1' per *prāṇa*”, and probably this was Āryabhaṭa's original reading. Further whom were the generality of astronomers condemning, if it was not Āryabhaṭa and people like him? Thus the scientific mind of Āryabhaṭa, always seeking simpler assumptions, must have arrived at the view that the rotation of the earth is simpler to assume, like the Greek, Heracleides of Pontus before in c. 350 B.C. But he could not have borrowed this idea from Heracleides since the latter's view had died out even in Greece long ago.

6. Certain Specialities in the use of Trigonometry in Astronomical Work

In Hindu astronomy, trigonometrical functions came to be used only after the time of the *Paulīśa siddhānta* condensed in the *Pañcasiddhāntikā*. Upto the time of the

saṃhitās and *Paitāmahasiddhānta* there was no need for them, when only mean planets were given and rough rules to compute the shadow, *lagna*, time, etc. were considered sufficient. The later *Vāsiṣṭhasiddhānta* found the equation of the centre as mentioned before by an arithmetical summation. To get the point of contact in the lunar eclipse, it used the related degrees of angles themselves instead of their sines (*P.S.* VI. 7-8). The *Pauliṣa* also does not seem to have used the sine etc. Here, in the case of the sun, empirical values for 30°-intervals are given to make it true. Its moon is common with that of the *Vāsiṣṭha*. To find the moon's latitude it uses the degrees of argument of latitude, instead of its sine, though this is not explicitly stated. In dealing with the solar eclipse in chapter VII. 2-4, three rules are given to represent the parallax correction for latitude as a correction to the moon's nodes, and in all, degrees are used instead of their sines as multipliers. Only in one place, (VII.1), is the sine used to correct the time of conjunction in longitude for parallax. But this is most probably Varāhamihira's own, to secure tolerable accuracy. The same verse occurring as VII. 9, in connection with the *Romaka*, confirms this. As for chapter IV, where a table of *R* sines is given, and spherical trigonometrical problems are dealt with, that is the author's own, and, if it must go with any *siddhānta*, it must be the *Sūryasiddhānta* there. We see evidence of the use of sines etc. from the time of the *Romaka* and in *Sūryasiddhānta* mentioned above and the later classical works beginning with the *Āryabhaṭīyam*.

As in modern trigonometry, the Hindus use the *vyārdha* or half-chord, calling it *vyā*, (a synonym of the word *śiṅginī*, from which the word sine is derived), unlike the Greeks who used the full chord. The Hindu genius which invented the place-value, decimal notation, and for its sake, the symbol zero, saw the convenience in using

the half-chord, and used it. But, instead of taking the radius as unity like the moderns, the Hindus measure it in certain natural or assumed units, so that the half-chords and arcs are measured in the same units. So, if the modern word 'sine' is to be used, the Hindu half-chords are to be called R sines. The *Āryabhaṭīyam* is the first extant work in which an R sine table is given. Āryabhaṭa uses a natural measure, i.e., he takes the circumference to be 21600 units, so that a minute of arc is the unit. Then the radius will be 3438 units, being the number of minutes in a radian. Many *siddhāntas* adopt this measure. But some, for the sake of convenience, assume an arbitrary value for the radius. For example, Varāhamihira assumes it to be 120 units, because in all computations with R sines etc. R occurs as a multiplier or divisor.

Usually the R sines are tabulated for intervals of $3^\circ 45'$, thus giving twentyfour of them in all for the quadrant. The *Vaṭeśvarasiddhānta* tabulates 96 R sines (plus two more near 90°), to secure greater accuracy in interpolation. With the R sine table, an R versine table is given by most *siddhāntas*, and R cosines are got by subtracting these from R , or taking the R sines of the complements. Āryabhaṭa gives a geometrical construction to derive the R sine of $\frac{\theta}{2}$, when R sine θ and its versine is known, by using the Pythagoras theorem, (*Ārya. Gaṇita*. 11). This can be carried to a very small fraction of an angle where R sine θ is indistinguishable from $R\theta$. He has used this to find the circumference of a circle, the radius being given. He says: When the diameter is 0000 units, the circumference is 62832 units, thus giving $\pi = 3.1416$, which is correct to four decimal places. Varāhamihira gives two formulae to find the R sines, (*P.S.* IV. 2-5) of successive half-angles, and thence all the twentyfour, tabulated. Using the first tabulated R sine $7' 51''$, he could have arrived at $\pi = 3.14$, which is correct to 2 places of decimals. Using the value of $R=3438$, the

circumference being 21600, the New *Sūryasiddhānta* could have found $\pi = 3.1414$, and so also Brahmagupta using his radius. But all these three give what comes to $\pi = \sqrt{10} = 3.1623$, so incorrect, it passes my comprehension why. If, for ease of computation, the fraction $\frac{22}{7} = 3.1429$ is simpler to use, and far closer to the correct value. The *Āryabhaṭīya*, *Gaṇita*. 12, gives an alternative method to compute the *R* sines, and the *Sūryasiddhānta* repeats it. It depends on the fact that the second differences of the *R* sines vary as the *R* sines. But the constant divisor of the *R* sines, the number 225, seems to have been assumed empirically by a supposed identity with the first *R* sine. If it had been found from *R* sine $(\theta - a)$, *R* sine θ , and *R* sine $(\theta + a)$, by calculation, they would have arrived at the correct 234. Bhāskarācārya has appended a whole section for getting the *R* sines etc. by various methods reminiscent of modern trigonometry.

These *R* sines etc. are used in computing the equation of the centre and the equation of conjunction which involve the solution of plane triangles, and in solving spherical triangles, which occur in problems like finding the right ascension and declination, and the polar latitude and longitude of heavenly bodies when their latitude-longitude co-ordinates are known, as also in problems requiring the solutions of the spherical triangle SPZ, (S representing the position of the body, P being the pole and Z being the zenith), problems like finding the length of day-light, orient or meridian ecliptic point, amplitude, azimuth, zenith distance, and hour angle of bodies, from which shadow, time, etc. are found. But, instead of using the spherical triangle formulae directly, Hindu astronomy uses plane right angled triangles whose sides correspond to the sides of spherical right angled triangles. These plane triangles are formed by perpendiculars dropped on to the horizontal, prime vertical, and meridian planes, as the sides containing the right angle. These are

all similar to the right-angled triangle formed with the equinoctial shadow and the twelve-unit gnomon, as its base and perpendicular, since in all of them the latitude of the place is involved, and therefore in every one the corresponding sides and hypotenuse are proportional. These are eight in number and are called *akṣakṣetras*, the equinoctial shadow triangle being the archetype. Problems are solved using the proportionality of these triangles. *Siddhānta Śiromaṇi*, *Gaṇita*, *Tripraśna*, 13-17 define these, and the *Vāsanābhāṣya* thereon explains them exhaustively. From the earliest *siddhāntas*, viz., the *Āryabhaṭīyam* and the *Pañcasiddhāntikā*, it is this method of solving spherical triangles that is used, and so it must have been developed by the Hindus earlier still, and is a special feature of Hindu astronomy.

THE VĀSIṢṬHA-PAULIṢA VENUS IN THE PAÑCASIDDHĀNTIKĀ OF VARĀHAMIHIRA

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In 1889 Thibaut and M. M. Sudhakara Dvivedi (TS) edited the *Pañcasiddhāntikā* with translation and notes. In the XVIII chapter dealing with the star-planets according to the *Vāsiṣṭha* and the *Pauliṣa siddhāntas*, they have committed several serious mistakes, professing also ignorance of the meaning of several verses. In my paper, 'Some mistakes and omissions of Thibaut and Sudhakara Dvivedi in their edition of the *Pañcasiddhāntikā*', presented at the First World Sanskrit Conference, 1972, (VIJ. XI, 1973, Hoshiarpur),¹ I pointed out the errors without going into the details. About the same time, O. Neugebauer and D. Pingree (NP) brought out an edition of the *Pañcasiddhāntikā*. In that they have improved upon TS's interpretation in some places, but committed worse mistakes in others. TS and NP have also failed to understand how the equation of the centre has been computed and applied to the equation of conjunction of Jupiter and Saturn. Since they must do something with the verses giving this, they have altered the verses in all sorts of ways, to yield what they thought the meaning might be. Even in the case of Venus, whose computation has been simplified by neglecting the small equation of the centre, they have committed several errors. I shall begin with the correct interpretation of the verses dealing with Venus, and point out their errors.

Verse 1. *hitvā munijala (? jalamuni)candrā-
dyu(n dyu)gaṇāḍ vedāṣṭa-bhūta-hṛtalabdhāḥ |
śukrodayā guṇāptaiḥ
sārdhāḥ pañcālino bhogāḥ ||*

1. See pp. 102-117 in this book.

Verse 2. *kanyāṃśāḥ śa(n śa)dvimsati-
mitvā śukro'parēṇa yātpḥu(tyu)dayam |
udayaikadaśabhagān
dīneṣu datvā tatastā(ścā)rāḥ ||*

1. Subtracting 174 from the days from epoch the quotient got by dividing the remainder by 584 are the heliacal risings of Venus. Its motion during the periods is 7^r 5° 30' 20'', each.

2. Having gone to 26° of Virgo, i.e., at, 5^r 26°, Venus rises in the west (for the first time after epoch). Adding an eleventh of the quotient (given in verse 1) to the remaining days, the motions (are to be taken from the Table given in verses 3-5).

(The translation is according to my emended text. Material emendations, which require explanation, are given with a question-mark (in brackets). Emendations of mere scribal errors are not).

174 days after epoch, Venus rises heliacally in the west. We have emended *munijala* as *jalamuni* because 147 will not agree with the longitude of Venus at rising given as 5^r 26°. If Venus is 5^r 26°, the sun must be 5^r 18°, to satisfy the 8° given for Venus's heliacal rising in verse 58. But since at epoch the sun is in the neighbourhood of 358°, it can be only near 4^r 25° after 147 days, i.e., 38° from Venus. After 174 days from epoch, the sun would be near 169° according to the *Vāsiṣṭha-Paulīśa*.¹ This gives the elongation as 7° instead of the required 8°. But this small discrepancy can be put to an accumulated error in computing from the original. TS have not felt the need for this emendation because they have emended *kanyāṃśān* into *kālāṃśān* and omitted the necessary *kṣepa*, i.e., the constant equal to 5^r 26°, for beginning the

1. 'The epoch of the Romaka.....', *Indian Journal of History of Science*, 13 (1978) ii. 155-58 (*vide infra*).

motion. They have not understood the meaning of the word *kālāṃśān* which they have brought in. It is the same as the 8° mentioned above, for Venus. NP have interpreted *kanyāṃśān* correctly. But since they have kept 147 days to be deducted intact, they find a serious discrepancy expressed by them on page 124 of Part II. However they derive satisfaction from the fact that the September 10th position of the sun would agree with the time of Venus's rising and their longitudes. This at least must have shown them that the rising takes place only more than 20 days later. They have made all sorts of unnecessary emendations, but they have failed to do this necessary one.

We can infer from the instruction to add an eleventh of the quotient, that one synodic revolution takes $583\frac{10}{11}$ days. In this period the sun has moved 1 revolution $7^{\circ} 5' 30' 20''$ and Venus, 2 revolutions $7^{\circ} 5' 30' 20''$. From this we can infer that the sun takes 365-15-25 days for a sidereal revolution. From the methods given from the other planets also we can see that the sidereal period of the sun used is c. 365-15-30, which is an evidence for the system given here being connected with the *Pauliṣa* also, as in the case of the moon.

TS have unnecessarily emended *bhogāḥ* into *bhāgāḥ* and taking *sārdhāḥ* to mean 'together with', instead of 'with half', have given a motion of $7^{\circ} 5' 20'$ per synodic period. They are unaware that this would make the sidereal year c. 365-22 days, so wrong. The error of $10' 20''$ in Venus would accumulate by 1° every nine years.

NP interpret *sārdhāḥ* correctly, but take *guṇāptaiḥ* to mean 'with $\frac{1}{3}$ degree' and give $7^{\circ} 5' 50'$, which would make the sun's sidereal year c. 365-3-0, so very wrong. By this the error in Venus would accumulate by 1° every 5 years.

Next, the days for segments of motion in the synodic cycle are given by 3-5.

Verse 3. *śaṣṭitrayeṇa vedāgniyaamayutām aṁśasaptatiṁ bhuṅkte |*
ardhā (? arthā) śṣa [kair dvā] viṁśati (? triṁśataṁ)
viṁśatyai (tyā) [vi] stri(tri) bhis sapādāṁśam (? dārthān) ||

Verse 4. *vakram atas tithibhir dvau*
pañcabhir evaṁ tato'parāstamitaḥ |
daśabhiḥ prāguditas syān nakhaiś ca jaladhīn mitā (tān)
gatvā ||

Verse 5. *anuvakri parigatvā*
viparitam astamaityaityāyā(ṇdryām) |
śaṣṭyāṁśapañcasaptatiṁ itvā 'parato bhṛgur dṛśyaḥ ||

3. In three periods of 60 days Venus moves 74°, 73°, 72° respectively. In 40 days he moves 32° and in 17 days, 5½°.

4. From here retrograde motion (begins). In 15 days these are 2°; in 5 days the same, i. e., 2°. Then, setting in the west, it rises in the east after ten days. Venus is in follow-up retrograde for 20 days, moving 4°.

5. Then continuing the (direct) motion round, in the order of days and motions reversed, Venus sets in the east. Then moving 75° in 60 days, it becomes visible in the west.

The argument to be used in the above table of motions are the days left over, together with the eleventh of the quotient, as mentioned. It can be seen that in my emendations of some of these I have done very little violence to the text. I have been guided in these by the actual motion that must have been observed, putting it to observational or other error, where the numbers are clear, but deviate from the actual. The ratio of Venus's distance from the sun to the earth is *c.* .72 as given by all Hindu and modern astronomy, and this I have used to compute the segments of actual motion for comparison.

TABLE I
Motion on the synodic circle (computed)

Superior conjunction		Retrograde begins		Retrograde ends	
30d.	37½°	9d.	—2½°	243d.	258°
Rising West		5d.	—2½°	Setting East	
60d.	74°	Setting West		30d.	37½°
60d.	73°	5d.	—2½°	Superior conjunction	
60d.	68½°	Inferior conjunction		Total 514d.	575½°
27½d.	26½°	5d.	—3°		
12½d.	9½°	Rising East			
17d.	7°	5d.	—2½°		
6d.	½°	9d.	—2½°		

Another guide is that the days and motions from the superior to the inferior conjunction must add up to half of the whole, i.e., 292 days, and 287½°, since the equation of the centre has been dispensed with.

72° motion for the third 60 days is about 4° in excess. For the next 40 days the motion has to be 36°, and I could have filled up the lacuna by [*khaiṣṣat*] instead of [*khairdvā*] to get this. But the *siddhānta* seems to have compensated the earlier 4° excess by the 4° defect here, which of course is an error. From this we can see that the emendation of '*dvimśat*' into '*triṃśat*' is necessary lest the motion be reduced to 26°, which is too small.

Next, 1½° for 23 days is too too small to be correct. Further, the correct total of days and degrees clearly given by the numbers will be spoiled by this. So I have given the meaning as 5½° in 17 days, which fairly agrees with the actuality, by introducing a (*vi*) for the defect of two *mātrās* and emending *sapādāṃśam* into *sapārdhān*.

Since the motion is only 15' for the 6 days near the stationary point as seen in the actual, the

siddhānta is justified in combining this with the $-2^{\circ} 13'$ for the next 9 days and giving -2° for 15 days. But, for the next 5 days, the motion is about $-2\frac{1}{2}^{\circ}$ and not -2° , and this is an observational error. For the 5 days forming the half period of invisibility till the inferior conjunction, the actual motion is about -3° but we are constrained to make it -2° for agreement with the other numbers, especially when it is left to be understood, no motion being given by the text. A glance at the comparative table will make everything clear.

TS have made the serious mistake of thinking that the segments given begin with the superior conjunction instead of the rising on the west (vide the scheme given in the Sanskrit Commentary, p. 123). By the total of 610 days, and $610\frac{1}{4}^{\circ}$ given, they have given that the rising takes place 26 days after superior conjunction, passing $22\frac{1}{2}^{\circ}$, which is absurd because it should be $37\frac{1}{2}^{\circ}$ in 30 days, i.e. half the time and degrees given for the time from setting to rising. They do not realise that this is the period of the quickest motion. Within the scheme they get the 77° for 85 days, by making a drastic change in the wording of the text. Further, for 85 days in that part, the motion would be more than 91° . Giving $1\frac{1}{4}^{\circ}$ for 3 days near the stationary point is wrong since it should be practically zero. -4° for 5 days in retrograde is too great. But they have given the same -4° for the 10 days in the *ativakra* region where the rate should be the greatest.

As for NP they have correctly interpreted that in the three 60 day periods after rising, the motions are 74° , 73° and 72° , that from setting in the east and rising in the west there are 75° for 60 days, half before the superior conjunction and half after, and that near the inferior conjunction there are 60 days of retrogression and -12° , half of each falling on each side of the point, as given by the text. Adding these we can account for $250\frac{1}{2}^{\circ}$ in 235

days. Since we should get $287\frac{3}{4}^\circ$ for the 292 days from the superior to the inferior conjunction, we have still to account for $37\frac{1}{4}^\circ$ in 57 days. This we must seek in the second half of verse 3. By some likely emendations we can secure this, as I have done. But NP have drastically changed the text as

arthāṣṭakaviṃśatyā viṃśatyam[śahā]sribhis sa pādāmśam,

also sinning against prosody, and given only 28° for the $27\frac{1}{2}$ days, after the third sixty-day period, and $1\frac{1}{2}^\circ$ for the next 3 days, thus, not accounting for 16° and $30\frac{1}{2}$ days.

ardhāṣṭakaviṃśatyā cannot mean $27\frac{1}{2}$, besides being an un-Sanskritic formation. Further, for the $27\frac{1}{2}$ days in that part of the synodic circle the motion should be more than 26° and for the next 3 days, more than $2\frac{1}{4}^\circ$ as can be seen by examining the actual. This error of 16° and $30\frac{1}{2}$ days is doubled for the whole cycle, and the weight of this error of 32° and 61 days has been carried by them to the 60-day period of invisibility and drawn the remark on page 121, Part II: “.....or 54^d and 32° more than for $\Xi \rightarrow \Psi$, a rather implausible conclusion. At any event, the description of the motion of Venus as given in our text seems incomplete”. The footnote here is uncalled for.

VĀSIṢṬHA - PAULIṢA JUPITER AND SATURN IN THE PAÑCASIDDHĀNTIKĀ

Introduction

In chap. XVIII (of the edition of Dr. Thibaut and M. M. Sudhakāra Dvivedi (TS) of the *Pañcasiddhāntikā* (PS) the computation of Jupiter and Saturn follow next to Venus. This is because their treatment is next simple, on account of their small mean motion and equation of conjunction, owing to their great distance. TS and O. Naugebauer and D. Pingree (NP) in their edition of the PS (chap. XVII) have expressed inability to understand the part of the computation where the equation of the centre is obtained and applied, before the application of the eq. of conjunction. Still, they have attempted to interpret the concerned verses, changing the wordings drastically, to yield their fancied ideas. In getting the eq. of conj., too, they have made several mistakes.

As in the case of Venus, here too, the true motion is traced from one heliacal rising to the next. The method of getting the true anomaly of the eq. cent. is similar to that of the moon given by the Vāsiṣṭha in chap. II, and based on the same theory of the uniform increase and decrease of the rate of motion, forming a linear zigzag. Even the same technical term, *pada*, is used here. All these are reminiscent of the Babylonian astronomy of the Seleucid period as I have suggested in my article dealing with Venus. (Ind. J. of Hist. of Sci. Vol. 14, no. 2, Nov. 1979).¹ As between Jupiter and Saturn, their treatment is exactly similar, so that explaining one would suffice for both. Verses 6-13, deal with Jupiter and 14-23 with Saturn. My main aim here is

1. (*vide Supra*, pp. 141-147).

to state and explain the procedure in the computation, a thing not understood by investigators. The verification of the epoch constants depend mainly on comparison with other systems and modern astronomy. So this will be done in a separate paper.

The better reading of the two original manuscripts (printed by TS in the left hand column) has been taken by me. In writing the verses below, the correction of obvious scriptory errors are put in brackets, but the more important ones are indicated also by a question mark.

Jupiter (verses 6-13)

6. विचतुस्त्रिंश द्विगु(द्युग)णं
नाडीभिस्तावताभिरपि च गुरुः(रोः) ।
ह(ह्र)त्वा नवनवदहनै-
तु(रु)दया लब्धास्थि(स्थि)ता दिवसाः ॥
7. उदयनचांश(शे) दत्त्वा
दिनेषु षड्वर्गसंगुणैरुदयः(यैः) ।
एकनवांशि छन्ने(च्छिन्नैः)
व(प)रमिति साष्टादशं शेषम् ॥

6. The days of Jupiter from epoch minus 34 d. 34 m. divided by 399, give the number of risings. The remaining are days (after rising).

7. Add to these days a ninth of the number of risings. Multiply the number of risings by 36, add 18, and divide by 391. The remainder here are *padas*.

We can conclude the following from these two verses : i. At 34 d. 34 N from epoch, the first period from rising to rising begins. ii. The interval between the risings, i.e., the synodic period is $399 - 1/9 = 398 \frac{8}{9}$ days. iii. 391 *padas* make one full sidereal revolution of Jupiter, i.e., 360° of mean motion. In one synodic period, Jupiter moves 36 *padas*. One *pada* = $55' 15''$. 36 *padas* =

36° 9'. At epochal days 34 - 34, Jupiter's longitude is 18 *padas* (=16° 35'.) But NP have taken the 18 given here as degrees. This is wrong. The difference of 1° 25' is too small to show itself in their verification, Table 32, Part II. But for Saturn this *pada* constant is 89, and for Mars, 85. Taking these as degrees have resulted in big differences and puzzled them. See Part II, page 124. iv. In 391 syn. revolutions there are $391 + 36 = 427$ solar sidereal revolutions = 36 Jupiter's sid. revolutions. ∴ one sid. rev. of Jupiter takes 4332 - 22 - 48 days, and one sid. rev. of the sun = 365-15-32 days. The latter being very near Pauliśa's 365-15-30, we conclude that, there too, as in the moon, it is mixed up with the Vasiṣṭha's.

TS and NP give the same interpretation though making more than necessary emendations. In their verification, TS use the rough syn. period of 399 days instead of the correct $398 \frac{8}{9}$, making the sid. period = 4333-35-0.

8. क्रमशो मध्य(ध्य) स्फुटश्च

खण्डो [कार्यो] तथो(यो)श्च विशेष(श्ले)षात् ।

स्फुटहनां द्युषु दद्या-

त्तमध्यत् (न्मध्यात्) सौरे(र्ये)ऽन्यथा हानिः ॥

8. One after another, mean and true segments are to be arranged. Taking their difference, if the true is less than the mean, the difference is to be added to the group of Jupiter's days (left over in the synodic cycle as the remaining days). Otherwise, (i.e., if the true is more, the difference is to be subtracted.

'True' here means 'true as corrected for the eq. of the cent'. How to get these true positions is given in verses 9-11, and the segments are to be got using these. So, this verse seems to have strayed from after verse 11. The mean positions are to be got by using the remaining *padas*, extending the work done in verse 7,

कार्यौ is introduced to make up for the mātrās wanting. सौरे would mean 'pertaining to either Sun or Saturn'. But we are dealing with Jupiter (सुरिः) here. सौर्ये would alone mean, 'pertaining to Jupiter'. One ms. has no द्वि.

Computing and arranging the mean and true segments against each other is to facilitate interpolation to any required day. It will also be useful to prepare an ephemeride. The example worked will make things clear.

TS have expressed doubts about their translation, since they have not understood verses 9-11. They have retained हि, not supplied the wanting mātrās, and not noticed the grammatical error in सौरे. NP have made three drastic emendations, quite unrelated to the lettering of the text, "निहिः", "मण्डलः" and "तन्मध्यखण्डे", though generally following TS.

9. रसविषयकृत्तशशाङ्काः

क्षयखण्डे वि(ख)धृनयः पदे यावत् ।

विषयरसोना(? रसेश) वृद्धौ

जीवः स्यात्पञ्चनवतिशतात् ॥

10. षड्वसुमनवो हानौ

तृतीयखण्डे गुरुस्तु षोडशके ।

पञ्च(? द)गुणिते इयप्रक्रमा-

जिते कलाः पूर्वतोऽभ्युदिते ॥

11. नव सार्धाः कन्यांशाः

प्रथमे खण्डे द्वितीयखण्डे रफु(र्युः) ।

चक्रार्धं च गुणां(?) युगां शाः

दश श(च) कला देवपूज्यस्य ॥

Ver. 9. Jupiter being in the diminishing-motion-sector upto 180 *padas*, there is the constant 1456 (to work with, in order to get the eq. cent-corrected-Jupiter). Being in the increasing-motion-sector in the next 195 *padas* (i.e. 181 to 375), there is the constant 1165.

Ver. 10. Jupiter being in the diminishing-motion-sector (again) in the next 16 *padas*, there is the constant 1486. (After subtracting or adding the *padas* for which we want computation from these numbers, in the respective sectors), multiplying them by the *padas* and dividing by 24, minutes of arc are got, (as the eq. cent. corrected total motion in the respective sector) at the rising in the east (and also thereafter if wanted).

Ver 11. The total of such motion of Jupiter in the first sector is $5^{\circ} 9' 30''$. In the second sector, it is $6^{\circ} 4' 10''$.

Briefly, expressed as formulae, the eq. cent. corrected Jupiter is given by :

- i. If *padas* are from 0 to 180, $(1456 - \text{padas}) \times \text{padas}' \div 24$.
- ii. If *padas* are in the next increasing sector, i.e. from 181 to 375, $(1165 + \text{padas}) \text{padas}' \div 24 + 5^{\circ} 9' 30''$, where the *padas* used are those gone in that sector.
- iii If the *padas* are in the next following sector, i.e. 376 to 391, $(1486 - \text{padas}') \div 24 + 5^{\circ} 9' 30'' + 6^{\circ} 4' 10''$, where the *padas* used are those gone in that sector.

Though the instructions are laconic, comparison with the moon's computation makes things clear. The increasing-motion sector is obviously the 180° from apogee to perigee, where the rate of motion is supposed by this siddhānta to increase uniformly from a minimum to a maximum. The apogee is at 180 *padas* ($= 166^{\circ}$) and the perigee at 376 *padas* ($= 345^{\circ}$). The last 16 *padas*, continued by the first 180 *padas* form the diminishing half circle where the rate of motion diminishes uniformly from the perigee to the apogee. Differentiating the formula, $(\text{constant} \mp \text{pada}) \text{pada}' / 24$, the increase or decrease in the rate of motion is found to be $\frac{2'}{24} =$

1'/12 per *pada*. There may be a small hiatus at the junction, apogee and perigee, owing to the unequal division of 391 into 196 and 195, to avoid half *pada*. But the average of the rates at apogee and perigee, $(1165' \text{ and } 1486')/24 = 55' \frac{1}{4}$, agrees with the mean motion forming one *pada*. (Incidentally, this justifies our amendment of विषयरसोना into विषयरसेशाः. There are other justifications also, as we shall show later). Further, the first sector being a continuity of the third, the rate during the first *pada* in the first sector must follow next to the rate during the 16th *pada* of the third sector. Since $1486'/24$ is taken as the motion of the first *pada*, the motion of the 16th is $(1486-30)/24 = 1456'/24$. This must be the commencement of the third sector, and this is what is given. We can also see that the fastest rate, (at perigee), is $1486'/24 = 62'$, and the slowest, $1165'/24 = 48' \frac{1}{2}$, (at apogee), giving the mean value $55\frac{1}{4}$, of the *pada*, already found. But the rate for the 196th *pada*, ending which there is the apogee, is, $(1486 - 195 \times 2)/24 = 1096'/24$. But the minimum motion falling at apogee is given as $1165'/24$. This hiatus must also be due to the fact that the $2'/24$ increase in the rate per *pada* is only approximate, and the actual is a little less than $2'/24$. But the formulae are so given that the total of the three sections must be exactly 360° . Thus: the total of the first sector is $(1456 - 180) 180'/24 = 5^\circ 9' 30'$, as given. The total of the second sector is, $(1165 + 195) 195'/24 = 6^\circ 4' 10'$. The total of the third sector is, $(1486 - 16) 16'/24 = 16^\circ 20'$. These add up to 12 rāśis, exactly, as they should. Incidentally, this justifies my emendation of गुणांशाः into युगांशाः, पञ्चगुणिते into पदगुणिते, and giving the meaning of त्र्यष्टक as $3 \times 8 = 24$. The justification for correcting विधृतयः into खधृतयः to get 180, and रसोना into रसेशा to get 1165, are also reinforced by this perfect agreement found here.

TS and NP also give खधुनयः, seeing the reason for that. TS emend रसोनाः into रसेना (=1265), which will give the total $6^{\circ} 17' 43''$, far from the correct $6^{\circ} 4' 10''$. The text itself gives $6^{\circ} 3' 10''$, one degree off. TS give 6 rāsis exactly, not knowing the peculiarity of this Siddhānta. Using चक्रार्धे thus, they are left with गुणांशः दश च कलाः. This they interpret as 13° (wrongly, for it can mean only 30 or 103). Emending दश च कलाः into परशक्ले, they say that this 13° is the total motion of the third sector. They do not realise that the 16 *padas* of the third sector is near perigee, and the total motion must be greater than the mean motion, $14^{\circ} 44'$. Not knowing the nature of the method here, they think that the total of the third sector also should be given. It has no use, and Varāhamihira has not given it.

About पञ्चगुणिते त्र्यष्टकभाजिते, I have emended पञ्च into पद्, to delete the one mātrā in excess, and to give the agreement already seen. त्र्यष्टक is 24, as already said. TS retain the पञ्च, but emend त्र्यष्टक into अष्टक, making it $5/8$, leading nowhere.

As for NP, they generally follow TS's emendations. But, for the divisor 8 they suggest the alternative 83 (त्र्यष्टक). Unlike TS, they realise that the three sectors must add upto 12 rāsis and make their own emendation of the last part of verse 11, as द्विगुणांशः दशा सदलाः, interpreting it as $20^{\circ} 30'$. NP have given the gist of verse 8 correctly, but making a lot of unnecessary emendations. They have wondered in Part II, why such small units, as *padas*, have been taken. This is because, they seem to think, that the three sectors are each taken wholly to get intermediate values by interpolation. An examination of the total of each sector would show how wrong it would be. The true eq. cent. corrected Jupiter is given for the end of any *pada* we want. We are expected to use these to get the true motion through any segmentation of the total *padas*, for correct interpolation,

and the ends of the segments may fall anywhere, from *pada* 0 to *pada* 390. Therefore the small *pada* segments are used. I shall work out an example at the end to make everything clear.

I shall explain the rationale of the instruction in verse 8, of adding or subtracting the difference. The eq. cent. corrected Jupiter is subtracted from the sun to get the anomaly of conjunction. So, a positive eq. cent. means, less anomaly of conjunction. The days left over represent the anomaly of conj. with the 399 days of the synodic period, corresponding to 360° of anomaly. So the day is taken as *roughly equal* to the degree of anomaly, and the difference in degree subtracted. Vice versa for the eq. cent. corrected Jupiter, it being less than the mean. Varāhamihira is too astute to confuse day and degree, as NP think. (In verses 64-81 too, there is no confusion in the author's mind, as NP seem to think. There he has deliberately chosen the time taken by the sun to move one degree as the unit of time, and call it 'day', for convenience. This is patent on the face of the synodic periods given, though TS have not even seen it, and are perplexed. We have reason to think that verses 64-81 are by somebody else). (Cf. item 26 of my paper 'Some errors and Omissions etc.' *Vishveshvaranand Indological Journal*, Hoshiarpur, Vol. XI. 1973).

Ver. 12 :

12. दिन षष्ठ्यंश(षष्ट्यांशान्) द्वादश
 खकृतैर्वेदाः(दान्) कृताश्विमिद्वौ च ।
 सप्ताष्टकेन वकी
 षड् वर्गाः (भागान्) षष्ठितः षट् च ॥

Ver. 13 :

13. अनुवकी(को)ऽशीत्यार्का-
 ह्री(न्द्र्यू)नार्घ्यु(श)तेन नव ततोऽस्तमितः ।
 स्थित्वा सैकं मासं
 स्फुटोदयाष्टात्तरं मासं (?योऽष्टोत्तरैर्द्वैः) ॥ बृहस्पतिः ॥

12: By 60 days, (Jupiter moves) 12° , by 40 days 4° and by 24 days 2° . Becoming retrograde, by 56 days he moves 6° (i.e. -6°) and by 60 days, 6° (i.e. -6°).

13: Following after retrograde, he moves 12° in 80 days, and 9° in 48 days. Then setting, staying so for a month plus one day, he clearly rises moving $6^\circ 8'$. Ends Jupiter.

The Scheme given

Days	Rising in the east	60	40	24	56	60	80	48	Setting West	31	Rising East	= 399
Degrees		12°	4°	2°	-6°	-6°	12°	9°		$6^\circ 8'$		= $33^\circ 8'$

These values agree well with actualities, considering that whole days and whole degrees are given, excepting the last $6^\circ 8'$, given to complete the value for the synodic cycle. $6^\circ 12'$ would be better at that region and for the whole number, 399 days. 56 days for -6° , and 60 days for the same -6° must be explained by the intention to give whole degrees and segmentation. चर्गा: is an obvious mistake for भगान्, and so corrected. TS have interpreted सप्ताष्टकेन to mean 15, which such an expression never means. It can mean either 56 or 87. They understand another 60 days by the word च used. All this, to make up the wrong scheme used by them, based on the mistaken idea that the statement of motions here begins with conjunction and ends with the rising in the east after the next conjunction. The following is their scheme:—

Days	Conjunc- tion	60	40	24	15	60	60	80	45	30	Rising east	= 414
Degrees		12°	4°	2°	0°	-6°	-6°	12°	9°	Setting West (15°)		= 42°

दीनार्धशतेन is emended by TS into ध्यूनार्धशतेन, but how can this word mean their 45? As for the last part, स्थित्वा सैकं मार्सं, they have taken it to mean 30 days instead of the correct 31 days. Let that be. They have

not given any motion for it in their interpretation. It cannot be left to be guessed and completed by an ordinary computer. They, who can be expected to know, have guessed, quite wrongly, 15° motion for 30 days, not realising that it can be only 6° and a few minutes more. For the 414 days from conj. to the rising after the conjunction, the total can only be about, $33^\circ 9' + 3^\circ = 36^\circ 9'$, and not the 42° given by them.

As for NP, they have emended दीनार्धशतेन into दिनार्धशतेन to mean 50 days. Since they take 30 days for the setting, i.e. one day less, they make the total of days, 400. They give 7° motion for the 30 days (which they make even 29 days in the last part). They have changed the wording to some ununderstandable form here, अवन्त्येमासस्य. Further, the 7° is far too much for 30 days. But there is no 7° in the text. They have corrected the text सैकं into “<श्वं>”, thinking that अश्वं in bhūtasāṅkhyā means 7° .

Incidentally, one other matter may be considered here, viz., the degrees of heliacal rising, for Jupiter. During the set-period of 31 days, the sun moves about $30\frac{1}{2}^\circ$ degrees, and Jupiter, about $6^\circ 8'$, and the relative motion is $30\frac{1}{2}^\circ - 6^\circ 8' = \text{about } 24^\circ$, from setting to rising. This gives about 12° , for the heliacal rising of Jupiter, which is fairly accurate, especially for very high latitudes. (Classical Hindu astronomy gives 11°). Verse XVIII. 58 gives the Vasiṣṭha-Pauliṣa's degrees of heliacal rising as 12° , 14° , 12° , 15° , 8° , 15° from moon onwards, by चन्द्रादीनां द्वादशमनुरवितिथ्यष्टतिथिसंख्यैः. 15° for Jupiter given here is too much, and 14° for Mars is too low. (Classical Hindu astronomy gives 17° for Mars). So, the scribe seems to have made a small change in the order, and the correct order is “चन्द्रादीनां द्वादशतिथि-मनुरव्यष्टतिथिसंख्यैः”, 12° , 15° , 14° , 12° , 8° , 15° , with only one change of place.

Example : Find the True Jupiter at 2415 days from epoch.

i. The beginning of the first cycle after rising next to the epoch is 34-34 days later.

The days after this, required to find the number of cycles gone = $2415 - 34 - 34 = 2380 - 26$.

Dividing by 399, cycles gone = $\frac{2380-26}{399} = 5$, with 385-26 remainder.

Adding $5 \times \frac{1}{3}$ days, (=0-33), we have 385-59 days left over after 5 cycles gone.

ii. The *padas* at 5 cycles gone = $18 + 5 \times 36 = 198$.

$$\text{Mean Jupiter} = 198 \text{ padas} = \frac{198 \times 360^\circ}{391} = 6^\circ 2' 18'.$$

True Jupiter:—

For the 198 *padas*, 180 *padas* forming the first sector has gone and 18 *padas* are left over in the second sector.

$$\therefore \text{True Jupiter} = 5^\circ 9' 30' + (1165 + 18)18'/24 = 5^\circ 9' 30' + 14^\circ 47' = 5^\circ 24' 17'.$$

$$\text{Eq. cent.} = \text{True} - \text{Mean} = 5^\circ 4' 17' - 6^\circ 2' 18' = -8^\circ 1'.$$

iii. The *padas* at 399 days in the cycle, i.e., the beginning of 6 cycles gone = $198 + 36 = 234 = 180 + 54$.

$$\text{Mean Jupiter} = 234 \times 360 \div 391 = 7^\circ 5' 27'.$$

$$\text{True Jupiter} = 5^\circ 9' 30' + (1165 + 54)\frac{54'}{24} = 6^\circ 25' 13'$$

$$\text{True} - \text{mean} = \text{Eq. cent.} = 10^\circ 14'.$$

$$\text{Eq. cent. at 0 day of 6th cycle} = -8^\circ 1'$$

$$,, \text{ 399 days of } ,, = -10^\circ 14'$$

$$,, \text{ at remaining days (385-59) } = \\ = (385-59) \times -2^\circ 13' \div 399 + -8^\circ 1' = -10^\circ 10'.$$

iv. *True Jup.* is less than *Mean Jup.* by $10^{\circ} 10' \therefore$ days of Anomaly of Conj. = $385-59 + 10-10 = 396-9$.

v. *True an. of conj.* =

for 60 days	+ 12°	
for 40 days	+ 4°	
for 24 days	+ 2°	$\frac{28-9}{31} \times 6^{\circ} 8' = 5^{\circ} 32'$
for 56 days	- 6°	
for 60 days	- 6°	
for 80 days	+ 12°	
for 48 days	+ 9°	
Total 368 ...	+ 27°	
for 28-9 ...	$5^{\circ} 32'$	
396-9	$32^{\circ} 32'$	

vi. *True Jup.* = *Mean Jup.* at 0 day of An. of conj.

+ eq. cent. + true ano. of conj.

$$= 6^{\circ} 2^{\circ} 18' - 10^{\circ} 10' + 32^{\circ} 32' = 6^{\circ} 24^{\circ} 40'.$$

Note 1: The need for interpolating the eq. cent. to the remaining days in the cycle can be seen by working for 399 days of the 6th cycle and 0 day of the 7th cycle and comparing. They must be the same.

Note 2: The eq. cent. is computed for 0 day of each cycle, i.e., for intervals of 36 *padas* = $33^{\circ} 9'$. Interpolation using these as we have done, can be only rough. To get better interpolations, we can divide the 36 *padas* into desired segments, find the eq. cent. of each, and use. We can form an ephemeride, giving the values at the ends of these smaller segments, each. Or we can form an ephemeride of values at the ends of the day segments given, 60, 40, 24 etc. and use for interpolation. All these logically follow from the instructions, though not specifically stated.

Saturn (Verses 14-20)

As I have already said, the treatment of Saturn is similar to that of Jupiter. So there will be little need for fresh explanations.

14. अर्धशतं श(स)त्र्यं-
 शमपनयेत्सूर्यजस्य दिवसेभ्यः ।
 वसुमुनिगुणोष्ट्र(दधृ)तेभ्यः
 स्थितं(ता) दिनाद्यास्त(स्स)मभ्युदयात् ॥
15. जहाद्यु(दु)दयदशांशं
 द्युभ्यो नवसंगुणान्(न्म)जेदुदयात्(न) ।
 षड्विषयमैः शेषं
 पदैर्युतं तन्त(न्न)वाशीत्या ॥

14. Regarding Saturn, 150-20 days are to be subtracted from the days from epoch. These being divided by 378, the remainder are the days from the rising gone, the quotient being the number of risings gone.

15. One tenth of the risings, (i.e., the quotient), in days, is to be subtracted from the remainder. The number of risings got is to be multiplied by 9, and divided out by 256. The remainder plus 89 *padas* form (the *padas* required for using in the computation). (The idea is that 89 is to be added to (quotient \times 9), and then divided by 256, to find the *padas* for use).

In (15), I have emended संगुणाद् and रुदयात् into संगुणान् and रुदयान् to agree with भजेत् requiring accusatives as also NP. But TS have kept them. In NP's emendation दिनाद्यान्तं, आप्तं does not agree with the word स्थिता and, the meaning also is redundant. Both TS and NP have emended पदैः into पदे, thinking that नवाशीतिः is degrees. Even this they doubt as seen in the translation, because as mentioned by them in Part II, page 124, it has led to disagreement. पदैः, as it is, clearly

says that the 89 is *padas*. So is the 18 of Jupiter and the 85 of Mars.

We understand from the instructions that the synodic revolution of Saturn takes $378\frac{1}{10}$ days, that in one synodic revolution Saturn moves 9 *padas*, that 256 *padas* make nine sidereal revolutions of Saturn, that there are $256 + 9 = 265$ sidereal revolutions of the sun in 256 synodic periods of Saturn, and that at 150-20 days from epoch, Saturn's mean longitude is 89 *padas*. (NP give in their translation, "89", as mentioned already. Therefore, one sidereal revolution of Saturn takes $378\frac{1}{10} \times 256 \div 9 = 10754.84$ days. One sid. revolution of the sun = $378\frac{1}{10} \times 256 \div 265 = 365-15-32$.

Again, the Sun's sid. period got is Pauliśa's.

One *pada* = $360^\circ/256 = 84' 22''.5$. The motion in one synodic revolution = $9 \times 84' 22''.5 = 12^\circ 39' 22''.5$.

Mean Saturn at 150-20 days after epoch = $89 \times 84' 22''.5 = 125^\circ 9'.4$.

16. षड्रूपवेदपक्षाद्
वृद्धिस्त्रिंशत्पदानि सौरस्य ।
नवरूपविषयमला-
ह्ला(द्घ्रा)सः स्वरभास्करपदाख्य(न्तः) ।
17. प्रचयः स्वराग्निखयमा-
न(न्न)वनवत(ति)स्त्रिघनभागलिप्तानाम् ।
क्षयवृद्धि(द्धी) द्विगुणपदै-
रेकगुणघ्नः शन्नै(ने)रू(रु)दयः ॥
18. षोडश कृषभस्यांशा
नवलिप्तावर्जिताः प्रथमखण्डाः(ण्डे) ।
विषयास्त्रिघन(ना)स्त्रिंश-
च(च्च)तुर्युता मध्यमे खण्डे ॥

Ver. 16 Regarding Saturn, there is an increase (of the rate of motion) for thirty *padas*, from 2416. Then, there is a decrease for 127 *padas* from 2519.

Ver. 17 Next there is an increase for 99 *padas* from 2037. The amount of decrease and increase are by the *padas* multiplied by 2. The divisor of the total minutes is 27, its multiplier being one.

Ver 18: The total of the first sector is $1^r 15^{\circ} 51'$ and the total of the middle sector is $5^r 27^{\circ} 34'$.

Note: The multiplication by one is unnecessary, but given to clear the doubt that may arise by the instruction to multiply the *padas* by two for subtraction and additions coming before.

The meaning is clear, and no material change has been needed. I shall give what is given in the form of formulae:

The total motion upto any *pada* in the first sector
 viz., $(1-30) = (2416 + 2 \times \text{padas}) \text{ padas} \div 27$, in minutes.
 -do- -do- second sector., viz.,
 $(31-157) = (2519 - 2 \times \text{padas}) \text{ padas} \div 27$, in minutes.
 -do- -do- third sector, viz.,
 $(158-256) = (2037 + 2 \times \text{padas}) \text{ padas} \div 27$ in minutes

The total of the whole of first sector given, $1^r 15^{\circ} 51'$ can be verified thus :

$$(2416 + 2 \times 30) 30 \div 27 = 2751' = 1^r 15^{\circ} 51' \text{ given.}$$

The total of the whole second sector =
 $(2519 - 2 \times 127) 127 \div 27 = 10654' = 5^r 27^{\circ} 34'$, given.

Being unnecessary, the total of the third sector is not given. But we can calculate it and use it to see if all those add up to 12 *rāśis*, as necessary, and this will verify every instruction given.

The total of the third sector = $(2037 + 2 \times 99) \times 99' \div 27 = 8195' = 4^r 16^{\circ} 35'$. Now, $1^r 15^{\circ} 51' + 5^r 27^{\circ} 34' +$

$4^{\circ} 16' 35'' = 12^{\circ}$. Examining the constants, we find that the maximum motion per *pada* is $2519' \div 27 = 93'.3$. The minimum rate is $2037' \div 27 = 75'.4$. The mean rate is $\approx 84'.35$ as already found, as the mean motion equal to the *pada*. Differentiating as before, the increase or decrease in the rate is $4'/27$. Actually it is slightly less than this, the multiplier being slightly less than 2, given. $(2037 + 4 \times 98) = 2416$ shows this. The perigee falls at end of 30 *padas*, i.e., $1^{\circ} 14'$, and the apogee, 127 *padas* later, at $7^{\circ} 13'$.

The instruction how to use the result of these verses has not been given, because it is the same as that given in verse 8 for Jupiter. Indeed, the un-emended reading सौरे there means, "with reference to Saturn".

As in the case of Jupiter, here too TS and NP have not understood what exactly is given in these verses, how it is got by applying the three formulae, how the eq. cent. is got, and why the instruction to apply this to the days remaining is given, in the manner said.

So, their emendations of the readings, done without knowing the subject matter, need not be taken seriously. TS have emended the correct द्विगुणवद्भिः into द्विगुणहृत, meaning "divided by 32", applied to the risings and not to the number got in the formulae. NP have kept the reading, but given the translation as, "There is a subtraction or addition of 12 degrees and minutes, (i.e., $12^{\circ} 12'$). Multiply by 31 and divide (the product) by 32 (or by 32 *padas*.) (The result is) Saturn's rising." Where is $12^{\circ} 12'$ mentioned? They take the 32, not as a number, but as a segment of longitude equal to 32 *padas*, i.e., 45° . Again, how can this give the risings? And the risings have already been given in verse 14. All these show that they do not understand what is said.

19. षड् हतास्त्रीणां(?) कृत्या त्रीनंशान्
 मनु(मुनि)भिलिप्ताश्चतुर्(श्रेषु)गुणास्सप्त ।
 षोडशभिश्चाशीत(ति)
 कृतोनषष्ट्या द्विगुण(?वेदयम)पक्षान् ॥
20. वकी विभूतषष्ट्या
 त्रि(त्री)नंशान् षष्टितः कृतान् सौरः ।
 अनुगोऽर्कशतैर्ना(तेना)ष्टौ
 षट्कृत्या चास्तगे(गो) दहनम् ॥ शनिः ॥

19: Saturn (moves) 3° in 36 days, 35' in 7 days, 80' in 16 days, and 224' in 56 days.

20: Then becoming retrograde, he moves 3° in 55 days, and 4° in 60 days. Then following up direct, he moves 8° in 112 days, and setting, he moves 3° in 36 days in the set period, (i.e. rises in the east after that). *Ends Saturn.*

This is the scheme given

days	36	7	16	56	55	60	112	36	= 378
distance moved	Rising East 3°	35'	1°20'	3°44'	Retrogr -3°	-4°	Direct 8°	Setting West 3°	Rising East =

I shall now discuss the values given, justifying the three emendations I have made. The corrupt षड्-हतास्त्रीणांशान् has to be emended as 3° for 36 days, considering the position, and the fact that it must practically be equal to the rate between setting and rising, 3° for 36 days. The days must add up to 378 days from rising to rising, also as from conjunction to conjunction. All the numbers for days are clear. Therefore, the days for the second segment must be 7. So I have emended मनु into मुनि. The motion given there, 28', gives the rate 2', too absurd for that position, if the original 14 days are accepted, and it cannot be that the Siddhānta does not know the absurdity. Even for the emended 7 days, it is so too low, being only 4' rate, while the rate on both

sides is 5', and also consistent with facts. Therefore, श्वतुर्गुणा is emended into श्वेषुगुणा. Now, these three segments can be combined into 4° 55' for 59 days, without affecting the result. I do not know why the Siddhānta has broken it into such bits.

Next, the total for the 378 days must be the mean motion for the period, *i.e.*, 9 *padas*, equal to 12° 39'.4, roughly taken by the Siddhānta as 12° 39'. Therefore the motion for the fourth segment, 56 days, must be 224'. So I have emended द्विगुण into वेद्यम्.

In the case of Saturn, too, as in the case of Jupiter, TS and NP have thought that the unnecessary total motion for the third sector has been given. Finding no wording answering to that, they have changed drastically the first half of verse 19, and obliterated the first two segments of days and motion. They have emended the half verse into खण्डेऽन्त्ये सिंहाशामुनयो लिप्ताश्चतुर्गुणास्तप्त as if they are writing their own book. This means, in the last sector the total is 4r 7° 28'. But even this does not help to get 12 rāsis, the total coming to only 11r 20° 53'.

With the other half and the next verse, they make up the whole scheme as :—

days	16	56	55	68	60	105	36	=	396
motion	+3°	+232'	+4°	-3°	-4°	+8°	+3°	=	15°

changing अशीति into अंशाग्नीन्, not giving any word for the motion of 4° in 55 days, but simply putting the motion there, त्रींशान् into अष्टरसैस्त्रीन् newly introducing 68 days, and giving it the retrograde motion -3°, and अर्कशतेन into अर्थशतेन to mean 105 days. As in the case of Jupiter, they trace the motion from conjunction to the rising after the next conjunction, taking 396 days. But the total motions must then be, 12° 39' + 1° 30' = 14° 9' and not 15° given. They must know that 3° for 16 days,

giving the rate $11\frac{1}{2}'$ per day is very much wrong, when the rate is only $5'$ for the nearer segment got from the motion 3° for 36 days.

As for NP, they emend the first half of verse 19, as परिहीनाः स्त्री खांशाः मनुभिर्लिताश्चेषुगुणास्सप्त meaning “Zero degree of Virgo diminished by 14° , plus $35'$ ”, i.e., $4^\circ 16' 35'$. They are here better than TS because they have seen that the aim should be to get the total of 12 rāśis for the three sectors combined. They have also kept closer to the lettering of the text, though the manner in which they have got their total for the third sector is far-fetched. After this, they follow the text without changing it. Only at the end they interpret that the motion of 3° for 36 days comes before the setting, and leave the period set without any days or motion given. Thus, their scheme is:

days	16	56	55	60	112	36	?	Total	378
motion	Rising East $80'$	retrogr- rade $232'$	-3°	-4° direct	8°	3°	Setting West ?	Rising East	Total $12^\circ 39'$

To make up the totals, a motion of $3^\circ 27'$ for 43 days has to be given. But it must be at least $3^\circ 35'$. For the 43 days of the set period, the sun's motion is $42^\circ 20'$. Therefore, the degrees of Saturn for heliacal rising comes to $(42^\circ 20' - 3^\circ 27') \div 2 = 19^\circ 26'$. This is far greater than the 15° given in verse 58, and also in all Siddhāntas. Further, the opposition must occur at the middle of the period from rising to setting and also the middle of the retrograde period. The one falls 160 days after rising, and the other 130 days after, as great as 30 days off. I am sure NP have noted all these discrepancies, but given them as they understood the wording, just to mark time.

I shall now give an example, to make the method clear.

Example :

Find true Saturn at 5000 days gone from epoch.

i. Days 5000

To be subtracted 150-20

Dividing by 378) 4849-40 (12 = full cycles gone)

313-40 (= Remaining days)

Days to be deducted $\frac{12}{10}$: 1-12

312-28(Corrected remainder)

ii. Padas at 0 day of the 13th cycle : $\frac{89 + 12 \times 9}{256} = 197$
remainder

Mean longitude = 197 padas = $9^r 7^\circ 2'$

197 = 30 + 127 + 40 (in the third sector)

Eq. cent. corrected mean longitude :—

= $1^r 15^\circ 51' + 5^r 27^\circ 34' + (2037 + 2 \times 40)40' \div 27$

= $1^r 15^\circ 51' + 5^r 27^\circ 34' + 1^r 22^\circ 16' = 9^r 5^\circ 41'$

Eq. cent = $9^r 5^\circ 41' - 9^r 7^\circ 2' = -1^\circ 21'$

iii. Padas at 378 days gone in the cycle = $197 + 9 = 206$

Mean longitude = 206 padas = $9^r 19^\circ 41'$.

206 padas = 30 + 127 + 49 (in the third sector)

Eq. cent. corrected mean longitude =

$1^r 15^\circ 51' + 5^r 27^\circ 34' + (2037 + 2 \times 47)\frac{47}{27} = 9^r 18^\circ 0'$

Eq. cent. = $9^r 18^\circ 0' - 9^r 19^\circ 41' = -1^\circ 41'$

Interpolated for days 312-28, the eq cent =

$-1^\circ 21' - 0^\circ 17' = -1^\circ 38'$.

iv. Correcting the remaining days 312-28 by this,
312-28 + 1-38 = 314-6 days, to be used to find
anamoly of conjunction.

v. 36 days	+ 3°	
7 ...	+ 0° 35'	Mean Sat. at 0 day 9 ^r 7° 2'
16 ...	+ 1° 20'	Eq. cent. — 1° 38'
56 ...	+ 3° 42'	An. of conj. + 7° 40'
55 ...	— 3°	
60 ...	— 4°	True Saturn = 9 ^r 13° 4'
Remaining 84-6	+ 6° 1'	
<hr/>	<hr/>	$\frac{8}{11\frac{1}{2}} \times 84-6$
314-6	+ 7° 40'	As per Ephemeris : 9 ^r 11°.7
<hr/>	<hr/>	

THE VĀSIṢṬHA-PAULIŚA MARS IN THE PAÑCASIDDHĀNTIKĀ OF VARĀHAMIHIRA

This paper on Mars is a continuation of my two earlier papers “Vāsiṣṭha-Pauliśa Venus” and “Vāsiṣṭha-Pauliśa Jupiter and Saturn”, contained in verses 21-35 in Chap. XVIII of Thibaut and Sudhakara Dvivedi’s (TS) edition, (Chap. XVII in O. Neugebauer and D. Pingree’s (NP) edition) of the *Pañcasiddhāntikā* (PS) of Varāhamihira (VM). As I have already said, Mars and Mercury need elaborate treatment owing to certain peculiarities about them, and so are reserved to the end of *PS* by VM. The synodic period of Mars on which the equation of conjunction depends is 780 days, during which there are more than two revolutions of the Sun and one revolution of Mars, so that one full anomalistic period of Mars is contained within this period. This, with the large equation of the centre, and the large equation of conjunction causes large variations in its motion from sign to sign and even in the same sign, according to the different types of motion governed by the anomaly of conjunction, like fast, slow, retrograde etc. Hence is the need for detailed treatment.

Further, we have reason to think that the various motions given are all empirical, based on long observation, synodic period after synodic period. The separation into the equation of the centre, and the equation of conjunction is yet to come, it seems, unlike the cases of Jupiter and Saturn, where it is easy. This would explain certain discrepancies found in the values given.

Regarding the constants given, some can be verified by mutual comparison and corrected where necessary when there is a doubt about the reading itself. But some, like the epoch constants, which are peculiar to the

Siddhānta itself, cannot be so verified and corrected when necessary. Only in such cases, where we can argue that no siddhānta is likely to give such wrong values and those so far from the real, that we can make some plausible corrections. Therefore, not only in the case of Mars, but also Venus, Jupiter and Saturn, I mean writing a separate paper, on their epoch constants.

TS and NP have not understood the nature of the motion of Mars, just as they have not understood Jupiter and Saturn. While TS have not even attempted translating some verses, wrongly interpreting those attempted, NP have attempted translating all, but many wrongly. I shall point out these after my own translation and discussion of the verses, step by step.

Verse 21 :

21. द्युगणे षट्कं व(पञ्च) यमान्
विहाय पञ्चाष्टकं च नाडीनाम् ।
गगनाष्टमुनिभिरुदया
लभ्यन्ते प्राङ् महीजस्य ॥
22. उदयगुणिता विनाडयः
स्वरतिथयोऽब्धा (ध्य)न्विता दिनक्षेपः ।
धृतिगुणितास्याम्रीदु(तांस्त्र्यग्नीन्दु)भि-
रुदयान्ह(न् ह)त्वा स्थितो तोस्माः(तस्मिन्) ॥
23. पञ्चाशीतिं कृत्वा
प्रतिराश्य मध्यमः क्रमशः ।
राशिप्रमाणतोऽस(स्य)
स्फुटता चा (तद्व)रक्रमं कुर्यात् ॥
24. स्फुटमध्यमविशेषां(श्लेषां)-
शान्क्षिपेन्मध्यमे [ऽधिके] द्युभ्यः ।
मध्यमहानौ जह्यात्
गतितोऽथ चाराम (न)भिघास्ये ॥

21 : 'Subtracting 256-40-0 days (-nādis-vinādis) from the days from epoch, and dividing by 780, the synodic risings of Mars in the east are got.'

22-23: '(157 plus 4) vināḍis, multiplied by the risings got, are to be added to the remaining days. Multiply the risings (got by verse 18), and adding 85, divide by 133. The remainder, converted into rāśis is Mars at rising. According to the whole or portions of rāśis, the true motions are to be taken one after another, and pieced together.'

24: 'The difference between the mean and true degrees should be added (to the remaining days got in verse 21), if the mean is greater. If the mean is less, the difference should be subtracted from the remaining days. This done, I shall give the true motions according to each type of motion.'

From these verses we learn the following :

(1) 256-40-0 days from epoch, Mars rises in the east, after which the counting of risings begin.

(2) One synodic revolution takes 779-57-19 days (= 780 days minus 161 vināḍis). The addition of vināḍis multiplied by revolutions, is for taking the synodic period as approximately 780 days.

(3) For this period of 779-57-19 days, we get $1 + 18/133$ sidereal revolution of Mars, and $2 + 18/133$ sidereal revolutions of the Sun. So, in one synodic period Mars moves $408^{\circ} 43'.3$.

(4) In 133 syn. periods = 103734-3-7 days, there are 151 sid. rev. of Mars and 284 sid. rev. of the Sun. From this, the sun's sid. period got is 365-15-38 days and Mars's 686-58-50 days. The sun's period is 38 vināḍis more than that given for it by the Vāsiṣṭha, and near the 365-15-30 of the Pauliśa. Therefore, like the moon, Venus, Jupiter and Saturn, Mars also is common to Pauliśa.

(5) At the first rising when calculation commences, mean Mars = $85/133$ rev. = $7^{\circ} 20' 4'.5$.

(6) The addition or subtraction of the difference from the remaining days has been already explained with reference to Jupiter and Saturn.

No method, however, is given to find the equation of the centre. Now, the true motion is affected by both the equation of the centre and equation of conjunction. The segments of motion given in verses 25-26 are as affected by the equation of conjunction alone. By making the days given for true motion in 27-35, conform to the segments, we can get the degrees and through that the days affected by the equation of the centre alone.

This can be of use only for the remainder of days. But the equation of the centre at the beginning of each cycle must be given. It has not been given by any rule. Since its period is about 687 days, and it has its own rise and fall of about 11° from perigee to apogee and back, it cannot be associated with the synodic period of 780 days. So this is an omission.

I have corrected the corrupt तारुवाघ्रीदुभिः into तारुवग्नीन्दुभिः, to mean 133. This is necessary for agreement with the actuals, and the effect of my emendation is seen in my discussion (3) above. TS have made it बाणेन्दुभिः. How can बाण, with such different lettering, come in here? Further, this will give $18/15$ rev. = 1 rev. 72° as the mean motion of Mars in one syn. period, 23° wrong per period. They have made पञ्चाशीतिं कृत्वा into पञ्चांशोने कृत्वा and thus shut out the position constant of Mars on the first day where reckoning begins, viz., the point of time 256-40-0 days from epoch. (It will be remembered that in every case, the Moon, Venus, Jupiter and Saturn, they have made this mistake). By this emendation they reduce the motion by $86^\circ 24'$, and make the mean motion of Mars $34^\circ 36'$ per syn. period of 780 days !!

As for NP, they have made the correct emendation *ज्यम्नीन्दुभिः* giving correctly 151 revolutions of mean Mars in 133 syn. periods, and identified it with that given by the Babylonian astronomy of the Selucid period.

But they have not seen that *पञ्चाशीति*, meaning 85, is correct as it is and gives the constant $7^{\circ} 25' 4''.5$ at 256-40-0 days from epoch, (see item (5) above). They think it is the constant in degrees, though no word meaning degrees is found here. (This kind of mistake they have made in the case of Jupiter and Saturn also, as we have shown). But 85° would not do; so they have substituted *सत्रिराशि* for *प्रतिराश्य* and made it 175° . But even this would not do, and therefore they have changed the days from Epoch itself into 216-40-0, by emending *षट्कंवयमान्* into *षट्कैकयमान्*. But this has led to other troubles, leading to their remark, "For Mars this would mean a longitude of 175° (instead of 194° derived on the basis of *a.* in table 32). This longitude would correspond to September 27, and a solar position at 186° , hence to an elongation of $11''$ (124, Part II). It is to be noted that 11° for the first visibility of Mars is given by nobody. It is in the range of 14° to 17° . (For details, see separate paper below, "The epoch constants of the Vāsiṣṭha-Pauliśa star planets").

Verse 25 :

25. प्रागुदये षट् चस्तस्तेक (चत्वार्यैक)-
मष्टादशम(ग)स्ततो वक्रम् ।
अत्यर्थं च ततः शीघ्रा-
द्युना षष्टिस्ततोऽस्तवितः ॥
(? अद्यर्थं च शतं शीघ्रां-
स्ततोऽस्तमितो द्युनां षष्ट्या) ॥
26. समतीत्य दशत्रियुता(तं)
निरंशगतो विंशतिं (? तत्त्रिंशतं) व्यतीत्य कुजः ।
उदयमुपयाति वक्ष्ये
गतिचारदिना(न)क्रमे चातः ॥

25-26: 'After rising in the east, Mars moves 146° (in quick motion) and then 18° each (of slow motion), retrograde, and follow up after retrograde (*anuvakra*) come, and after that, 150° of quick (*śighra*) motion. Then setting, it reaches conjunction (i.e. *niraṁśagataḥ*) in 60 days, moving 13 plus 30 (=43) degrees. Then it rises, (moving the same degrees in the same days). Beginning from here I shall mention the series of motions, with their days.'

Rises east				
+	176°	{	$146^\circ = \text{I}$	type of motion (<i>śighragati</i>)
		{	$18^\circ = \text{II}$	„ „ (<i>mandagati</i>)
—C.	18°	{	$7^\circ = \text{III}$ (<i>vakragati</i>)
		{	$11^\circ = \text{IV}$ (<i>ativakragati</i>)
+C.	18°		V (<i>anuvakragati</i>)
+	150°		VI (<i>śighragati</i>)
Sets west				
+	43°		VII (in 60 days)	(<i>atiśighragati</i>)
Conjunction				
+	43°		VIII (in 60 days)	„
Total	<u>412°</u>		Rises east	

The numbers I have given in the scheme are practically what are found in the text, without emendation, excepting three. In verse 25, I have emended चत्वार्येक into चत्वार्येक to get 146° , the most plausible value. TS have made it सप्तत्येक, meaning 176° which is too large. See discussion following. 150° is given by अर्धशत, where चत is emended into शत. This is necessary to make up the total 410° motion in 780 days. Secondly, 43° motion for the 60 days given from setting to superior conjunction is required to agree with the 17° usually given for heliacal rising. This is made up by emending विंशति into त्रिंशत, with the 13° given by दशत्रिंशत added. I shall now show that the motion of Mars is near 43° in 60 days, in the region of the conjunction. For its

distance, nearly 1.53 that of the sun given by modern astronomy and also as computed from Hindu astronomy, the equation of conjunction at this region is $12'$ per day, (as can be verified) which, plus the daily mean motion of $31'.4$, gives $43'.4$ per day, making, in 60 days, roughly 43° . This also agrees with the angle for heliacal rising of Mars, nearly 17° , given by Hindu astronomy. (In 60 days the sun moves 59° . So the elongation is $59^\circ - 43^\circ = 16^\circ$ roughly. If विंशति is taken as it is, we get $20^\circ + 13^\circ = 33^\circ$, which is 10° short of the actual 43° and which also gives the angle for heliacal rising as great as 26° , so far from the 14° – 17° given by all).

In the mean, the motion from setting to conjunction must be equal to the motion from conj. to rising. That is why it is not given by the text separately. That the motion segments given in the two verses is mean is also clear, since no position of Mars from its apogee is taken into account. So the total motion must be equal to 409° . But the total got by adding the segments is 401° . This must be due to the defective method of the original or the empirical nature of the motions, and rounding off to whole degrees, as seen from 43° being given for $43^\circ.4$. The opposition must fall at the mid-point of the retrograde motion, -18° , and divide it into -9 , -9 . The total motion from conj. to opposition must be equal to that from opposition to conj. But actually, $43^\circ + 146^\circ + 18^\circ - 9^\circ = 198^\circ$, and $-9^\circ + 18^\circ + 150^\circ + 43^\circ = 202^\circ$, is given. It may be that the angle segments given are empirical, and also there are errors in the apparently correct numbers giving the segments, needing emendation.¹

1. It is only in the case of Mars does VM give these eight types of motion. In II. 12–13 of the later *Sūrya Siddhānta*, a set of eight types of motion is given. But they cannot be equated to these, each to each. So we have only to guess when in doubt,

The days on the synodic cycle taken to pass each type of motion must be nearly equal to the average of the days given in 27-35 for that type of motion. This has been used to check the degrees of each type. But the synodic period, as also the mean motion of $1+18/133$ revolution during it, are very near accurate, and they must have been got by analysis of the observed motions. So the Siddhānta must have known that the motions and times are half and half on both sides of the opposition.

Another point: Beginning from rising, type I is *śīghra* (quick) motion. II is *manda* (slow) motion. The distinction seems to be 'faster than the mean' or 'slower than the mean'. So, the dividing point must be where the tangent from the earth touches the synodic circle. Since the distance of Mars is 1.53 times that of the earth from the sun, this point falls about 189.5 degrees from conjunction. Subtracting 43°.5 from conjunction to rising (given as type VIII) 146° is left for I. This segment extends upto the point where retrograde begins. As the planet is stationary here, a small error of observation can make this lesser or greater than the actual. The text seems to give it as 18°. Types III and IV form the retrograde motion. III is called *vakra* (retrograde) and IV, *ativakra* (faster retrograde). This text is defective here, and we cannot fix the exact extent of the retrograde segment. But III and IV seem to be divided as 5:7 of the total. V is *anuvakra*, is (i.e., 'follow up after *vakra*'). In the detailed motions given, this is the sum of segments III and IV, but direct motion. This must be the counterpart of II. Type VI is *śīghra*, and so the counterpart of I. Its extent is given as 150°. Type VII is the very quick motion (*atiśīghra*) from setting to conjunction, and given as 43° in 60 days. Type VIII (*atiśīghra*) is the counterpart of VII, from conjunction to rising.

These divisions are mostly based on convention. But as these divisions are given only in the case of Mars,

and Classical Astronomy does not give them, we have only to guess regarding the segments. To add to the difficulty, the text is corrupt in the places giving the numbers.

TS have expressed inability to understand *verse 25*. Still they have made some emendations which do not give any cogent meaning. No translation is given. There is only a question mark. In *verse 26*, they give 20° motion from conjunction to rising. This can give only 28 days, as against the 60 days given by the text. By this the elongation for heliacal rising would be $7\frac{1}{2}^\circ$, so absurdly low.

As for NP, in both verses, they have needlessly emended correct forms, wrongly emended the corrupt ones, some in faulty Sanskrit, and given an untenable scheme. The following is their scheme : Rising east/ 186° motion/ 18° retrograde motion/ 180° motion / Setting / 30° motion / Conj. / 30° motion / Rising east. They have made the emendations and substitutions with their eye on the total motion of 409° in the synodic period. They make the total 408° , nearly correct. But they do not identify the vestiges of the different types of motion found in these verses. Further, 30° motion from setting to conj. and then from conj. to rising, is short by $13\frac{1}{2}^\circ$ from the actual $43\frac{1}{2}^\circ$. The time required to move 30° is 42.2 days, and the sun would move $41^\circ.5$ during this time, giving an elongation of $11^\circ.5$ for heliacal rising, far short of the actual, especially for such high latitudes as the Vāsiṣṭha-Pauliśa envisages.

Verse 27 :

27. चत्वारिंशश(शछ)शिनम(ग)
 [मु]ध्य(न्य)ष्टयमान्विता विपक्षा च ।
 प्रथमगतौ कुर्यादि (? क्रमदि)वसा
 मीनाद्वाशिद्वयसमानाः ॥

27 : In the I type motion, there are $40 + 1 (= 41)$, $40 + 7 (= 47)$, $40 + 7 (= 47)$, $40 + 8 (= 48)$, $40 + 2 (= 42)$, $40 - 2 (= 38)$, days per motion of 30° each, respectively, in each month of the diad of rāsis beginning from Mīna, (i.e. Pisces).

This means, that for 30° of motion, the time taken is 41 days in the rāsis is मीन (Pisces) and मेष (Aries), 47 days in ऋषभ (Taurus) and मिथुन (Gemini), 47 days in कर्कटक (Cancer) and सिंह (Leo), 48 days in कन्या (Virgo) and तुला (Libra), 42 days in वृश्चिक (Scorpio) and धनुस् (Sagittarius), and 38 days in मकर (Capricorn) and कुम्भ (Aquarius).

An examination of the rate shows that the perigee is situated at the end of मकर, and the apogee at the end of कर्कटक, which both fairly agree with the actual.

TS say that they do not understand this verse, and no translation is given, its place being taken by a question mark. NP translate thus : "In the first gati 240 plus 28 minus half ($= 267\frac{1}{2}$) (days). One should calculate days for every two signs from Pisces." It can be seen that they do not see that this verse gives the detailed rate of motion of the I gati in the diads of rāsis from Pisces, as affected by the equation of the centre. They think that the first motion given in verse 25, 186° according to them, takes $267\frac{1}{2}$ days, as given by them here. If so, what is the use of the instruction to calculate for "every two signs from Pisces"?

विषय[रस]स्वर सप्त(? रस) तु-
पञ्चकाद(न्द)शगुणान् द्वि[ती]यगती ।
सहितान्स्वरैकपक्ष-
तु चन्द्रशीतांशुभि क्रमशः ॥ २८ ॥

28 : In the II type motion, in the same order, (i.e., for each month of the diads, Pisces-Aries, etc.), 18° takes

$5 \times 10 + 7$, $6 \times 10 + 1$, $7 \times 10 + 2$, $6 \times 10 + 6$, $6 \times 10 + 1$, and $5 \times 10 + 1$ days.

This gives 57 days each to move in each of the signs Pisces-Aries, 61 days for each of Taurus-Gemini, 72 days for each of Cancer-Leo, 66 days for each of Virgo-Libra, 61 days for each of Scorpio-Sagittarius, and 51 days for each of Capricorn-Aquarius. From the days given it can be seen that there is a slight tilt in the apogee towards Leo, and in the Perigee towards Aquarius. This small difference from the findings in verse 27 shows that the values are empirical.

As for the readings, [रस] has been inserted because we want six numbers for the six diads, and one is wanting. Symmetry requires that it must be रस (=6) there. Also, two *mātrās* are wanting. सस is emended into रस because, 76 for Virgo-Libra, with 72 on one side, and 61 on the other, will take the apogee to the end of Virgo, 60° off from its place.

The average in the II type motion is 30° in 61 days, which is less than the mean rate. From this we can conclude that the I type motion is faster than the mean, and the II type slower, as we have surmised.

TS have expressed inability to interpret this verse also, and not translated it. Yet they have made an emendation which need not be taken seriously, since it has been done without understanding.

NP have interpreted the verse as giving 57, 71, 72, 66, 61 and 51, by inserting ऋतु, as the fourth. But symmetry shows that the second number 71 is wrong, and it must be 61, to avoid the jump from 57 to 72. At any rate, read with their interpretation of verse 27, we can see they do not understand the use of this series of numbers. They do not even say that these are days.

Verse 29 :

क्षत्रवृश्चिकाजवा (?वा) पे
 पुत्रकेषु(वक्रं) षट्सप्तकेन वव(? नय)भागं(गान्) ।
 वि(द्वि)कृतेन दिनग(? तैर्नवा)ऽतिवक्त्री
 दिनषष्ट्या षोडशानुगतिः ॥ २९ ॥

Verse 30 :

गोमिथु गतौलिङ्गन्या-
 नु । सनैश्च(? स्वग्निमागरेः स्व)रानंशान् ।
 ख ? त्रिकृतेर्दश त्रिषष्टो(ष्टया)
 सप्तदश यथाक्रमं वक्राशा(त्) ॥ ३० ॥

Verse 31 :

कर्कटमिहयोर्वेद-
 सागरैस्सप्त सप्तवर्णवैश्च दिवसान्(? रत्नार्णवैः शिवानंशान्)
 षट्षा(ष)ष्ट्याष्टादश
 क्रमात्कुजो वक्रपूर्वासु ॥ ३१ ॥

Verse 31 :

घटमृगयोर्यम(? नंग)दहनैः
 पङ्कभागानववद्ग (? अत्रहु)ताशनैरे(न)वच (?च) ।
 मुनिविषयैः पञ्चदशां-
 शकांश्च तद्वत्त्र(त्त्र)येऽप्यारः ॥ ३२ ॥

Verse 33 :

वक्रैर्दिनत्रिभागे-
 नवांशयुततुल्यजिनैर्भुक्तैः ।
 अतिवक्रैः च गतीं
 वक्रमनुवक्रगच्छ्यंशम् ॥ ३३ ॥

Ver. 29 : In the signs Pisces, Scorpio, Aries and Sagittarius, Mars moves 7° in 42 days when retrograde (vakra), and 9° in 42 days when extra-retrograde (ativakra). In the follow up after retrograde (anuvakra) Mars moves 16° in 60 days.

Ver. 30 : In the signs Taurus, Gemini, Libra and Virgo, Mars moves 7° in 43 days retrograde, 10° in 43

days extra retrograde, and 17° in 63 days in the follow-up-after retrograde.

Ver. 31 : In Cancer and Leo, Mars moves 7° in 44 days, 11° in 46 days, and 18° in 66 days, respectively in the three types retrograde etc.

Verse 32 : In Capricorn and Aquarius, Mars moves 6° in 37 days, 9° in 39 days, and 15° in 57 days, respectively in the three types of motion.

Verse : 33 : ? ? ? ?

Types III, IV and V, called respectively retrograde (*vakra*), extra-retrograde (*ativakra*) and follow up-after-retrograde (*anuvakra*), are given in these verses. The first two are actual retrograde motion, and the third is the slow direct motion following. They are shown hereunder in a tabular form :

Signs	Pis-Aries	Taur-Gemi	Can-Leo	Virg-Libr	Scor-Sagit	Capri-Aquar
Type						
III	—7°/42 d	—7°/43 d	—7°/44 d	—7°/43 d	—7°/42 d	—6°/37 d
IV	—9°/42 d	—10°/43 d	—11°/46 d	—10°/43 d	—9°/42 d	—9°/39 d
V	+16°/60 d	+17°/63 d	+18°/66 d	+17°/63 d	+16°/60 d	+15°/57 d

The division into the three types is arbitrary, based on some convention. By examining the table we can see two things to be noteworthy. The total of I and II is equal to III though III is positive. The days for I and II are the same, except for Cancer-Leo, and Capri-Aquarius. There is symmetry on both sides of these sets. Guided by the above, I have emended certain

numbers which glaringly go against these points. In verse 29 नव for *vakra* is corrected into नग since it must be less than 9° given for *ativakra*, and both equal to 16° , clearly given for *anuvakra*. In verse 30 the corrupt स्तर is changed into स्वर to make up the total 17° . The corrupt नुवासनैः is amended into निसागरैः, guided by symmetry. खकृतैः is emended into त्रिकृतैः since the number should be greater than 42, by symmetry, In verse 31 the corrupt सप्तखार्णवैश्च दिवसान् makes no sense and is corrected into रसाणवैः शिवानंशान्, because it is required to make up the 18° total for *anuvakra*. सप्त is a repetition, and खार्णव, 40, does not fit since maximum is wanted there, and 46 eminently fits. In verse 32 यम is corrected into नग since यमदहनैः ($=32$) is too short a period, and far from the 42 days on both sides, and the number should also be a little less than 39. खेच corrupt is emended into नव च which will make up the total 15° of *anuvakra*.

As for verse 33 the words in it are all perfect, without any corruption. But they do not make any sense. It seems that some rules are given here for the division into the three types with their days, and the proportion is roughly 5:7:12 of the degrees of all three combined. At any rate, this instruction does not seem to serve any purpose.

Ativakra represents the faster retrograde motion near opposition plus the slower *vakra* motion on the other side. That is why it is greater and faster. But why exactly the same number of days? This seems to be a convention. But this is against logic. For, only in Cancer-Leo and Capricorn-Aquarius, there is a small excess of days for *ativakra*, but even this is far too small. The sum of *vakra* and *ativakra* is 18° and a maximum at Cancer-Leo, and minimum 15° at Capricorn-Aquarius, and fairly evenly distributed in between. But actually, at Capricorn-Aquarius, the sum is near 9° , as a compari-

son with the motion of Mars given in the *Vākyakarana*¹ will show.

TS have translated verses 29-32, omitting verse 33 as obscure. But they think that all three types are retrograde motion, (while only III and IV are retrograde, and V is direct motion). This would make the range of the retrograde motion from 36° in Cancer-Leo to 30° in Capricorn-Aquarius, while the range given is 18° to 15°, and actually the latter is even as small as 9°. Also they do not see that the IV type should be greater than the III. So they give the numbers as they get from the words, instead of emending them appropriately. So, in verse 29, नव भागान् as emended by them should be correctly नवभागान्, नगान्वक्त्री should be नवातिवक्त्री. In verse 30 their emendation अन्धिसमुद्रैः should be अशिसागैरः. खकृतैः should be त्रिकृतैः. In verse 31 their खर्णवैर्दिवसैः, giving no degrees at all for the days, and defective in *mātrās*, should have been emended into रसार्णवैः शिवानंशान्. In 32, symmetry requires our emendation of यमदहनैः into नगदहनैः, while TS have kept it.

As for NP, they have understood that type III gives retrograde and type IV, extreme retrograde, though the numbers they give for degrees and days are untenable in many cases. Seeing that the degrees of V are the sums of those of III and IV, they think that V is the total of the retrograde motions, while actually V is direct motion. They do not see that if the degrees of V are the total of III and IV, the days too must be the sum of the days, and therefore V is not the total retrograde. They have translated verse 33, but this does not give any sense.

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1. Cf. *Vākyakarana*, Kuppaswami Sastri Research Institute, Madras, 1962, Appendix III, Kujavakyas, in the cycles where 'Retrograde' means in the signs Capricorn-Aquarius.

एकेन्द्रियवसुशिवमनु-

[मनु]भव त्रिवर्गं तु पक्ष (? गंतुपञ्च)(शं)संयुक्तम् ।

शीघ्रगतौ पञ्चाष्टक-

मूनं च शशाङ्ककृतवेदैः ॥ ३४ ॥

Verse 34 : In the quick motion following (= *śighragati*) (type VI), there are the days, 40+1, 40+5, 40+8, 40+11, 40+14, 40+14, 40+11, 40+9, 40+5, 40-1, 40-4, and 40-4, for every 30 degrees.

Obviously, these days are given for each one of the signs beginning from Pisces. The numbers are almost perfectly symmetrical on both sides of the inter-section of Cancer-Leo and Capricorn-Aquarius, reinforcing the conclusion that the former intersection is apogee, and the latter perigee.

[मनु] for Leo is a glaring omission and is inserted. Symmetry requires 45 for Scorpio, and so पक्ष is emended into पञ्च. The number-words forming a 'dvandva' compound, त्रिवर्गं is wrong for त्रिवर्गं and so has been emended. तु is removed being an extra *mātrā* and purposeless. This verse is of the same kind as 27-28 combined, giving types I and II.

TS have not understood what is given here and its purpose, as they have not understood the corresponding verses 27-28. But they have translated this verse as the words go, and wrongly too, the numbers given by them forming a mere jumble, without any instruction, 2+1, 2+5, 2+8, 2+14, 2+11. 2+9, 40-1, 40-4, 40-4. No wonder, they append this with a question mark.

NP have emended the already correct पञ्चाष्टकं, meaning 40, into पञ्चवर्षि which they think would mean $5 \times 60 = 300$. पञ्चवर्षि would mean only 65. To mean 5×60 , the form should be पञ्चवर्षयः (nominative) or

पञ्चषष्टीः (accusative). The trouble is that they have not understood that the days given are for every 30°, everywhere, except in types III, IV and V. That is why they make such unauthorised corrections.

षट्त्रिंशत्संयुक्ता

द्विकलाह्वार्क (?) ह्यनलाह्वार्कं त्रिवर्गगुणशून्यैः ।

दिवसास्सप्तमगत्यां

चावो (रो) य[द]त्र [त]द्वदष्टम्याम् ॥

Verse 35 : In the VII type, (i.e., *atisighra*), for every 30° of the diads Pisces-Aries, Taurus-Gemini etc., there are the days, 36+3, 36+9, 36+12, 36+9, 36+3, 36+0. The motion as given for the VII type, is for the VIII type too. *Ends Mars*

Here, too, the symmetry on both sides of the apogee and perigee is necessary, and the corrupt द्विकलाह्वार्क, is corrected into ह्यनलाह्वार्क corresponding to त्रिवर्गगुण. This forms the motion from setting to conjunction. As the VIII type, forming the motion from conjunction to rising is exactly the counterpart of the VII in the synodic cycle, it follows that type VIII is the same.

TS have emended द्विकलाह्वार्क into द्वयनलाह्वार्क resulting in 7 quantities of days, while only 6 are wanted for the 6 diads of signs. NP, here too, as in verse 34, have made an unauthorised correction under the same misapprehension. They have substituted षष्टि<मि>श्च for the correct षट्त्रिंशत्. They do not see the contradiction this leads to. From setting to conj. or from conj. to rising the days they give are not less than 60, 66, on the average. For this, the average motion would be about 46°, in this region of the synodic cycle. But they have interpreted that it is 30°, in verse 26. They have not seen the contradiction.

As I have done for Jupiter and Saturn, here too, I shall work out an example, to make my explanations clear.

Before closing, I wish to say something about Mercury. The case of Mercury is as involved as that of Mars, but in a different way. In one sidereal year Mercury traverses three full synodic cycles, and more. But it moves in the zodiacal circle together with the sun in the mean, and its equation of the centre depends on its position in the 12 signs. So the motion types vary from synodic cycle to synodic cycle in the same year, needing a large number of day-groups, and these are given by VM. But it is these numbers that are spoiled by scribes most, and the reconstruction is a tedious job. (The methods though sometimes peculiar, can be guessed and explained). I have neither the leisure nor the equanimity of mind to undertake this work at present, *and hope someone else would do it.*

Example : Find true Mars at 800 days from Epoch.

Verses 21-23 Days from Epoch	800- 0-0
Subtract days at first rising	256-40-0
	<hr/>
	543-20-0
	<hr/>
No. of revolutions gone	543-20-0
	<hr/>
	780
Remainder	543-20-0
Correction for revolutions gone	0- 0-0
	<hr/>
After rising, remaining days	543-20-0
	<hr/>
Mean Mars at Rising =	$\frac{18 \times 0 + 85}{133}$ revolutions = 230°

Verse 24-35. Motion of mean Mars in 543-20 days = $408^{\circ} 43' \times 543\frac{1}{2} \div 780 = 284^{\circ} 42'$.
 Starting from 230° at rising, true Mars moves in 543-20 days, 20° in वृश्चि 543-20
 Type I 146°

	To go	10°	in	वृश्चि	takes 14 days.	Remaining days	
	"	30°	in	धनु	42	"	529-20
	"	30°	in	मकर	38	"	487-20
	"	30°	in	कुम्भ	38	"	449-20
	"	30°	in	मीन	41	"	411-20
	"	16°	in	मेष	21-52	"	370-20
	"	14°	in	मेष	26-36	"	348-28
II 18°	"	4°	in	वृष	8-8	"	321-52
	"	-4°	in	"	24-36	"	313-44
III- 9°	"	-5°	in	मेष	30-0	"	289-8
	"	-9°	in	"	42-0	"	259-8
IV- 9°	"	$+16^{\circ}$	in	"	60-0	"	217-8
V 18°	"	$+2^{\circ}$	in	वृष	7-25	"	157-8
	"	28°	in	वृष	44-48	"	149-43
VI 150°	"	30°	in	धुमि	51-0	"	104-55
	"	$29^{\circ}-57'$	in	कट	53-55	"	53-55
	"					"	0

True motion in 543-20 days (form 20° वृश्चि $29^{\circ} 57'$ कट) = $249^{\circ} 57'$.
 Mean motion = $248^{\circ} 42'$.

Days to be added = 34-45.

True motion for 34-45 days = $30^{\circ} \times 34\frac{1}{2} \div 54 = 19^{\circ} 18'$.

True Mars: $29^{\circ} 57'$ in कट + $19^{\circ} 18' = 19^{\circ} 15'$ in सिंह.

THE EPOCH OF THE ROMAKA SIDDHĀNTA IN THE PAÑCASIDDHĀNTIKĀ, AND THE EPOCH LONGITUDES OF THE SUN AND MOON IN THE VĀSIṢṬHA-PAULIṢA

I. Romaka Epoch

Varāhamihira (VM) gives in chap. I.8 of his *Pañca-siddhāntikā* (PS) the Epoch of the *Romaka Siddhānta* as mean sunset at Yavanapura (Alexandria in Egypt) ending Sunday and beginning Monday, close to the beginning of the Hindu *Caitra Śukla* of *śaka* 427 elapsed, equivalent to 6 p.m. local mean time at Alexandria on the Julian Sunday 20th March, 505 A. D. He says this is the Epoch of the *Pauliṣa Siddhānta* as well. Since in III. 13 he says that the local mean time at Avanti (Ujjain) is 7-20 *nāḍis* in advance of that of Yavanapura, the moment of the Epoch is 37-20 *nāḍis* from mean sunrise at Ujjain on Sunday 20th March, 505 A.D. Thibaut and Sudhakara Dvivedi (TS), the first editors of the PS, agree with this. But Neugebauer and Pingree (NP) in their edition of PS (Kobenhavn 1970, 1971) say (Part I, p.8) that it is one day later, i.e., Yavanapura, Monday/Tuesday, equal to 6 p.m. 21st March. This is wrong, and I shall show in this paper that the Sunday / Monday, one day earlier, is the Epoch.

The matter can be clinched by comparing the mean new moon of the *Romaka* with those of the other *Siddhāntas* and modern astronomy. They should be reasonably near each other. Since the *tithi* is independent of the origin, the new moon is eminently fit for comparison. VM says that the *Romaka tithi* is tolerably

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accurate. The epoch constants of PS I. 9-10, can also be used to check the agreement.

The following table gives the mean sun and moon at 37-20 *nāḍis* from mean sunrise at Ujjain on Sunday 20th March, 505 A.D. (corresponding to sunset at Yavanapura, Sunday/Monday) according to various *siddhāntas*; and modern astronomy. The modern values are tropical, but since the *Ayanāṃśa* (precession) is near zero and the *tithi* is independent of the origin, this will not affect the result. The *Kali* days of the point taken is 1,317,122-37-20 days from the first day of mean *Kali*, i.e., Friday mean sunrise at Ujjain, 18th February 3102 B.C.

The Julian days of this point is 1,905, 588-9-43. The titles of the columns I to III are I, Modern Astronomy (Newcombe, Brown), II, *Brāhmasphuṭasiddhānta*, and *Siddhānta Śiromaṇi*, III, Modern *Sūryasiddhānta*, *Āryabhaṭṭya*, and the *Sūryasiddhānta* of PS. Columns IV to VII are for the *Romaka* at 37-20 *nāḍis*, at Ujjain on Epoch day ; IV if the moon's constant is 1984, given for Ujjain sunset, V if the moon's constant is 1984 for Ujjain at 37-20 *nāḍis*, VI, the constant 10984 given for Ujjain at sunset, VII the constant 10984 given for Ujjain at 37-20 *nāḍis*.

	I	II	III	IV	V	VI	VII
Mean Sun	359° 37'	0° 42'	359° 49'	359° 42'	359° 34'	359° 42'	359° 34'
Mean Moon	354° 48'	355° 49'	355° 6'	0° 56'	359° 19'	357° 49'	356° 12'
Moon—Sun	—4° 49'	—4° 53'	—4° 43'	+1° 14'	—0° 15'	—1° 53'	—3° 22'
Moment of New Moon (in <i>nādis</i>)	23-41 later	24-0 later	23-11 later	6-4 earlier	1-15 later	9-11 later	16-33 later
Beginning of Mean Solar Year (in <i>nādis</i>)	23 later	43 earlier	11 later	18 later	26 later	18 later	26 later

The following must be noted : (1) The positions of the sun and moon are apparently greater by about one degree in the *Brāhmasphuṭa Siddhānta* and *Siddhānta Śiromaṇi* because these take the tropical zero-point about 100 years later (c. 628 A.D.) as the zero-point of *Meṣa Rāśi*. But, as we have already said this will not affect our investigation.

(2) The constants for the sun and moon in PS IX. 1-2 are taken for mean noon at Ujjain on Sunday 20th March, 505 A.D. This agrees with the *Khaṇḍakhādya*.

(3) Columns II and III exhaust all main classical *siddhāntic* schools. The *Vāsiṣṭha* and *Pauliṣa* will be treated separately.

We see from these tables that the new moons of all the classical *siddhāntas* and modern astronomy fall within one *nāḍi* of one another, showing their accuracy. Their moments are 23 to 24 *nāḍis* after the time taken. Since the classical group includes the *Sūryasiddhānta* of PS, the point of the time taken, Yavanapura sunset at Sunday/Monday, is the epoch. Now the *Romaka*, described as nearly correct compared to the *Pauliṣa* and *Sūrya* in its *tithi* (cf. PS I. 4) must have its new moon also near other new moons, may be not very close. We must rule out its falling, say about 30 *nāḍis* earlier or later, which can be due to a mistake in the epoch data given. There is a doubt about the time of day for which *Romaka* constants of the sun and moon are given. Is it 37-20 *nāḍis* after Ujjain sunrise (sunset at Yavanapura) as normally it should be? Or is it sunset at Ujjain (7-20 *nāḍis* earlier), taking the time for the constant of the moon's anomaly given in PS VIII. 5 for these also? Columns IV and V of the table give the results of the two cases. Both are unsatisfactory. In the former case, if Sun/Mon is the Epoch, new moon falls 1-15 *nāḍis* later than the point of time, about 22 *nāḍis* earlier than the

rest. In the latter, new moon falls 6-4 *nādis* earlier than the point of time, 29 *nādis* earlier than the others. If Mon/Tues is taken as epoch, in the former case new moon falls about 38 *nādis* later, and in the latter about 30 *nādis* later. Both are unsatisfactory and so the constants must be wrong.

There is fair agreement between the *Romaka* mean sun and that of the others. But the mean moon differs by several degrees if the constant is taken as 1984 (*kytāṣṭanavakaika*). By emending the word into (*kytāṣṭanavakhaika*, we get the constant as 10984. With this constant, given for Ujjain sunset, we get 357° 49' for the mean moon, and 356° 12' if it is given for Ujjain 37-20 *nādis*, i.e., at the epoch time of day. The former gives the new moon 9-11 *nādis* later, while the latter gives the new moon 16-33 *nādis* later, as close as 7 *nādis* from the rest. This is the best we can get. If the epoch is taken as Mon/Tues, these two new moon moments will be 46 *nādis* and 53 *nādis* later. Hence the epoch day is Sun/Mon.

This new moon, coming 19-33 *nādis* after zero, Sun/Mon, agrees with the statement *Caitra-śuklādau*, the absence of the constant for *adhimāsa*, and the constant 514 given for *avama*. The sun being at 350° 34', the solar year begins after 26 *nādis*, i.e., about 9-30 *nādis* from new moon, for which the *adhimāsa* constant will be as small as one, and is therefore neglected in PS 1.9. It is also the beginning of *Caitra* as stated. If the new moon is taken to the next day, 50 *nādis* in the new solar year would have passed, the new moon would initiate the *adhika-Vaiśākha*, and the *adhi*-constant would be as large as 222, disagreeing with the above instruction. These confirm the Sun/Mon epoch. As for the *avama* constant 514, it represents an *avamaśeṣa* 43-52 *nādis* giving the end of new moon as 16-8 *nādis*

from epoch¹ which agrees fairly well with the 16-35 *nādis* got after Sun/Mon.

All these show that the epoch is Sun/Mon and not Mon/Tues. (The *Vāsiṣṭha-Pauliṣa* constants also give this result, as we shall show later). This is confirmed by I. 17-20, which give the Lords of the *sāvana* year, month and day. At the moment of epoch, (i.e., zero day gone) we have to work with $2227+0-2227$. Dividing by 2520, the remainder is 2227. Dividing this by 7, the remainder is 1. The instruction is that the Lords are to be counted from the Sun (I. 19). So we get, Sunday ending at Zero Epoch, and the next day beginning is Monday. So the Epoch is Sun/Mon. Now, in part II, p. 14, NP add one to 2227 to get the Lord of the *first day after Zero Epoch*, and dividing out 2228 by 7, get remainder 2 all right, which is correctly Monday, counting from Sunday as one. But they mistake it to be Tuesday, and incorrectly take Mon/Tues to be the Epoch.²

Now, we come to the actual day mentioned in PS I. 8. The reading of MS. ∞ is '*saumya-divasādye*', that of β is '*bhaumya-divasādye*'. Bhaṭṭotpala's reading is '*soma-divasādye*'. It is this reading, meaning 'beginning Monday' that is correct. But NP discredit it as one likely to have been emended by Sudhākara Dvivedi in

1. cf. Siddh. Śir., Gola. Madhya, 16b-18a :

darśagrataṣaṁkramakāḷataḥ pr-k
sadaiva tiṣṭhatyadhimaśaśeṣam /
tithyantasūryodayayos tu madhye
sadaiva tiṣṭhayavamavaśeṣam //

2. Though it is irrelevant for our purpose here, I shall clarify one point. In the footnote on p. 13, NP are correct in observing that the rule for Lord of the month as given in verse, I. 19 is wrong, but their suggestion to correct it will not serve the purpose. "Subtract one" is the additional instruction required there, as given by Bhaṭṭotpala's reading... '*vyekāḥ*' for '*kāryāḥ*'.

his edition. Whatever it is, the discussion in this paper shows that it is correct. The word *saumya* also can be taken to mean Monday. It has two meanings, (1) 'the son of Moon', i.e., Budha. If this is taken it would denote Wednesday, which is two days later and obviously wrong ; (2) 'realated to Moon' i.e., moon's, day, i.e., Monday. The reading, '*bhaumya*' is in the worse vitiated manuscript β , and a corrupt form of '*saumya*'. Further, *bhaumya* is meaningless, and has to be corrected into *bhauma* to mean Tuesday.

The emendation into *bhauma* is due to Dikshit. In Part I. 18 NP say : "Dikshit concludes that the Epoch of VM is Tuesday 22nd March, 505, but that according to the *Sūryasiddhānta*, the *kṣepakas* (epoch constants) in IX. 1-4 are for the noon of Sunday 20th March, and the *kṣepakas* in XVI. 10-11 are for midnight 20/21 March in the same year. The *kṣepakas* in VIII. 1, 4-5, 8 are computed for sunset of 20th March 505, and this is not the epoch of the original *Romaka Siddhānta*, which he claims was written between the time of Hipparchus and A.D. 150." Now, the days for computing are from the zero day of Epoch, which is precisely the time of the Epoch, and the *kṣepakas* also are for the time of Epoch. So, according to these statements of Dikshit himself, the *Romaka* Epoch is sunset, Sunday 20th March, near Yavanapura sunset, ending Sunday, beginning Monday, for the Julian date 20th March. Similarly the *Sūryasiddhānta* Epoch for the sun and moon is Ujjain noon on the same sunday, 22 *nāḍis* before the *Romaka* Epoch, and the Epoch for the star planets in PS XVI is 8 *nāḍis* later than the *Romaka* Epoch, which is the julian Sun 20/Mon 21, March. Then how could Dikshit say that the Epoch is Tuesday 22nd, when the purpose of the Epoch is only to give the beginning of the time to be taken in a computation? To compute modern values for comparison, NP go even to 5 p.m. Tuesday 22nd March, one day

later than the Epoch they themselves have fixed, and two days later than the correct *Romaka* Epoch.

Thus Sun/Mon is the Epoch. As already mentioned the constants of the *Vāsiṣṭha-Pauliṣa*, and *Sūrya siddhānta* (in XVI), also agree with this.

II. The Epoch Longitude of the Sun And Moon in the *Vāsiṣṭha-Pauliṣa*

In II. 1 the sun's Epoch constant is given by *yaṣamamṛtuyuta*, corrected into *mṛtuyuta*, and thus taking +6. This verse gives the true sun. So the true sun at Epoch is $(0 \times 4 + 6) \times 30^\circ / 127 = 1^\circ 25'$. Since on examination we see that the apogee falls near the middle of *Mithuna*, we may roughly take the equation of the centre at the beginning of *Meṣa* to be $2^\circ 10'$, (the maximum being $= 135'$) and subtracting this from the true sun, the mean sun is $359^\circ 15'$ at Epoch.

The mean moon at Epoch can be got from the constant +1936 days, given in PS II. 2. The days to work with is Epoch days $+1936 = 0 + 1936 = 1936$. The mean moon = the mean motion in 1936 days + a constant (PS II. 2-4) $= 309^\circ 25' + 1^\circ - 14^\circ - 29' = 353^\circ 54'$. Obviously this constant is the mean moon

1. We get $1^\circ 14' 29''$ by emending *muni* in *śaśimuninavayamāśca rāśyadyāh* into *manu*. Then the moon and new moon agree reasonably with those of the other *Siddhāntas* (Cf. the Table above). As it is, the moon will be less by 7° , and therefore 8° less than the modern value if the epoch is Sun/Mon and 21° less if it is Mon/Tues. This would mean that the new moon would be later 40 *nādis* and 105 *nādis*, respectively, which is impossible. In an earlier paper ('*Vāsiṣṭha* sun and moon', supra pp. 1-28. I was not willing to amend even obvious errors, concluding that the *Vāsiṣṭha* moon must have been given for sunrise at Ujjain. But in the absence of any special instruction to that effect, only the Epoch time should be normally taken. So later I emended *muni* into *manu*, which gives such good agreement.

at the point 1936 days before epoch. This is only 54' less than the modern, given in the table, and need not be ascribed even to the error of the *siddhāntas* (its mean motion being remarkably equal to the modern) but to its first point being forward, by this amount from the first point of τ (i.e. the vernal equinox) of 505 A.D. as it should be about 60 years earlier. (Its mean sun at Epoch $359^\circ 15'$, confirms this idea.) Mean sun *minus* mean moon $5^\circ 21'$, the new moon falls 21-18 *nāḍis* later than the epoch time, and if it should fall in line even approximately with the other *siddhāntas* and the modern, the Epoch itself should be Sun/Moon. Since the *Vāsiṣṭha* moon is the same as the *Pauliṣa*, the Epoch constants of the *Pauliṣa* got from PS I. 11 and III. 1-3 should confirm this, and they do, as we shall see.

In part II, p. 22, NP go back 1936 days from 5 p.m. 22nd March 505 A.D. arriving at Dec 3, 499, and state that the mean moon of that day at 5 p.m. at Ujjain is $2^\circ 9' 20''$. This being $= 2^\circ 9' 7' 1''$ (the constant given in this text by *śaṣimuninavayamāś ca rāśyādyāḥ*) they say that the 5 p.m. 22nd March, and the constant given, are verified at one stroke. But the 5 p.m. 22nd March is nearly one day later than the Epoch they fix, namely Mon/Tues Ujjain 37-20 *nāḍis* on 21st March. This new Epoch is given nowhere in the text of *Vāsiṣṭha* or *Pauliṣa*.

The following is their mistake : The correct date to go back is Dec. 1st 499, 6 p.m. Ujjain, and at that time the mean moon is nearly $1^\circ 15' 23''$ by modern astronomy. This is only one degree off the emended text value $1^\circ 14' 29''$. which error can be reasonably attributed to the *Siddhānta*. It should be noted that the apparent agreement brought about by NP is due to two mistakes made by them, one equal to the other. First the two days error of about 26° mentioned above. Secondly, they have interpreted *śaṣimuninavayamāśca rāśyadyāḥ* as $2^\circ 9' 7' 1''$, not paying attention to the word *rāśyādyāḥ*.

meaning 'in the order beginning from *rāśis* etc.' The correct value, as given by the emended text is $1^{\circ} 14' 29''$. So the error here is nearly 25° . (cf. the previous footnote).

Next we take the *Pauliṣa*. Since no separate instruction is given, the new moon is the same as the *Vāṣiṣṭha*. To fix the mean sun at Epoch without any doubt, the whole of III. 1-3, giving the true sun must be studied, *since these verses have not been understood* by TS or NP. Like the *Vāṣiṣṭha*, the *Pauliṣa* also begins with giving what is actually the true sun at the beginning of the year, and calling it the mean sun. Therefore, the equation of the centre at the beginning of the year, about $2^{\circ} 13'.5$ minus the $7'$ got by the instruction to subtract $11' \times 20/30$, equal to $2^{\circ} 6'.5$, is included in the so-called mean sun. I shall translate the verses PS III. 1-3.¹

III I. Multiply the days from Epoch by 120, subtract 33, and divide by 43831. The revolutions etc., of the mean sun is got. This, plus 20° is the *kendra* (i.e., anomaly, but here used in the sense of the argument to be used).

III. 2-3. For each *rāśi* of *kendra* subtract continuously, one for one, $11', 48', 69', 70', 54', 25'$, and then add $10', 48', 70', 71', 54'$ and $25'$. The mean sun becomes true.

This straight interpretation gives clearly the method of computation. Example : Let the days from Epoch be 620. Then $(120 \times 620 - 33) / 43831 = 1 \text{ rev. } 8^{\circ} 10' 52''$. This plus 20° gives $9^{\circ} 0' 52''$ as the *kendram*. So the true sun = $8^{\circ} 10' 52'' - 11' - 48' - 69' - 70' - 54' - 25' + 10' + 48'$

1. This was given as item 1 of a paper by me in the *Vishveshvaranand Indological Journal*, Hoshiarpur 1973, first presented at the World Sanskrit Conference, 1972. See *Supra* pp. 103-4.

$+70' + 2'$ (the last term for the $52'$ left over) $= 8^{\circ} 8' 25'$.
(This is similar to the method given in the *Vākyakaraṇa*).

I pointed out TS's mistakes in the paper mentioned in the previous footnote. I subsequently found that in their edition (Part II, p. 21) NP have improved upon TS' Translation, but have made a great mistake in concluding that the sun's maximum equation of the centre to be $72'$. This is due to their mistaking the $11', 48'$ etc. as actual values, instead of differences, of the equation of the centre and that the anomalies taken are 290° onwards. But actually the apogee is taken to fall at 70° , i.e., 10° Gemini, not at 80° (i.e., 20° Gemini) as they say. The 70° may appear to be quite wrong, but it may be the relic of a very early period when the vernal equinox was forward by about 5° (the equivalent of 360 years). This shifting of the first point backwards, together with the actual small movement of the apogee, could have been the reason for taking the differences as $-11', -48'$, etc. instead of the correct original differences for $270^{\circ}, 300^{\circ}$ etc., namely $-19', -51', -70', -70', -51', -19', +19', +51', +70', +70', +51', +19'$.

On this basis, we can fix the mean sun of the *Paulīṣa* at zero day Epoch. The so-called 'mean sun' is $(0 \times 120 - 33)/4383' = -16'.5$. This contains the equation of the centre of that point, $133'.5$ less the $7'$ got by computation, i.e., $126'.5$. The real mean sun at Epoch is, $-16'.5 - 126'.5 = -2^{\circ} 23' = 357^{\circ} 37'$ ¹. The mean sun will reach zero *Meṣa* 2 days 25 *nāḍis* later than the Epoch. Mean Sun - Mean moon $= 357^{\circ} 37' - 353^{\circ} 54' = 3^{\circ} 43'$. So the new moon occurs 18-17 *nāḍis* later than the Epoch. This is 5 *nāḍis* before the classical and modern

1. This large deviation from the zero point of c. 505 A.D., confirmed to by other *Siddhāntas*, need not surprise us since this will be consistent with an earlier time by about 200 years. There is a similar shifting of the zero point in the *Vāsiṣṭha* also.

new moon, and closer than the *Romaka*. The days corresponding to the *adhimāsaśeṣa* are from mean new moon to mean zero point of *Meṣa*, 2 days, 6 *nādis*, 43 *vinādis*.

That these findings are correct will be confirmed by the *kṣepas* (constants) for *adhimāsa* and *avama*, in PS I. 11, according to the *Paulīsa*. The former is 698, giving 698/9761 of a lunar month, nearly equal to the 2 days, 6 *nādis*, 43 *vinādis* found above.

The *avama* constant is given by the last foot, both Mss. combined giving the reading, *trikṛtadinānyavamasamkṣepaḥ*. '*trikṛta*' is corrupt, as taken by all. So, whatever is given by '*trikṛta*', that is the *avama* constant given. I emend it into '*vikṛtakṛtāny*' and take it to mean "the *avamakṣepa* is less (than the *kṣepa* given before, i.e. 698) by 44'. So the *avamakṣepa* is 654. This gives very nearly 41-43 *nādis*, which subtracted from 60 *nādis* gives nearly 18-17 *nādis* as the time of new moon after epoch, giving perfect agreement.

I shall justify my emendation. There must be an *avamakṣepa*, in the absence of which the *Paulīsa* new moon will have to be taken as falling at Epoch itself, so far away from others, and so wrong. We cannot take the *Romaka kṣepa* for the *Paulīsa* since in that case both new moons will fall at the same moment, very unlikely, and also the *Paulīsa*'s will be as wrong as the *Romaka*'s. Therefore the *kṣepa* is given, and that must be 654, if we want the word *kṛta* to be kept. TS and NP also have to emend *kṛta* to mean 6. TS make it *ṛta*. (NP's *ṣat* is very unlikely, and unnecessary). Their emendation means "63 days are the *avamakṣepa*". What they mean is that the lunar days are to be divided by 63 days to get the *avamas*. But where is the *kṣepa*? They neglect the word '*kṣepaḥ*'. We can understand the lunar days to be divided. But these have not become days yet,

and there is no meaning in saying 'divide by 63 days'. If 'days' mean lunar days here, then it should be 64. Further, the instrumental case is required to instruct division, not the nominative. My emendation sets everything right.

This detailed discussion, though lengthy, has been required to fix the Epoch and the Sun and Moon at Epoch precisely, removing any doubt. This is necessary for the determination of the Epoch constants of the star-planets of the *Vāsiṣṭha-Pauliṣa*, and to compare them with the constants of the later *Siddhāntas*, and modern astronomy, which will be done in another paper.

THE EPOCH CONSTANTS OF THE VĀSIṢṬHA— PAULIṢĀ STAR PLANETS

In my paper, 'The Epoch of the Romaka Siddhānta' (See above, pp. 188ff.), I have shown that the epoch of the Romaka-Pauliṣā therein is sunset at Yavanapura, ending Sunday and beginning Monday, March 20, 505 A.D., (equal to 37-20 *nāḍis* from mean sunrise at Ujjain, as given, 20th March). I have also shown in Section II of the paper that the Vāsiṣṭha mean sun at that time is $359^{\circ} 16'$, taking the vernal equinox of circa 505 A.D. as the zero point, and the Pauliṣā mean sun, $357^{\circ} 37'$. The mean moon according to both is $353^{\circ} 54'$. In this paper the epoch-constants of the star-planets of the Vāsiṣṭha-Pauliṣā are sought to be determined.

Several of the words giving the numbers are corrupt owing to the carelessness of the scribes, and some have been deliberately tampered with inadvertently owing to ignorance. Yet it is possible to determine many of these numbers by mutual comparison, aided by a knowledge of the methods that are being used and the related information given in other verses. But some cannot be checked in these ways. A knowledge of the original siddhāntas alone can help, but they have not been found yet. In these cases we can only see whether our conclusions agree with other siddhāntas and modern astronomy tolerably well, explaining small disagreements, and rejecting those that are too far away to be correct. In my three papers dealing with the Vāsiṣṭha-Pauliṣā Venus, Jupiter-Saturn, and Mars, I was mostly content with correcting the numbers in so far as they mutually agreed, so that my purpose, *viz.*, to explain the method of computation was not impeded by too much extraneous details. Hence is this paper, where I have attempted to fix the constants as correctly as possible.

1. The Epoch Position of Venus

In my paper on Vās-Paulīśa Venus, I emended the word giving the days of first rising after epoch, *munijala-candrān*, (=147) into '*jalamunicandrān*' (=174), as doing the least violence to the text. At 174 days from epoch, the mean sun is = $16\frac{1}{2}^{\circ}$ degrees according to Vāsiṣṭha and 16° degrees according to Paulīśa. Since Venus is 176° , given, the elongation is $6\frac{1}{2}^{\circ}$ degrees and 7 degrees, respectively, a little less than the 8° given in verse 58 of the chapter. So the emendation into *jalamuni* is plausible, the words having interchanged places as in III. 4. Since the Vās-Paulīśa sun has been fixed near 358° at epoch, the sun would be near $169\frac{1}{2}^{\circ}$ at 174 days, and the elongation of Venus would be near the 8° required. The small difference can be attributed to a small error accumulated through a long period.

This also agrees with the 30 days given from setting to superior conj. and 30 days again from sup. conj. to rising, the motion given being $37\frac{1}{2}^{\circ}$ (verse 5). The sun's longitude would be 30° less, i.e., 139° . Venus would be $37\frac{1}{2}^{\circ}$ less, i.e., $138\frac{1}{2}^{\circ}$ mean longitude, both nearly equal. The time of sup. conj. is 142 or 143 days, respectively, from epoch. Subtracting the mean motion for the respective days, we get the mean position of Venus at Epoch as $138\frac{1}{2}^{\circ} - 227\frac{1}{2}^{\circ} = 271^{\circ}$, or $138\frac{1}{2}^{\circ} - 229^{\circ} 6' = 269^{\circ} 24'$, respectively. Modern astronomy as well as the *Sūryasiddhānta* of the PS give 267° . The difference is not great, and can be put to the variation from siddhānta to siddhānta itself.

2. The Epoch Position of Jupiter

The numbers giving the Epoch constants are clear. So, we have to put any discrepancy to other causes than

1. Cf. the Paper 'The Epoch of the Romāka and Paulīśa' above, pp. 188ff.

wrong numbers. The first rising is 34-34 days from Epoch (verse 6). The Pauliṣa¹ mean sun of date is $34^\circ + 357^\circ 37' = 31^\circ 37'$. The mean Jupiter, as corrected for the equation of conjunction alone is 18 *padas*² = $16^\circ 35'$. The equation of the centre at this point is that for the 18 *padas* at rising, minus $16^\circ 35' = (1456-18) 18'/24 - 16^\circ 35' = 1^\circ 24'$ (by verses 8, 9) \therefore the true Jupiter at this point is $16^\circ 35' + 1^\circ 24' = 17^\circ 59'$, i.e., the same as $(1456 - 18) 18'/24$. The elongation for heliacal rising got is $31^\circ 37' - 17^\circ 59' = 13^\circ 38'$, which is reasonably near that given in verse 58, especially when we see that we have used only the mean sun here, so that the treatment may be general. The elongation for heliacal rising as computed from the time given from setting to rising, 31 days, and the motion for that, $6^\circ 8'$, (verse 13), viz. $(30^\circ 32' - 6^\circ 8')/2 = 12^\circ 12'$, is fairly close, when we consider that various values, from 11° in Hindu siddhāntas to 14° in verse 58, are given.

Continuing, Mean Jupiter $15\frac{1}{2}$ days before 34-34 days = $16^\circ 35' - 3^\circ 4' = 13^\circ 31'$. Subtracting the mean motion for the 19-4 days remaining to reach the epoch, mean Jupiter at Epoch is $13^\circ 31' - 19-4 \times 5' \text{ (per day)} =$

1. I use the Pauliṣa sun because the sidereal year computed from the data given for all star-planets is Pauliṣa's.
2. These are *padas* and not degrees as Neugebauer and Pingree (NP) mistake them. The instruction in verse 7 is to add 18 to the *padas* remaining after division by 391, and after that the equivalent *rāṣis* etc. are to be found. So these are also *padas*. The difference caused by the mistake is not great here. But in the case of Saturn and Mars it is 36° and 145° . They have tried to minimise this large difference by changing the numbers giving the days as 216-40, (*ṣaṣṭkaikayamān*) in verse 21, in the case of Mars, to make up for the large difference. But they are still not satisfied and remark, "For Mars this would mean a longitude of 175° , (instead of 194°). For Saturn, however, no such agreement seems obtainable for the given numbers."

11° 56'. Modern astronomy gives 9° and the *Surya siddhānta* of the PS 8°.

3. The Epoch Position of Saturn

Here too there is no doubt about the numbers giving the epoch constants. The method of procedure is similar to that of Jupiter: The first rising is 150-20 days (verse 14) after epoch, and the mean sun then = the epoch position of the sun + 150-20 days' mean motion = $357^{\circ} 37' + 148^{\circ} 12' = 145^{\circ} 49'$. The mean longitude of Saturn as corrected for the equation of conjunction = 89 *padas*, = $125^{\circ} 9'$. The equation of the centre-corrected mean Saturn = $1^{\circ} 15' 51'$ (for the first 30 *padas* + $(2519 - 2 \times 59) 59'/27$, (for the remaining 59 *padas*) = $45^{\circ} 51' + 87^{\circ} 27' = 133^{\circ} 18'$, (verses 15, 16). The equation of the centre = $133^{\circ} 18' - 125^{\circ} 9' = 8^{\circ} 9'$. (Since 8-9 days will have to be subtracted from the 0 day (verse 8), the actual conjunction will fall 8 days later). The uncorrected elongation = $145^{\circ} 49' - 125^{\circ} 9' = 20^{\circ} 40'$. Taken to the date of actual conj., and corrected for its equation of the centre, the sun = $145^{\circ} 49' + 8^{\circ} 2' - 2^{\circ} 10' = 151^{\circ} 41'$. True Saturn carried to this point of time = $133^{\circ} 18' + 41' = 133^{\circ} 59'$. The elongation is $151^{\circ} 41' - 133^{\circ} 59' = 17^{\circ} 42'$, fairly near the 15° for Saturn given by verse 58, when we consider that the values might be empirical.

Continuing, mean Saturn, 18 days before rising, = $125^{\circ} 9' - 1^{\circ} 30'$, (= 18 days motion at the rate $5'$ near conjunction) = $123^{\circ} 39'$. Subtracting the mean motion for the 132-20 days remaining to reach epoch, at mean motion $2'$ per day, mean Saturn at epoch = $123^{\circ} 39' - 4^{\circ} 15' = 119^{\circ} 24'$. Modern astronomy gives 122° , and the *Sūryasiddhānta* of PS $122 \frac{1}{2}^{\circ}$.

4. Epoch Position of Mars

In the case of Mars in verses 21-23, the days of first rising after epoch is given by *ṣaṭkamvayamān*, (which is

corrupt) plus 40 *nāḍis*. TS and I have emended it into *ṣaṭpañcayamān* (= 256) while NP have corrected into *ṣaṭkaikayamān* (= 216). But 216 is certainly wrong since the true longitude of Mars at that time is given as 85 *padas* (= 230°), unmistakably. For, 216 days would give the sun in the neighbourhood of 211°, but it must be greater than 230° by about 14° to 17°, for heliacal rising to take place at that time. So we have to take the emendation *ṣaṭpañcayamān*, in the absence of any other *plausible* one. There is doubt only about the mid-digit and no other number can fit, as can be verified.

At 256-40 days from Epoch, the mean sun = Vās-Pauliṣa Mean Sun at epoch + 256-40 days' motion = $357^{\circ} 37' + 253^{\circ} = 250^{\circ} 37'$. The uncorrected elongation is seen to be $20^{\circ} 37'$, not far from the 14° to 17° given for Mars by verse 58, and Hindu astronomy. The mean longitude of Mars, 85 *padas* (= 230°) at rising, actually contains the $+12^{\circ}$ of the equation of conjunction at the point of rising. Since, according to the method of the text, this is constant, and the mean motion for the 780 days from rising to rising is 409° , (got from the 18 *padas* given plus the one revolution or 360° , understood) which is also constant, the 12° can be combined with this for ease of work. So the actual mean Mars is 218° at rising. Subtracting from this the mean motion for 256 days 40 *nāḍis*, equal to $409^{\circ} \times 256 \frac{2}{3} \div 780 = 134^{\circ} 36'$, we get $218^{\circ} - 134^{\circ} 36' = 83^{\circ} 24'$, as the mean longitude of Mars at epoch. According to modern astronomy it is about 75° , and according to the *Sūryasiddhānta* of the *PS* $75 \frac{1}{2}^{\circ}$. There is a difference of about 8° . This cannot be reduced by correcting the days alone, since both the 256-40 days, and the 85 *padas* will have to be corrected. But the 85 is given unmistakably. So it must be put to small errors accumulating over a long period and/or empirical values, and/or some mistake made by somebody somewhere.

I am not taking Mercury for investigation now.

SAURASIDDHĀNTA OF PAÑCASIDDHĀNTIKĀ: PLANETARY CONSTANTS AND COMPUTATION (PS XVI, XVII, XVIII)

Chapter XVI of the *Pañcasiddhāntikā* (PS) deals with the computation of the mean star-planets, Mars etc. according to the *Saura Siddhānta*, and chapter XVII of the true motions, with their heliacal risings and latitudes. The mean planets are made true by employing the method of epicycles, as in the case of the sun and the moon, in chapters XI and X. Of the five *siddhāntas* condensed by Varāhamihira (VM) the *Saura* alone uses epicycles, and there is no evidence of its use in any other. So, in the originals also only the *Saura* must have used epicycles, since V.M. follows the original as far as necessary. Thus the *Saura* is the most mature and may be considered to begin the highest developed stage of Hindu astronomy, represented by the *Āryabhaṭīya*, the *Brāhma-sphuṭa-siddhānta*, the Later *Sūrya Siddhānta* etc.

Though VM's *Saura*, being a *karaṇa*, does not use *yuga*-cycles for the planets, the original must have had them, and they can be re-constructed from the epoch-constants given, as we have done in the case of the sun, moon, moon's apogee, and nodes. These can be seen to agree with the corresponding parameters of the *Paulīśa* quoted by Bhaṭṭotpala in his commentary on the *Bṛhat-saṃhitā*, and with the *Ārdharātriṅga-pakṣa* of Āryabhaṭa, a work now lost, but reconstructable from its description given in the *Mahābhāskarīya*, chapter VII. 21-35, and from the *Khaṇḍakhādya* of Brahmagupta which latter expressly follows the *Ārdharātriṅga-pakṣa*. Not only the *yuga*-cycles, but also the *yuga*-days, and epicycles and apogee positions and nodes agree in these. Strangely enough, the *Later Sūrya Siddhānta* does not agree with the 'Old' in many things. In the matter of computing the

latitudes of the star-planets, the *Saura* gives the same method as the *Ardharātriaka-pakṣa* combining two types of latitudes, but the *Khaṇḍakhādya* follows the *Āryabhaṭīya* itself exactly as propounded in the *Mahābhāskarīya*, VI. 52-55.

As for agreement of VM's *Saura* with the other *siddhāntas* of the period, a perusal of the table appended will show this. But it must be noted that the agreement in mere number of cycles is not real agreement, because, the *yuga*-days being different, there will be difference in the calculated mean values. But, at the period we are considering, c. 500 A.D., the mean positions fairly agree with one another, and also with what would be got by modern astronomy, showing thereby the accuracy of their observations.¹ There is agreement in the degrees for heliacal rising and setting and the method of computing the star-planets between the *Saura* of the *PS* and the Later *Sūrya Siddhānta*, though the epicycles differ in many ways.

Another important matter should be mentioned. In XVI. 10-11, and XVII. 10-11a, VM gives corrections, which are his own, to secure agreement with observation to make the *Saura* fit for correct almanac-making, which naturally will be demanded by the literate. Thus, in XVI. 10-11, certain *bijas* are given to correct the mean

1. For example, for *PS*'s epoch, all except the *Later Sūrya Siddhānta* give nearly 236° for Rāhu, including the modern. This is seen only in the Rāhu of the *Later Sūrya Siddhānta*. Evidently there is error of reading here. In I. 35 of the *Later Sūrya Siddhānta*, the original should have been वस्वन्वियमाश्विनिखिद्रसका; instead of बह्वग्नि etc. This mis-reading must have occurred before the commentator Raṅganātha, for he gives अष्टरामकृतिरामद्विमिताः. If what I suggest is correct, 3° will be added to the 232° 29' got according to the wrong reading making Rāhu = 235° 29', giving fair agreement.

of Mars, Jupiter and Saturn and the *śighra* of Mercury and Venus. The corrections amount, in terms of *yuga*-cycles, to : Mars + 57; Mercury + 400, Jupiter — 33½; Venus — 150; and Saturn + 25. These corrections are similar and approximately equal to the famous *वाग्मावोन* correction on the *Āryabhaṭīya*, propounded by his successors in his school, to correct his cycles to agree with their observation. I do not suggest that VM was aware of the *वाग्माव* correction in that form, but the tendency to correct the earlier results with *bījas* based on observations, is found everywhere, whether north or south, a healthy sign of the growth of the science. In XVII. 10-11a, VM attempts to correct Mercury and Venus to secure agreement with observation. (See notes under 10-11a).

Another thing is to be noted. In (1) the *Āryabhaṭīya*, in (2) the *Ardharātrika-pakṣa* (which means *ipso facto* the *Khaṇḍakhādyaka*, VM's *Saura* and Bhaṭṭotpala-quoted *Paulīśa*) and in (3) the Later *Sūrya Siddhānta*, the *yuga* cycles are such that the mean planets are all zero at the beginning of Kali, the moon's apogee is 90°, and the moon's node 180°. Now, the *Āryabhaṭīya* has equal *yuga-pādas*, *Kṛta*, *Tretā*, *Dvāpara* and *Kali*, i.e., they are equal in length. The other *siddhāntas* have unequal *yuga* divisions, *Kṛta* being 4 parts, *Tretā* 3 parts, *Dvāpara* 2 parts and *Kali* 1 part. If the other *siddhāntas* also postulate, like the *Āryabhaṭīya*, that the planets were created and began to move from the beginning of the *Kalpa* from a zero position, then the cycles should be divisible by 20. But they are not so divisible in all. This necessity is avoided by postulating a time later than the beginning of the *Kalpa* called 'the time of creation of planets' by the Later *Sūrya Siddhānta*, as stated in the verse,

ग्रहक्षदेवदैत्यादि सृजतोऽस्य चराचरम् ।

कृताब्धिवेदा दिव्याब्दाः शतघ्ना वेधसो गताः ॥ (I. 24)

and by having both the number of cycles and *yuga*-days divisible by four. In the case of the moon's apogee, the cycles should be odd, and in the case of Rāhu, the cycles should be even, but not divisible by 4. These necessary conditions are indeed found in the Later *Sūrya Siddhānta* and its kind. Thus, if there is any observed difference in the mean planets, moon's apogee and nodes, they must be due to the 3600 years elapsed after Kali, for the period c. 499 A.D. But the observed differences should be only small, and due to error of observation. The cycles must have been, and have been, constructed with an eye to this also. In fact, the number of cycles have been determined by observation, and by using the Diophantine equation (*kuttaka*). The difference of just 300 days in the length of the *yuga*, (it does not matter much if it is 328 days, as in the Later *Sūrya Siddhānta*), to secure equality at c. 499 A.D., between the *Ārdharātrika-pakṣa* and the *Āryabhaṭīya*, which is called, for distinction, the *Audayika-pakṣa*, meaning the type beginning the day from mean sunrise at Ujjain, provided the number of cycles are the same. (See table). There is a difference of just a quarter of a day accumulated from zero Kali to c. 499 A.D. and the difference is made zero at this point of time. We shall now proceed to examine the verses, one by one, in chapter XVI and XVII.¹

एष निशार्धेऽवन्त्यां
ताराग्रहनिर्णो(? निर्णयोऽ)कसिद्धान्ते ।
तत्रेन्दुपुत्रशुक्रौ तुल्यगतौ
मह(? मध्य) मार्केण ॥ 1 ॥

'The following is the determined position of the star-planets at midnight at Ujjain according to the *Saura Siddhānta*. For their computation, the mean sun should be taken as the mean Mercury and Venus.'

1. For full explanations and details, see 'The main characteristics of Hindu Astronomy', *supra*, pp. 118-40.

Note : I follow TS's emendations.

Example : Find the mean Venus at 1,20,553 days after epoch for the star planets, viz. 427 Śaka elapsed midnight at Ujjain. This is the mean sun at 1,20,553.5 days from midday, of epoch taken for the sun (IX. 1). Therefore the mean Venus = the mean sun = $(1,20,553.5 \times 800 - 442) \div 2,92,207 = 17^\circ 18' 27''$.

जीवस्य शताम्यस्तं

द्वित्रियमाग्निचि (? त्रि)सागरैर्व(?वि)भजेत् ।

द्युगणं कुजस्य चन्द्रा-

ऽऽहतं तु सप्ताष्ट[षड्]भकम् ॥ 2 ॥

सौरस्य सहस्रगुणा-

द(द)तुरस(शून्य)र्तुषट्कमुनिखैकैः ।

यल्लब्धं ते भगणा

शेषा मध्या(ध्य)ग्रहाः क्रमेणैव ॥ 3 ॥

दश दश भगणे भगणे

संशोध्यास्तत्पराः सुरेज्यस्य ।

मनवः कुजस्य देयाः

शनेश्च बाणा विशोध्यास्तु(स्तु) ॥ 4 ॥

राशिचतुष्टयमंशद्वयं-

कला विंशतिर्वसुसमेताः ।

नव वेदाश्च विलिप्ता

शनेर्धने(नं) मध्यमास्थे(मस्थे) वा ॥ 5 ॥

अष्टौ भाया(गा) लिप्तर्त्त(प्ता

कृत)वः खमुक्षो(पक्षौ) गुरौ विलिप्ताश्च ।

क्षेपः कुजस्य ज(य)मतिथि-

पञ्चत्रिंशच्च राश्याद्याः ॥ 6 ॥

शतगुणिते बुधशीघ्रं

स्वरनवसप्ताष्टभाजिते क्रमशः ।

अत्रार्धपञ्चमाः त-

त्पराश्च भगणहत(ताः) क्षेपः ॥ 7 ॥

सितशीघ्रं दशगुणिते

द्विगुणे भक्ते स्वराणवाश्रि(श्वि)यमैः ।

अर्कै(र्धै)कादश देया

बिलिप्ता(सिका) भगणसंगुणिता ॥ 8 ॥

सिंहस्य वसुयमांशाः

स्वरोद(रेन्द)वो लिसिका क्षशीघ्रघनम् ।

शो[ध्याः] सितस्य विकलाः

शशिरसनवपक्षा(क्ष)गुणदहनः ॥ 9 ॥

To get the mean Jupiter, multiply the days from epoch by 100, and divide by 4,33,232. Revolutions etc. are got. Deduct 10''' per revolution. Add 8° 6' 20'', the mean at epoch. (This is called *Kṣepa*.) (A *bīja* correction is given by VM, to this, for which see verses 10-11 below.)

To get mean Mars, divide the days by 687. Revolutions etc. are got. Add 14''' per revolution. Add 2^r 15° 35' 0'', the mean at epoch. (See verses 10-11, below for *bīja* correction.)

To get mean Saturn, multiply the days by 1000 and divide by 1,07,66,066. Revolutions etc. are got. Deduct 5'' per revolution. Add 4^r 2° 28' 49'', the mean at epoch. (See verses 10-11 below for *bīja* correction).

To get the Śighra of Mercury, multiply the days by 100 and divide by 8797. Revolutions etc. are got. Add 4½''' per revolution. Add 4^r 28° 17' 0'', for the *śighra* at epoch. (See verses 10-11, below for *bīja* correction.)

To get the Śighra of Venus, multiply the days by 10 and divide by 2247. Revolutions etc are got. Add 10½''' per revolution. Add 8^r 27° 30' 39'', the *śighra* at epoch. (See verses 10-11 below for *bīja* correction).

Note i. I follow TS's emendations, except in vese 6. where I have read खमक्षौ as खपक्षौ instead of their खमक्षौ,

which is meaningless. But their meaning, 20, is all right. In 7, the word क्षेप can stand, and need not be emended as done by them.

Note ii. The word मध्य with reference to Mars, Jupiter and Saturn is mean planet in modern parlance, and शीघ्र with reference to Mercury and Venus is mean planet according to modern terminology.

Note iii. How to get the days from epoch has already been explained, and it should only to be brought to the midnight following, to be used here.

Example 1. Find the mean Mars at 1,20,553 days from the midnight following the Romaka epoch, which is the epoch given for the star-planets.

$$\begin{array}{rcl}
 1,20,553 \div 687 & = & 175 \text{ revolutions,} \quad 5^{\circ} 21' 52'' 40'' \\
 \text{The revolution correction} & = & 175 \times 14'' = \quad + 41'' \\
 \text{Kṣepa or mean at epoch} & = & 2^{\circ} 15' 35' 0'' \\
 \text{Mean Mars at required date} & = & \underline{\underline{8^{\circ} 7' 28' 21''}}
 \end{array}$$

Example 2. Find the Śīghra Venus at 1,20,553 days for epoch;

$$\begin{array}{rcl}
 1,20,553 \times 10 \div 2247 & = & 536 \text{ revolutions,} \quad 6^{\circ} 2' 19' 23'' \\
 \text{Revolution correction} & = & 536 \times 10\frac{1}{2}'' = \quad + 1^{\circ} 33' 48'' \\
 \text{Śīghra at epoch} & = & 8^{\circ} 27' 30' 39'' \\
 \text{Śīghra of Venus at 1,20,553 days} & = & \underline{\underline{3^{\circ} 1' 23' 50''}}
 \end{array}$$

Note iv. The rules given to find mean planets etc. depend on the fact that there are approximately 100 revolutions of Jupiter in 4,33,232 days, one revolution of Mars in 687 days, 1000 revolutions of Saturn in 1,07,66,066 days, 100 Śīghra (truly mean) revolutions of Mercury in 8,797 days and 10 of Venus in 2247 days. The

revolution corrections make these exact. The epoch constants are the means at epoch.

Note v. From the rules given we can reconstruct the *yuga* cycles of the original *Saura-siddhānta*, of which the *Saura* of the *PS* is a *karāṇa*, and from these the epoch constants. These we shall do now. The *yuga* days of the original *Saura* are 1,57,79,17,800, as computed from the short *Saura-yuga* given in I. 14, from which it can be computed that in 1,80,000 years there are 6,57,46,575 days, since the *yuga* is 43,20,000 years, being 24 times the short *yuga*.

We might now verify, by calculation, the *yuga* revolutions (*yuga-paryaya*) and epoch constants (*kṣepa*) of the several planets.

Jupiter : Yuga revolutions

$$\begin{array}{rcl}
 1,57,79,17,800 \times 100 \div 4,33,232 & = & \\
 & 3,64,220 \text{ rev.,} & 0^{\circ} 17' 25'' 1'' \\
 \text{Revolution correction} & = & \\
 & 3,64,220 \times 10'' & = -16^{\circ} 51' 43'' \\
 \hline
 \therefore \text{The number of rev. etc. in the } yuga & = & \\
 & 3,64,220 \text{ rev.,} & 0^{\circ} 0' 31' 18'' \\
 & & \hline
 \end{array}$$

The error in the *Karāṇa* method is 31' 18'' in 43,20,000 years, which is negligible when we consider that the rule is given in a *karāṇa*, which is not intended to be used for such a long period. The *yuga* revolutions 3,64,220, is indeed that given in the original, as seen from the *Ārdharātriṅka-pakṣa* and Bhaṭṭotpala's *Paulīṣa*, and *Khaṇḍakhādyaka*.

Epoch constant (kṣepa) for Jupiter

The epoch is 427 Śaka, i.e. $427 + 3179 = 3606$ years from zero Kali, i.e. midnight. -3 *nādis*, 9 *vinādis*. For

$$3606 \text{ years the motion is } 3,64,220 \times \left(\frac{1}{1200} + \frac{1}{1200 \times 600} \right) \\ = 304 \text{ rev.,} \quad 0^{\circ} 8' 6'' 36''$$

$$\text{Subtracting the motion for } 3 \text{ } n\ddot{a}\ddot{d}is, \\ 9 \text{ } v\ddot{in}\ddot{a}\ddot{d}is \quad \quad \quad -16''$$

$$\text{Jupiter's epoch constant got} \quad = \quad \underline{\underline{0^{\circ} 8' 6'' 20''}}$$

This is exactly what is given above in verse 6.

Saturn: Yuga revolutions

$$1,57,79,17,800 \times 1000 \div 1,07,66,066 = \\ 1,46,564 \text{ rev.,} \quad 0^{\circ} 3' 26' 12''$$

$$\text{The cycle correction} = 1,46,564 \times 5'' = \underline{\underline{- 3^{\circ} 23' 34''}}$$

$$\therefore \text{The } yuga \text{ cycles got} = 1,46,564 \text{ rev.,} \quad \underline{\underline{0^{\circ} 0' 2' 38''}}$$

This is indeed the *yuga* cycles given in the *Ardharātri-ka-pakṣa* etc. neglecting the small error of 2' 38'' accumulating in 43,20,000 years, due to the *karāṇa* roughness.

Epoch constant for Saturn

$$1,46,564 \left(\frac{1}{1200} + \frac{1}{100 \times 1200} \right) = 122 \text{ rev.} \quad 4^{\circ} 2^{\circ} 28' 55''$$

$$\text{Deducting for } 3 \text{ } n\ddot{a}\ddot{d}is, 9 \text{ } v\ddot{in}\ddot{a}\ddot{d}is \quad \quad \quad - 6''$$

$$\text{The epoch constant got} \quad = \quad \underline{\underline{4^{\circ} 2^{\circ} 28' 49''}}$$

Mars: Yuga revolutions

$$1,57,79,17,800 \div 687 = 22,96,823 \text{ rev.} \quad 6^{\circ} 29^{\circ} 4' 59''$$

$$\text{Rev. correction} = 22,96,823 \times 14'' = \underline{\underline{+ 428^{\circ} 52' 5''}}$$

$$\therefore \text{Yuga-cycles} \quad = \quad 22,96,823 \text{ rev.} + \underline{\underline{11^{\circ} 27^{\circ} 57' 4''}}$$

= 22,96,824, in round numbers, being short only by 2° 3', negligible in the long period. We see agreement with the original.

Epoch constant for Mars

$$\begin{aligned}
 \text{The epoch constant is } 22,96,824 \left(\frac{1}{1200} + \frac{1}{600 \times 1200} \right) \\
 = 1917 \text{ rev., } \quad 2^{\circ} 15' 36'' 43'' \\
 \text{For 3 } nādis, 9 \text{ } vinādis \quad \quad \quad - 1' 39.5'' \\
 \text{less } 1917'' \div 5 \quad \quad \quad \quad \quad - 6'' \\
 \hline
 \text{The epoch constant} \quad \quad \quad = \quad \underline{\underline{2^{\circ} 15' 34'' 58''}}
 \end{aligned}$$

There is agreement.

Mercury: Yuga revolutions

$$\begin{aligned}
 1,57,79,17,800 \times 100 \div 8997 &= \\
 &1,79,36,998 \text{ rev. } \quad 11^{\circ} 21' 41'' 33'' \\
 \text{Rev. correction} &= 1,79,36,999 \times 4\frac{1}{2}'' = + \quad \underline{\underline{1^{\circ} 13' 41'' 15''}} \\
 \text{The yuga cycles} &= 1,79,37,000 \text{ rev.,} \quad \underline{\underline{0^{\circ} 5' 22' 48''}}
 \end{aligned}$$

There is fair agreement with the *Ārdharātriṅka-pakṣa* etc. with an excess of $5^{\circ} 22' 48''$ in the *yuga*, which need not be considered great in a *karaṇa* rule. $4 \frac{7}{16}$ instead of $4\frac{1}{2}''$ would have taken this difference also into account.

Epoch constant for Mercury

$$\begin{aligned}
 1,79,37,000 \left(\frac{1}{1200} + \frac{1}{600 \times 1200} \right) &= \\
 &14972 \text{ Rev., } \quad 4^{\circ} 28' 30'' 0'' \\
 \text{Subtracting for the excess 3 } nādis, 9 \text{ } vinādis &- \quad 23' 53'' \\
 \text{For } 1/16 \text{ repeat the correction} &+ \quad \underline{\underline{16''}} \\
 \text{Epoch constant} &= \quad \underline{\underline{4^{\circ} 28' 17'' 23''}}
 \end{aligned}$$

Here the constant seems to have been given to the nearest minute.

Venus: Yuga revolutions

$$\begin{array}{rcl}
1,57,79,17,800 \times 10 \div 2247 = & & \\
& 70,22,331 \text{ Rev.,} & 1^r \ 8^\circ \ 55' \ 54'' \\
\text{Revolution correction} = & & \\
& 70,22,331 \times 10\frac{1}{2} = 56 \text{ Rev.,} & 10^r \ 21^\circ \ 47' \ 52'' \\
\text{yuga cycles} = 70,22,388 & & \underline{\underline{0^r \ 0^\circ \ 43' \ 46''}}
\end{array}$$

There is a small error of 43' 46'', negligible in the long period of *yuga*, due to the *karaṇa* rule. 10 85/178 would have been very correct.

Epoch constant for Venus

$$\begin{array}{rcl}
\text{Epoch constant, } 70,22,388 \left(\frac{1}{1200} + \frac{1}{600 \times 1200} \right) = & & \\
& 5861 \text{ rev.,} & 8^r \ 27^\circ \ 35' \ 38'' \ .4 \\
\text{Less for 3 } nādis, \ 9 \ vinādis & - & 5' \ 2'' \\
\text{Extra in the correction} & + & 2' \ 4'' \\
\text{Epoch correction (in full agreement)} = & & \underline{\underline{8^r \ 27^\circ \ 30' \ 39''}}
\end{array}$$

In the sun, moon, Rāhu, and moon's apogee too we see such exact agreement with the *Ārdharātriṇī-pakṣa*, *Khaṇḍakhādya* and *Bhaṭṭotpala*-quoted *Pauliṣa*, from which we can conclude that the source of VM's *Saura* is the *Old Saura Siddhānta*.

क्षेप्याः स्वरेन्दुविकलाः

प्रतिवर्ष(र्षे) मध्यमक्षितिजो(जे) ।

दश दश गुरोर्विशोभ्याः

शनैश्चरे सार्धसप्तयुताः ॥ 10 ॥

पञ्चाब्द(ब्ध)यो विशोभ्याः

सिते, बुधे स्ता(खा)श्विसप्तयुताः ।

खखवेदेन्दुविका(क)लिकाः

शोभ्याः स्युः सुरपूजितस्य मध्याः स्युः ॥ 11 ॥

'Deduct 10" per year from mean Jupiter. Add 17" per year to mean Mars. Add $7\frac{1}{2}$ " per year to mean Saturn. Add 720" per year to the *Śighra* ('mean, according to modern parlance,) of Mercury. Subtract 45" per year from the *Śighra* (modern 'mean') Venus. In addition, subtract 1400" or 23' 20", constant, from Jupiter's mean.

Note 1. I follow TS's corrections.

Note 2. These corrections are obviously VM's own, to secure agreement with observation, because VM sees the *Saura* used widely for almanac making (besides himself being its follower) and uses these *bīja* corrections to the *Saura*. Being VM's own, we cannot verify the numbers used, but we can compare these corrections with those given by the followers of the *Āryabhaṭīya* belonging nearly to his time. Note how close they are, and commend the tendency to observe and correct, instead of blindly following the masters.

The Kerala school following the *Āryabhaṭīya* gives the famous *Vāgbhāva* correction :

वाग्भावोनाच्छकाब्दाद् घनशतलथहान्मन्दवैलक्ष्यरागैः

प्राप्ताभिर्लित्तिकाभिर्विरहिततनवश्चन्द्रतत्तुङ्गपाताः ।

शोभानीरुढसंविद्गणकनरहतान्मागराप्ताः कुजाद्याः

संयुक्ता ह्यारसौराः सुरगुरुशुक्रजौ वर्जितौ भानुवर्जम् ॥

(*Kaṭapayādi* notation is used here.) According to this, the corrections per annum are for Mars + 11.5", for Mercury + 105", for Jupiter - 12", for Venus - 39", and for Saturn + 5". See that these compare well with VM's.

Example : Give the *bīja* corrections for Mars and Venus at 1,20,53 days from epoch.

This is 330 years.

The correction for Mars = $330 \times 17''$ (positive) = $+ 1^\circ 33' 30''$. The correction for Venus = 330×45 (negative) = $-4^\circ 7' 30''$.

Thus *bija*-corrected mean Mars of date is $8^\circ 9' 1' 51''$, and *bija*-corrected Venus, $2^\circ 27' 16' 20''$.

शीघ्राख्योऽर्कोऽन्येषां

भौमादीनां तु परिधयोऽद्विगुणाः ।

पञ्चत्रिंशत्स(न्म)नवो-

ऽष्टयः सु(स्व)रास्त्रिंश[च्च भाग]ाः ॥ XVII. 1

'For the other planets (i.e. other than Mercury and Venus, viz., for Mars, Jupiter and Saturn), the sun is their *śighra*. The epicycles of equation of the apsis of Mars etc. are twice, 35° , 14° , 16° , 7° , and 30° , (i.e., of Mars 70° , of Mercury 28° , of Jupiter 32° , of Venus 14° and of Saturn 60° .)'

Note 1. I follow TS's emendation in पञ्चत्रिंशन्मनवः. But I read सुरा as स्वराः and not शराः, like TS, because स्वराः is nearer the given reading सुराः and also 14° is the epicycle given in the *Ārḍharātrika-pakṣa* etc. Five *mātrās* are wanting in the last foot, and it must be supplied with some such words as भागाः, as all numbers are already given. But TS make it षड्युगस्त्रिंशः and, strangely enough, translate it as 24, confusing the 'addition' mentioned by themselves for 'subtraction'. (However, on page XXIII of the Introduction Thibaut gives the correct 60° , $30^\circ \times 2 = 60^\circ$.)

Note 2. The first foot is to be read with verses 7 and 8 of chap. XVI where the *śighra* of Mercury and Venus have already been given. Properly speaking, the matter in this foot should have been given in chap. XVI.

रसभववसुदेवार्क

विंशतिगुणिताः कुजस्य दशकोणा(ना)ः ।

मन्दगतिनाम(?) भागाः

कुजबुधगुरुशुक्रसो(सौ)राणाम् ॥ 2 ॥

‘6, 11, 8, 4, 12, multiplied by 20, Mars’s being less by 10°, (i.e. 120°, 120°, 160°, 80°, and 240°) are the apogee positions of Mars, Mercury, Jupiter, Venus and Saturn.’

Note 1. I follow TS’s emendations. मन्दगतिनामभाषाः does not make any sense. But the meaning is obvious, it must mean apogee positions. Some drastic emendation of the word can be made to give this meaning, but I am against such an emendation.

Note 2. These positions agree with those given in the *Ārdharātrika-pakṣa* etc., as also in the *Āryabhaṭīya*. The correct positions according to modern astronomy are 128°, 234°, 170°, 290°, and 244°, respectively.

Note 3. The apogee 80° for Venus and the epicycle 14° are the same as given for sun. The apogee position 80° given is near the perigee position of modern astronomy, so far away. We shall explain this under verses 10-11a, below.

शीघ्रपरिधावथांशाः

कृतगुणपक्ष(क्ष)द्विवह्निशीतकरः ।

पक्षस्वरा ख(ख)षय(इय)-

मा(माः) खकृताः कुजादीनाम् ॥ 3 ॥

‘The degrees of epicycles of conjunction of Mars is 234, of Mercury 132, of Jupiter 72, of Venus 260, and of Saturn 40.’

Note 1. I have generally adopted TS’s corrections. But the text is corrupt in the third foot, and TS’s correction itself wants one *mātrā*. I would read the third and fourth foot thus :

पक्षस्वराश्च खं षड्यमा खकृताश्च कुजादीना ।

This would follow the original work.

Note 2. The values agree with the *Ārdharātrika-pakṣa*, *Khaṇḍakhādya* and Bhaṭṭotpala-quoted-*Pauliṣa* group, as to be expected.

Verses 4-9 give the method of computation of true planets.

शीघ्रान्मध्यमहीनाद्
वा(रा)शिञ्जितये गतैष्य(ष्य)दंशे(श)ज्ये ।
भुःकोटी तत्परतः
षड्भ्याः(भिः) पत(ति)ते स एव विधिः ॥ 4 ॥

खपरिधिगुणिते भाज्ये
खर्तुगुणै[स्ते] विपरिग(रिण)ते तच्च ।
कोटिफलं व्यासार्धं
मृगकक्ष्यादौ चापचयाः(यम्) ॥ 5 ॥

तद्भुजकृतियोगपदै-
भजियेत्रनभूजखं(भज्यं भुजं ख) पूर्वम ।
तच्चापार्धं मन्दे
हानिधनं शीघ्रकेन्द्रवशात् ॥ 6 ॥

स्फुटयित्वैवं मन्दं
मध्याच्च विशोध्य तस्य भुजम् ।
परिण स्य कार्मुकार्धं
तन्मन्देनैव धनहानी ॥ 7 ॥

मध्यात् पुरो(न)र्विशोध्य
स्त(त)स्माद् बाहुन(हुन) तस्य यच्चापम् ।
तन्मध्यमे क्षयधनं
कर्तव्यं मन्दकेन्द्रवशात् ॥ 8 ॥

एवं स्फुटमध्याख्यां(ख्यं)
शीघ्रात् संशोध्य पूर्वविधिनैव ।
आदिवदात्पे(सं)चापं
स्फुटमध्याख्योप(ख्ये) चयापचयः(यम्) ॥ 9 ॥

The first step:

‘Deduct the mean from the *śīghra*. If the remainder (called *śīghra-kendra*) is within 90° , sin. *śīghra-kendra* is called *bhuja*, and sin ($90^\circ - \text{śīghra-kendra}$) is called *koṭi*.

If *śīghra-kendra* is more than 90° and less than 180° , subtract it from 180° . Taking this as the *śīghra-kendra*

sin. *śighra-kendra* is *bhuja* and $\sin (180^\circ - \text{śighra-kendra})$ is *koṭi*. If *śighra-kendra* is more than 180° and less than 270° , deduct 180° from it and take this as *śighra-kendra*. Sin. *śighra-kendra* is *bhuja* and $\sin. 90^\circ - \text{śighra-kendra}$ is *koṭi*. If *śighra-kendra* is from 270° to 360° , deduct it from 360° and take its sine as the *bhuja* and $\sin 90^\circ - \text{śighra-kendra}$ is the *koṭi*. (The *bhuja* of *manda-kendra* is to be found in the same way using *manda-kendra* in the place of *śighra-kendra*.)¹

The *bhuja* and *koṭi* must be multiplied by the planet's epicycle of conjunction and divided by 360. Thus transformed, they are called *bhuja-result* and *koṭi-result* pertaining to the equation of conjunction. If the *śighra-madhya* is from 270° to 90° , the *koṭi-result* is to be added to 120 (the R. of the PS). If *śighra-madhya* is from 90° to 270° , the *koṭi-result* is to be subtracted from 120. Square this and add it to the square of the *bhuja-result*. Find its square root, and by this divide $120 \times \text{bhuja-result}$. Find arc-sine of this. Subtract half this from the longitude of apsis if the *śighra-kendra* is from 0° to 180° . Add if from 180° to 360° .

Second step: Half rectifying the apogee position, thus, deduct it from the mean. The result is to be used as the anomaly of the apsis in the second step. As we find the *bhuja* of the anomaly of conjunction (*śighra-kendra*), so find the *bhuja* of the anomaly of apsis. Multiply the *bhuja* by the *manda* epicycle and divide by 360 and get the transformed *bhuja-result* of the apsis. (This is sine equation of the centre.) Find its arc-sine. Add half this arc to the half rectified longitude of apogee if the anomaly of apsis is from 0° to 180° and subtract if 180° to 360° . Thus the apogee is rectified completely.

1. In modern usage, for all the above we can simply say $\sin. \text{śighra-kendra}$ is the *bhuja* and $\cos. \text{śighra-kendra}$ is the *koṭi*, without taking into account the sign + or —.

Third step: Subtract this rectified apogee from the mean and thus get the anomaly of apsis. Find its *bhuja* and multiply it by the epicycle of the apsis and divide by 360° . The *bhuja*-result, (this is the equation of the centre), is got. Find the arc-sine of this, and subtract the whole of this arc from the mean of the anomaly of apsis is from 0° to 180° , and add if from 180° to 360° . The result is rectified mean.

Fourth step: Deduct the rectified mean from the *sighra*. The anomaly of conjunction is got. Find the *bhuja* and *koṭi* of this in the same manner as we did in the first step. Multiply the *bhuja* by the epicycle of conjunction and divide by 360° . Sine anomaly of conj. is got. Multiply the *koṭi*, i.e., cos. anomaly of conjunction, by the epicycle of conj. and divide by 360° . The related cosine is got. Add this to 120 if the anomaly is from 270° to 90° and subtract from 120 if from 90° to 270° . Square this, add the square of the *Bhuja* (i.e. equation of conjunction) and find the square root. Divide the equation of conj. \times 120 by this square root. The arc sine of this is the result. Add this result to the rectified mean if the anomaly of conj. is from 0° to 180° . Subtract otherwise. The geocentric true planet is got.

Note 1. In verse 4, I follow TS's reading, except that I have emended षड्भ्याः into षड्भिः instead TS's षड्भ्यः because my reading allows us subtraction or addition, as is wanted. In verse 5, I follow TS, except in the second foot, where I give the *Byhatsaṃhitā* reading. Either reading gives the same sense. In verse 6, I follow TS, except in the second foot, where I have given भाज्यं for विभजेत्, as being more likely. But the meaning is the same. In verse 7, the text requires no emending, and TS's घनहानि is unnecessary. In verse 8, like TS, I have corrected पुनः into पुनर् but I have also corrected बाहु into बाहुर which is required by grammar. In verse 9, I have

corrected मध्याह्नं into मध्याह्न्यं, which is the reading of the other manuscript. Otherwise I follow TS.

Note 2. VM here, as elsewhere in the *PS*, uses his tabular sines, where R is $120'$, as given in chap IV. So we must use his tabular values to get the R sines and R cosines. Of course, we may use the modern table, or the Siddhāntic table with $R=3438$. But there the R , 1 for the modern tables, and 3438, for the Siddhāntic tables, is to be used instead of $120'$, which is instructed here. (VM uses *bhuja* to mean sine and *koṭi* to mean cosine, instead using the word *jyā*).

Note 3. The method is the same as what is found in the Later *Sūrya Siddhānta*, with some changes for convenience. But in the matter of the number or order of the steps, the *Āryabhaṭīya* and the *Siddhānta-Śiromaṇi* differ. This is because, correctly speaking, the first two steps are useless, and the last two steps alone are necessary. In essence, the third serves to get the true heliocentric position, and the fourth to convert the heliocentric position into geocentric. The earlier steps are in the fond hope of getting correct positions agreeing with observation, while the real trouble is in the inexact parameters followed by the *Siddhāntas*.

Note 4. The second and third steps are merely akin to finding the equation of the centre and applying to the mean. The first and fourth steps are conversion of heliocentric to geocentric positions, neglecting the latitude, which is small, and does not affect the result much.

Example 1. Find the true, i.e. geocentric, Mars at 1,20,553 days from epoch, given :

Mean Mars, already found with *bīja* corr. $8^{\circ} 9' 2''$

Śighra Mars = mean sun of date = $0^{\circ} 17' 18''$

Aphelion (apogee) of Mars assumed, $3^{\circ} 20'$
(for 120° given)

Epicycle of apsis = 70° , Apsis of conj. = 234° .

First step: Anomaly of conj. =

$$\begin{aligned}\text{Śighra—mean} &= 17^{\circ} 18' - 8^{\circ} 9' 2'' \\ &= 128^{\circ} 16' .\end{aligned}$$

This is more than 90° and less than 180° .

So, subtracting from 180° , *bhujāṁśa* is $51^{\circ} 44'$,
koṭiāṁśa = $38^{\circ} 16'$

$$\text{Bhuja} = 94' 2'' . \quad \text{koṭi} = 74' 17'' .$$

$$\text{Bhuja result} = 94' 2'' \times 234^{\circ} \div 360^{\circ} = 61' 14''$$

$$\text{Koṭi result} = 74' 17'' \times 234^{\circ} \div 360^{\circ} = 48' 17''$$

As anomaly of conj. is between 90° and 270° , this is subtractive from $120'$.

$$\therefore 120' - 48' 17'' = 71' 43'' .$$

$$120' \times \text{bhuja-result} \div \sqrt{71' 43''^2 + \text{bhuja-result}^2}$$

$$= \frac{120' \times 61' 14''}{\sqrt{71' 43''^2 + 61' 14''^2}} = 77' 55''$$

$$\text{Arc for } 77' 55'' = 40^{\circ} 30' .$$

$$\frac{1}{2} \text{ arc} = 20^{\circ} 15'$$

Subtracting from aphelion (since anomaly of conj. is from 0° to 180°) $110^{\circ} - 20^{\circ} 15' = 89^{\circ} 45'$, which is the half corrected aphelion.

Second step:

$$\text{Anomaly of apsis} = 249^{\circ} 2' - 89^{\circ} 45' = 150^{\circ} 17'$$

This is more than 90° and less than 180° .

$$\text{So, Bhuja degrees} = 20^{\circ} 43' . \quad \text{Bhujā} = 42' 24'' ,$$

Bhuja result (i.e. eq. of the centre) = $42' 24'' \times 70^\circ / 360^\circ = 8' 15''$.

Arc for this = $3^\circ 56'$; $\frac{1}{2}$ arc = $1^\circ 58'$.

This is additive because An. of apsis is between 0° and 180° .

Half rectified aphelion + $1^\circ 58' = 91^\circ 43' =$ full rectified aphelion.

Third step :

The corrected anomaly of apsis = $249^\circ 2' - 91^\circ 43' = 157^\circ 19'$. The *bhuja* degrees = $22^\circ 41'$. *Bhuja* = $46' 27''$. The *Bhuja-result* = $46' 17'' \times 70^\circ / 360^\circ = 9' 0''$. Arc of $9' 0'' = 4^\circ 18'$, deductive because an. of apsis is between 0° and 180° .

Mean-arc = $249^\circ 2' - 4^\circ 18' = 244^\circ 44'$ is corrected (for eq. centre).

Fourth step :

An. conj = $17^\circ 18' - 244^\circ 44' = 132^\circ 34'$. *Bhujāṁśa* = $180^\circ - 132^\circ 44' = 47^\circ 26'$, *koṭiāṁśa* = $42^\circ 34'$. *Bhuja* = $88' 20''$, *koṭi* = $81' 8''$. *Bhuja-result* = $88' 20'' \times 234^\circ \div 360^\circ = 57' 25''$. *Koṭi result* = $81' 8'' \times 234^\circ \div 360^\circ = 52' 44''$.

As An. of conj. is in 270° to 90° , this is deductive from $120'$. So, $120' - 52' 44'' = 67' 16''$. $120' \times \text{bhuja-result} \div \sqrt{\text{bhuja result}^2 + 67' 16''^2} = 77' 32''$.

Arc of this taken as sine = $46^\circ 16'$, additive because An. conj is from 0° to 180° .

So, corrected mean arc = $244^\circ 44' + 40^\circ 16' = 285^\circ 0' =$ geocentric true Mars.

Example 2. Find geocentric Venus at 1,20,553 days from epoch, given :

The *siḡhra* of Venus = $2^r 27^\circ 16' 20'' = 87^\circ 16'$

Mean Venus = mean sun = $17^\circ 18'$

Aphelion of Venus = $2^r 20^\circ = 80^\circ$.

Epicycle of conj. of Venus = 260.

Epicycles of the apsis = 14° .

First step :

An. conj = $87^\circ 16' - 17^\circ 18' = 69^\circ 58'$.

The bhuja degrees = $69^\circ 58'$. Koṭi degrees = $20^\circ 2'$.

Bhuja = $112' 42''$. Koṭi = $41' 5''$.

Bhuja-result = $112' 42'' \times 260^\circ \div 360^\circ = 81' 24''$.

Koṭi-result = $41' 5'' \times 260^\circ \div 360^\circ = 29' 40''$.

An. conj. is between 270° and 90° . So the *koṭi-result* is additive to $120'$.

So $120' + 29' 40'' = 149' 40''$.

Bhuja-result $\times 120' \div \sqrt{81' 24''^2 + 149' 40''^2} = 57' 20''$.

Arc $57' 20'' = 28^\circ 33'$. Half arc = $14^\circ 17'$.

This is subtractive to aphelion (as An. conj. is from 0° to 180° .)

$80^\circ - 14^\circ 17' = 65^\circ 43' =$ half corrected aphelion.

Second step :

An. of apsis = mean — half cor. aphelion = $17^\circ 18' - 65^\circ 43' = 311^\circ 35'$.

The *bhuja* degrees are $48^\circ 25'$.

Bhuja = $89' 44''$.

Bhuja result = $89' 44'' \times 14^\circ \div 360^\circ = 3' 29''$.

Arc. $3' 29'' = 1^\circ 40'$. Half arc = $50'$, subtractive as an. conj. is from 180° to 360° .

Corrected aphelion = Half corrected aphelion — $50' = 65^\circ 43' - 50' = 64^\circ 53'$.

Third step :

$$\begin{aligned}\text{An. of apsis} &= \text{mean corrected aphelion} \\ &= 17^\circ 18' - 64^\circ 53' = 312^\circ 25'.\end{aligned}$$

$$\text{Bhuja degrees} = 47^\circ 35'. \quad \text{Bhuja} = 88' 33''.$$

$$\text{Bhuja-result} = 88' 33'' \times 14^\circ \div 360^\circ = 3' 27''.$$

Arc sine $3' 27'' = 1^\circ 39'$, additive as An. of apsis is 180° to 360° .

So, mean Venus + arc = $17^\circ 18' + 1^\circ 39' = 18^\circ 57'$ is the eq. cent. corrected mean.

Fourth step :

$$\begin{aligned}\text{An. conj.} &= \text{Śighra} - \text{corrected mean} \\ &= 87^\circ 16' - 18^\circ 57' = 68^\circ 19'.\end{aligned}$$

$$\begin{aligned}\text{Bhuja degrees} &= 68^\circ 19'. \quad \text{Koṭi degrees} = 21^\circ 41' \\ \text{Bhuja} &= 111' 29''. \quad \text{Koṭi} = 44' 19''.\end{aligned}$$

$$\text{Bhuja-result} = 111' 29'' \times 260^\circ \div 360^\circ = 80' 31''$$

$$\text{Koṭi-result} = 44' 19'' \times 260^\circ \div 360^\circ = 32' 0''.$$

As An. conj. is from 270° to 90° , additive to $120'$.

$$\text{So, } 32' 0'' + 120' = 152' 0''$$

$$\text{Bhuja result} \times 120 \div \sqrt{\text{Bhuja-result}^2 + 152^2} = 62' 12''.$$

Arc sine = $62' 12''$. Half arc = $31^\circ 14'$, additive as an. conj. is 0° to 180° .

$$\text{Geocentric true Venus} = 18^\circ 57' + 31^\circ 14' = 50^\circ 11'.$$

In verse 11, below VM requires us to subtract $67'$ or $1^\circ 7'$ constant, as *bīja*-correction, after all work is over.

$$\text{Geocentric True Venus} = 50^\circ 11' - 1^\circ 7' = 49^\circ 41'.$$

सर्वे स्फुटाः स्युरेवं

ज्ञस्य तु शीघ्राद्विहाय रविमन्दम् ।

रविगिघिनतं बाहुं

बुधवत् (बुधेऽर्कवत्) क्षयघने कुर्यात् ॥ 10 ॥

शुक्रस्य सप्तव्याष्टि(षष्टि)लिप्ताः

शोध्याः स्फुटीकृतस्यै(स्यै)व ॥ 11a ॥

10. All star-planets are (geocentrically) made true in the above manner. But in the case of Mercury, this additional work is to be done : Subtract its apogee from the *śighra*, and using the sun's epicycle, find the *bhuja*-result and apply it to the mean Mercury (which, of course, is the same as the sun's), with the addition or subtraction done, as the sun's *bhuja*-result is additive or subtractive.

11a. From Venus, subtract 67', constant, after all the earlier *sphuṭa* work instructed has been done.

Note 1. The reading क्षयघने is better as it is, and T.S. need not have corrected it into क्षयघनम्. The reading बुधवत् is deficient by two syllables. Lalla's reading बुधेऽर्कवत् supplies these and makes the meaning more clear. So I have adopted it. TS's emendation बुधकरवत् with फल added for the two *mātrās* wanting, is not different in meaning from बुधवत्. Being an arbitrary rule, we cannot decide which gives the original meaning, बुधेऽर्कवत् or बुधवत्. But since Lalla's reading is not defective, at least as far as the *mātrās* are concerned, I have adopted it. Also, it is clear that it is not a substitute for any of the four steps because, if so, the separate epicycle for Mercury will be useless.

Note 2. It is clear that the rules given here are VM's own, to secure, in his opinion, better agreement with observation, because they are not given in the *Ārdharātriḥ-pakṣa* etc. and the original four steps are all in line with them, as also the Modern *Sūrya Siddhānta* and *Siddhānta Śiromaṇi*.

Note 3. The whole work of finding the true positions, especially of the star-planets, is defective in

Hindu astronomy in that the equation of the centre of Hindu astronomy neglects the second, third, etc. terms, which is considerable in the case of the moon, Mars, Saturn and Mercury, in which last case the second term is as large as 3° . In the case of Mercury and Venus it is applicable to the sun, instead of their *śighra* which is really their mean. In the equation of conjunction, the sun's true distance from the earth, and true longitude should be used, instead of the mean distance and mean longitude, as is done by Hindu astronomy. On account of these defects, computation does not agree with observation, and all sorts of hotch-potch rules are given in different astronomical works. The disagreement among themselves would itself show that they are beside the mark. When these defects are remedied, the third and fourth steps alone would be necessary, the third step giving the heliocentric true planet, and the fourth step converting the heliocentric position to the geocentric.

Note 4. In the case of Venus, there is another kind of defect. Its maximum eq. of cent. being small, it is confused with the sun's, and the sun's epicycle and apogee are given to Venus also. While its aphelion position is 290° according to modern astronomy, its apogee is given as 80° , the same as the sun's.

TABLE I

**Heliocentric Star-planets at Epoch,
(for mutual comparison)**

Planets	1 Modern astronomy	2 <i>Siddh. Śīromāṇi</i>	3 Later <i>Sūrya Siddhanta</i>	4 Earlier <i>Sūrya Siddh. of PS</i>	5 <i>Vāsiṣṭha- Pauliṣa of PS</i>	6 Interpola- tion of PS xviii
Mercury	151°	148½°	166°*	148°		161½°
Venus	269°	268½°	264°	267°	269½°	269½°
Mars	75°	76½°	78°	75½°	83½°	83½°
Jupiter	9°	9½°	9°	8°	12°	9°
Saturn	122°	122°	123½°	122½°	120°	118°

For values in column (5) see the papers 'Vāsiṣṭha-Pauliṣa epoch constants' (pp. 201-5) and 'Mars' (above). All values have been computed by me.

* This needs explanation: Perhaps the reading is *sūryāśvi* in *Sūrya Siddhanta* I. 31, which will reduce the degrees by 12. But the commentator Raṅganātha takes it as *śūnyartuḥ*.

TABLE II
Synodic periods of the Star-planets

Planets	1 Mod. Ast.	2 Sid. Śir.	3 New Sā. Siddh.	4 PS Sā. Siddh.	5 PS Vās.- Paul.	6 Interpol- ated section	7 Ptolomy
Mer. 115.	87747766	8784290	8780110	8785195	8791307	8750556	.879
Venus 583.	92136655	8968279	9001782	8975750	9092440	9060301	584.000
Mars 779.	93610175	9222494	9242712	9211734	9553326	9787326	.943
Jup. 398.	88404760	8894794	8891768	8891698	8891358	8852917	.886
Sat. 378.	09190150	0859936	0863874	0860183	0997090	1100185	.093

All values except column (7) have been computed by me. In column (6) the solar 'days' given have been converted into ordinary days.

वक्रानुवक्रकालौ

भुक्तिविशेषेण विज्ञेयः ॥ 11b ॥

11b. The times from the beginning of the retrograde motion to its end and the follow up period can be found by the daily motion (being negative,) during this period, and the convention regarding these.

Note : The terms *vakra* (retrograde) and *anuvakra* (follow-up at the end of retrograde) are technical. They are eight in number according to the *Sūrya Siddhānta*, (given by the verse) :

वक्रातिवक्रा कुटिला मन्दा मन्दतरा समा ।

तथा शीघ्रतरा शीघ्रा ग्रहाणामष्टधा गतिः ॥

The generally given reading वक्रानुवक्रा is wrong in my opinion and I have read it as वक्रातिवक्रा, and अतिवक्रा has taken the place of अनुवक्रा in the verse. The expression या वक्रा सानुवक्रा in the next verse makes it clear.

Generally the near ones are subsumed into one another. But in the case of Mars VM gives all these eight and their degrees and periods, (See paper, 'Vāsiṣṭha-Pauliśa Mars in VM's P.S.' above, pp. 169ff., under verses 33-34).

स्फुटदिनकरान्तरान्तरां(करान्तरां)शाः

चन्द्रादीनां च दर्शनी(ने) ज्ञेयाः ।

वि(र्विंशतिरु(रु)ना वसु-

शशि (delete) शिखिमुनिनवरुद्रेदि(न्द्रि)येः क्रमशः ॥ 12 ॥

12. The heliacal rising and setting of the moon, Mars, Mercury, Jupiter, Venus and Saturn are when their elongation (from the true sun) are 12°, 17°, 13°, 11°, 9° and 15°.

Note 1. I generally adopt TS's readings. But शशि is extra, and evidently a mistake which has crept into

TS's reading. To make up for this they have removed रुद्र, which is necessary, and this emendation has spoiled the correct agreement with other Siddhāntas.

Note 2. These are time-degrees, i.e. time expression degrees (*kālabhāga*) and are arbitrary in essence, and depend on the keenness of the observer's eyesight, as also the atmospheric conditions. The Later *Sūrya Siddhānta* gives 10° and 8° for Venus at superior and inferior conjunctions, and 14° and 12° for Mercury, respectively, while the *Sūrya Siddhānta* here and some others give the mean of each. (The *Mahābhāskarīya* gives even 4° or 4½° for Venus at inferior conj. and 8° at superior conj.)

मन्दग्रहान्तरज्या

स्वाष्टांशयुताऽर्किजीवशुक्राणाम् ।

सौम्यान्प(न्य)श्च पदोनां(ना)

विक्षेपोन्प(न्य)श्च शीघ्रविधौ ॥ 13 ॥

गुरुभूतनयाऽऽस्फुजितां

पादोना ज्ञयममयोमुशां(ज्ञयमयोस्तु सः)ष्टांशाः ।

त्रिज्याघ्नी कर्णाप्ता

वि(वि)योगयोःशस(योगश)विक्षेपः ॥ 14 ॥

13. Add one eighth of itself to the R (120') sine of (mean planet—apogee,) in the case of Saturn, Jupiter and Venus. For the two others, (i.e., Mercury and Mars) subtract one fourth of itself. (This is one part of latitude). There is another part of latitude using the Anomaly of conjunction.

14. From the R sine anomaly of conjunction of Jupiter, Mars and Venus subtract one fourth of itself. From that of the rest, (viz., Mercury and Saturn), add an eighth. Add both algebraically and note the direction, north or south. Multiply this by R (i.e. 120') and divide by the hypotenuse got in the last step. The latitude is got, its direction being that of the noted direction,

Note 1. This is a peculiar primitive way of finding the latitude of the star-planets. It is not found in the allied *Khaṇḍakhādya* and the quoted part of the Bhaṭṭotpala-quoted *Pauliṣa*. It is found in Āryabhata's *Ārdharātrika-pakṣa* given in the *Mahābhāskarīya* (VII. 28-33). But there are some differences between the two, and we cannot decide which follows the original *Saura* here, and which has slightly modified the original. They both mention two kinds of latitudes for each star-planet which are to be added algebraically. But there is a difference in the maximum latitudes and in the ascending nodes to be subtracted from the mean longitudes or *śighras*. VM's *Saura* implies the max. latitude 90', 90', 135', 135', and 135', for Mars, Mercury, Jupiter, Venus and Saturn, respectively, to be multiplied by sine anomaly of conjunction, and 90', 135', 90', 90', and 135' to be multiplied by sine anomaly of conjunction, no separate node being given, which means that the apogee itself is the node for the one kind of latitude, and the mean planet itself for the other. But the *Ārdharātrika* gives only one set of maximum latitudes for both, viz., 90', 120', 60', 120', 120'. It gives the nodes, 20°, 40°, 70°, 260°, and 150° for the former and 20°, nil, 70°, 260°, and 150° for the latter. Govindasvāmi's *Bhāṣya* and the *Mahābhāskarīya* being meagre, does not help us.

Note 2. By implication, we had better take the arguments of the eq. cent. used in the third step for the former, and the anomaly of conj. used and hypotenuse obtained in the fourth step for the latter.

Example: Find the latitude of Mars at 120,583 days from epoch.

In the third step of the earlier example, the sine of the argument of eq. conj. is 42' 24". As it is Mars, deducting a quarter of itself, the latitude is 31' 48", north, as this argument is between 0° and 180°. In the fourth

step, the sine of the argument of conj is $88' 20''$. For Mars one fourth is to be subtracted. So, the latitude due to this is $66' 15''$, again north, since the argument is from 0° to 180° . Adding, $31' 48'' + 66' 15'' = 98' 3''$, north.

The hypotenuse obtained there, in the fourth step, is $88' 52''$.

$98' 3'' \times 120' \div 88' 52''' = 132''$ north, is the true latitude of Mars for the day.

(As it is, this is far from the latitude obtained from using the later Siddhāntas.)

Note 3. In XVIII. 57-60 this latitude is used to correct the mean elongation given in verse 12, for the heliacal setting and rising. For this reason, we shall deal with those two verses here, to complete the *Saura*

ज्याविधिविक्षेपज्ञा-

च(च्च)रकालादम्बराष्टवेदांशम् ।

जह्या[त्] क्षिपेच्च याम्यो-

त्तरं(रे) ग्रहे स्वं यथाकक्षं(काष्टम्) ॥ XVIII. 57 ॥

एवं कृते ग्रहान्त-

रांशकैरस्तदर्शनं तेषाम् ॥ 58a ॥

57. Find the R sine of the latitude of the body. Multiply the (maximum) half-*cara* (i.e. the half difference between the half day-time or night-time from 15 *nādis*) in *vinādis*, by this R sine, and divide by 480. Add this or subtract this to or from the moon or star-planet if the latitude is north or south, according to the proper direction, i.e. according as the phenomenon (of setting or rising), takes place in the west or east respectively.

58a. When this is done, their setting or rising happens according to the interval in degrees between the sun. and the planet (given in XVII. 12, or the moon).

Note 1. There is a lot of lacunae in verse 57. The half-*cara-vinādis* meant is the maximum for the place. यथाकक्ष is meaningless here and corrected into यथाकाष्ठम्, i.e. according to the direction, but the direction is not mentioned. If north latitude, the addition is for the moon or planet in the west. Also, if north latitude, the subtraction from the moon or star planet is to be done for the east. If south latitude, the subtraction is for the west, and the addition for the east. These things can be got by a little reflection.

Note 2. The amount of degrees to be applied can simply be got by multiplying the degrees of latitude of the body by the tangent of the latitude of the place. (The equinoctial mid-day shadow of the 12" gnomon \div 12, is tan. latitude of a place.) The amount got is very rough.

The degrees wanted = Sin. half-*cara* (i.e., tan latitude of place \times tan. declination of the sun) \times sin (90°—angle for heliacal rising), nearly. The last term is neglected here, tan. declination is roughly taken as 48', half-*cara* is converted into degrees by division by 10, and the conversion into sine-function is applied to the latitude of the planet instead of the half-*cara*, as roughly equal.

Note 3. This application is what is technically called *Akṣa-dṛkkarma*. The *Āyāna-dṛkkarma* is neglected.

Note 4. The heliacal rising of the moon, and of Venus and Mercury when retrograde, takes place in the west. The heliacal setting of the moon and retrograde Venus and retrograde Mercury takes place in the east. Otherwise, all star-planets set in the west and rise in the east. (This *Siddhānta* does not envisage the setting or rising of Venus and Mercury when retrograde, no separate degree for that being given.)

चन्द्रादीनां द्वादश

मनुरवितिथ्यष्टतिथिसंज्ञैः ॥ 58 b ॥

58b. The setting and rising (mentioned in 58a) is by 12°, 14°, 12°, 15°, 8° and 15°, for the moon etc.

Note 1. This has to be taken with verse 58a. The degrees given here separately are according to the *Vāsiṣṭha-Pauliṣa*, which do not instruct the correction due to the latitude of the planet or for even the latitude of place (*ākṣa-valana*). The result will therefore be very rough.

Note 2. These degrees are necessary, as mentioned already, and the correctness of the numbers cannot be verified in the absence of the original *siddhāntas* which are now lost. But we can guess the probable values as we are sure of the relative luminosities of the planets. The numbers seem to have been misplaced. They should be *dvādaśa*, *tithi*, *manu*, *ravi*, *aṣṭa*, *tithi* (12°, 15°, 14°, 12°, 8°, 15° for moon etc.). All *siddhāntas* give 17° for Mars instead of 15°. The rest are nearly correctly given, according to one *siddhānta* or other.

त्रि(त्रि)शतविनाडीगुणितै-

घ(रु)दश(य)[वि]नाडीप्रमाणद्वैतैः ।

लब्धाङ्कप्रमाणा-

दुदयोऽस्तं वा स्फुटं वाच्यम् ॥ 59 ॥

59. Multiply the degrees by 300 and divide by the *vinādis* of oblique ascensional difference of the sign rising at that moment, (near sunset or sunrise, as the case may be), and get the respective degrees. When the distance between the sun and the planet is that much, the respective setting or rising takes place.

Note 1. This work is what is known as the conversion of time-degrees (*kālabhāga*) into degrees of distance on the ecliptic (*kṣetrabhāga*). Since the rule has to apply commonly to *Saura* on the one hand and the *Vāsiṣṭha-Pauliṣa* on the other, it has been placed last.

Note 2. Since the positions of the sun, moon and planets are given only on the ecliptic, this conversion is necessary to measure distances.

ज्ञसिताऽऽरेज्याकीं(क्यू)नाः
 शाशिनः प्रत्युत्तरं खरांशाना(शुश्र)।
 ज्ञात्वैव विक्षेपा-
 दादेशमनागतं कुर्यात् ॥ 60 ॥

60. (The rising takes place in the east when Mercury, Venus, Mars, Jupiter and Saturn are less in longitude than the sun, and the sun is less than the moon in the opposite direction, (i.e., west). Making the computation according to the instruction given above using the latitude etc, the phenomenon should be predicted.

Note. The verse is very corrupt. But knowing what it is about we can give the meaning, making possible corrections. The rising is mentioned here as it is more important for application to *dharmaśāstra* etc. But rising also envisages setting, with the word 'less' taken for 'more', and 'more' for 'less'.

Example 1. The latitude of the moon is 3° 45' N. The maximum half-cara of the place is 150 *vinādis*. The oblique-ascensional difference of the rising sign is 280 *vinādis* near sunset. Find the ecliptic distance between the sun and moon for the heliacal rising of the moon .

The time-degrees for the moon is 12°. The heliacal rising of the moon takes place in the evening. $R \sin 3^\circ 45' \times 150 \div 480 = 7 \frac{38}{80} \times 150 \div 480 = 2^\circ 23'.$

This is additive since the moon's latitude is north, and the phenomena pertains to the west. Therefore, the corrected time-degrees = 14°.

$14^\circ \times 300 \div 280 = 15^\circ$ is the distance on the ecliptic between the sun and the moon, required.

Example 2. For the same place, (i.e., max. half-*cara* 150 *vinādis*) find the ecliptic distance required for heliacal rising, given: the latitude of Mars 1° 15' N. and the oblique ascensional difference near sunrise at that time is 330 *vinādis*.

The latitude correction to the time degrees (17° for Mars) = $R \sin 1^\circ 15' \times 150 \div 480 = 2' 33'' \times 150 \div 480$ taken as degrees = 48'.

As the latitude is north, and the phenomenon pertains to the east (since it is the rising of Mars that is considered), it is subtractive.

$17^\circ - 48' = 16^\circ 12'$ is the corrected time-degrees.
 $16^\circ 12' \times 300 \div 330 = 14^\circ 44'$, is the distance on the ecliptic required.

आवन्त्यकः समासा-

छि(च्छि)व्यहितार्थं तमद्ग (?) स्फुटाङ्कसमम् ।

चक्रे वराहमिहिरः

ताराग्रहकारिकातन्त्रम् ॥ 61 ॥

प्रद्युम्न(म्न)भूमितनये

जीवे(वे) शौथवावी (?सौरेऽथ वि)जयनन्दिकृते ।

बुधेव नशा(च भग्नो)[त्साहः]

स्फुटमिदं करणं भजतां(तात्) ॥ 62 ॥

दृष्टं वराहमिहिरेण सुखप्रबोधं

.....

.....

.....

61. For the good of his disciples, Varāhamihira, born in the Avanti country (Ujjain region), wrote this section dealing with the star-planets, briefly and with the constants such as to agree with the originals.

62. A learner, discouraged by the computation of Mars by the astronomer Pradyumna, the computation of

Jupiter according to the *Saurasiddhānta*, and the computation of Mercury by Vijayanandi, can have recourse to this section of the manual.

63. By Varāhamihira has been seen, (i.e., written) (*this karaṇa*) easy to understand,

Note 1. Verses 61 and 62 clearly close the section dealing with the star-planets. Since VM says that he has improved on the earlier authors, he must be referring to chapters XVI and XVII, dealing with the *Saura*. His reference to his improvement on the *Saura* itself in the case of Jupiter must refer to the *bīja* correction made by him in XVI. Indeed, his dissatisfaction with the Jupiter of the *Saura* is reflected in his formula for computing Jupiter to give the years of the sixty-year Jovian cycle, given in his *Bṛhatsamhitā*, in the chapter dealing with the motion of Bṛhaspati (Jupiter). As for chap. XVIII he could not have meant the *Vāsiṣṭha-Paulīśa* star-planets there as an improvement, they being crude.

Note 2. Verse 63 evidently closes the *Pañca-siddhāntika*, as indicated by the *Vasantatilaka* metre instead of the regular *āryā* metre. But unfortunately the last three feet are missing. Perhaps it is a purposely done 'black-out' by a later astronomer-scribe, to append his spurious verses 64-81, and unfortunately only his manuscript has survived as the archetype of the few extant manuscripts.

PAÑCASIDDHĀNTIKĀ XVIII 64-81:
AN INTERPOLATION

That verses 64-81 of *Pañcasiddhāntikā* (PS) form only an appendage to a manuscript of the work is evident from its occurring after the work has closed in the customary way, with a concluding colophonic verse, with its metre changed to *Vasantatilakā* from the *Āryā* metre in which all the previous verses of the chapter had been couched and the author speaking about himself in the said concluding verse. It is also to be noted that this set of verses begin with a new salutation. Had these verses really belonged to the PS, the customary finish must come at its end, but there are no such finishing verses at the end. Further, in verse 65 it is said that VM considers this as a superior set containing a previous method or matter and that he was giving it, with a liberal mind, to the generality of astronomers without hiding it from them. But actually it is inferior stuff, and can give only very rough results since the equation of the centre is dispensed with, only the equation of conjunction being given, which makes it valueless. VM is alleged to boast here that he has made things easy, and takes credit for this which only a novice could have done. Fancy VM speaking thus, when in verse 62 he is so intent on accuracy that he says, "Let people who have been dissatisfied with the inaccuracy of astronomers like Pradyumna, Vijayanandi etc. have recourse to his treatment of the *Saura*." Further, there are mistakes in the computation of Venus and Mercury, unpardonable in any astronomer.

These spurious verses are dealt with below for the sake of completeness of the text as given in the editions of the work.

प्रस्तावेऽपि न दोषा(वान्)
 जानन्ना(न्न)पि न (delete) वक्ति यः परोक्षस्य ।
 प्रथयति गुणा(णां)श्च तस्मै
 सुजनाय नमः(मः) परहिताय ॥ 64 ॥

अष्टादशभिर्बद्धा-
 न्याताराग्रहतन्त्रमेतदार्याभिः ।
 वराहमिह(वरामति) वराहमिहिरो
 ददति निर्मत्सरः करणम् ॥ 65 ॥

63. Salutation to the good people, ever interested in the welfare of others, who even when knowing the faults of others, and even when there is an opportunity, do not mention their faults, but proclaim their good qualities.

64. In eighteen *āryā* verses, Varāhamihira, without feeling any jealousy, gives this manual to the world, ending with the treatment of the planets, thinking that it is good.

Note 1. The emendations are TS's also.

Note 2. Verse 64 is a paranomasia and means also, "Salutation to the good science of astronomy called technically *Parahita-gaṇita* (prevalent in Kerala in South India), which at the beginning deals only with mean motions, though knowing its defective nature as not being true motions, and which furnishes tabular values of equations going by the names, *mandajyā* (R sine table of the equation of the centre), *karkijyā* (R sine table of the equation of conjunction for the anomaly 90° to 270°) and *makarajyā* (R sine table of the equation of conjunction for the anomaly 270° to 90°).

Since this meaning being not obvious to laymen, I give the phrase by phrase meaning :

प्रस्तावे ग्रन्थारम्भे, परोक्षस्य अस्फुटग्रहस्य, दोषान् अस्फुटत्वादि-
 दोषान्, जानन्नपि यः न वक्ति जानन्नपि न वदति, (मध्यगतेरेव प्रकृतत्वात्.)

गुणान् मन्दण्या - कर्कण्या - मकरण्या इत्यादिगुणशब्दवाच्या ज्याः, प्रथयति प्रकटीकरोति गणयित्वा लिखतीति यावत्, तस्मै सुजनाय तस्मै शोभनजन्मने परहिताय परहितगणिताय नमः नमोऽस्तु ॥

Note 3. These two verses also form part of the 18 verses mentioned. So, actually there are only 16 verses (66-81), giving the computation.

भाकरणाद् रविभागा

दिवसाश्वा(श्च)रांशका रवौ कार्याः ।

अधिका र्य(य)दा दिते(ने)भ्यः

भागा ज्ञेयास्तदा चक्रात् ॥

66. From the epoch to the time of computation of the planet, find the sun's degrees passed. These are to be technically called '*days*' (and used in the computation). Find the remainder after dividing by the cycle number given for the respective star-planets. Take the '*days*' of motion corresponding to the set of motions given to the respective planet. These are degrees of planetary motion. Add this to the sun's longitude. The true planet is got.

Note 1. The '*days*' mentioned here is only what is called *sauradina* (sun's day) as distinguished from the *sāvana* or civil day, and are actually degrees. (This is like the word light-year, which is used as a unit of distance). This instruction is given with respect to all planets.

Note 2. The cycle given for each planet is only the period of the planet's synodic revolution converted into solar days, i.e., it is the synodic period $\times 360^\circ \div 365-15-30$, nearly. When so converted, we have :

Mercury Venus Mars Jup. Saturn

The regular

synodic days 115-52-45 583-55 779-57 398-53 378-6

Converted into

solar days 114 6/29 575½ 768-45 393 1/7 372¾

The second set is given for the respective planet, saying that the synodic period is so many 'days'. Not knowing this TS have remarked that they do not understand why there is so much difference in the periods from the regular days generally known. Also, they say they cannot dismiss them as wrong, since the numbers given are checked in the computation itself. (See pages ix-iii-lxiv of Introduction in TS's edition).

Pingree and Neugebauer have understood (as seen from their edition of the *PS*) that the cycles are in solae 'days'. But they have remarked that 'VM' has confused the days and degrees, not realising that there is a purpose in giving the cycles in the solar day units. These units have been used because, now, the 'days' and the 'degrees' will have the same meaning, and they can be combined without, at every point, instructing it. Thus, ultimately, the combined value is the degrees of the true planet.

Example. Days from epoch 1,20,553. Find the 'days', and assuming the sun at epoch as zero, find the sun at the end of 1,20,553 days.

$$1,20,553 \times 360 \div 365 - 15 \cdot 30 = 1,18,187.5 \text{ 'days'}$$

This plus zero, and divided out by $360^\circ = 17^\circ.5$, sun's longitude.

नवमयगृणानुहीना(गुणतुहीने)
 कृताहते(हते) विषयसप्तखाग्निहते ।
 भृ(भू)यो हतो(हते) चतुर्भिः
 विरंस(निरंश)दिवसा महीजस्य ॥ 67 ॥
 षट्त्रिंत्सवै(विषयै)स्तिथ्यु(थ्यू)नः
 दष्टवसुधृति[भि]रंशकाच्छ(ः ष)ष्टिः ।
 अष्टशतेन व (च) षष्टिः
 सप्तत्या द्दष(ज्य)धिकया नवतिः ॥ 68 ॥

षष्ठ्यो(ष्टया)टयुक्तया सं(श)त-

दलं च खाब्धि(श्वि)द्विकैः खराद्विघ्नाः(घ्नाः) ।

अस्तमितोऽतः सप्ताष्टकेन

तिथयो निरंशगशनिः (गतिः) ॥ 69 ॥

67. Subtract 6329 from the 'days'. Multiply the remainder by 4 and divide out by 3075. Take the remainder and divide by 4. These are the 'days' after conjunction (i.e., the anomaly of conj.) for Mars.

68-69. After 56 'days' he goes behind the sun by 15°, and becomes observable, (i.e., the heliacal setting ends). In 188, 108, 73, 68, 220, 'days' Mars lags behind by 60°, 60°, 90°, 50°, 70°, respectively. Then it sets heliacally, and in 56 'days' lags 15° behind and goes into conjunction.

Note 1. I generally agree with TS's emendations. But in verse 68, I give षड्विषयेः for षट्त्रिंशत्सवैः, which latter is both meaningless and has one *mātrā* extra. TS's षड्वर्गैः does not agree with the last सप्ताष्टकेन, for the numbers should agree or at least nearly agree. सप्तत्या द्व्यधिकया has been emended by me into सप्तत्या त्र्यधिकया, and खाब्धि into खाब्ध्वि. These will not only make the total correct, but also bring about agreement with the *Siddhāntas*, which all generally agree with the actual as given by modern astronomy.

Degrees moving

behind	-15°	-60°	-60°	-90°	-50°	-70°	-15°	-360°
'Days' given	56	188	108	73	68	220	56	769
Actual 'days'	54	188	106	72	75	220	54	769

The great difference in 'days' between the 68 given and 75 actual must be explained by their following next to be retrograde period, where even a large number of days can produce a very small difference in degrees. So,

correction to whole degrees can produce this difference in days.

Note 2. Verse 67 means that 6329 'days' after epoch, there is conjunction, which repeats after each synodic cycle. The cycle for Mars is $768\frac{3}{4}$ 'days'. So instead of dividing by $768\frac{3}{4}$, we are asked to multiply by 4 and divide by 3075. To get back the true remainder, the remainder here is divided by 4.

Note 3. (a) In the case of all star-planets the total 'days' should be equal to the days of the respective cycle.

(b) In the case of the superior planets, (Mars, Jupiter and Saturn), the degrees are all negative and add upto -360° . When the given degree is numerically greater than the corresponding 'days', (for eg., -90° for 73 'days' here) the planet is retrograde.

(c) The heliacal setting and rising are at the beginning and end of the cycle for all. But for the two inferior planets, Mercury and Venus, there is another setting and rising at inferior conjunction, when the two are retrograde.

(d) The rising and setting are given by observation at different regions and different conditions of the atmosphere, and therefore vary among the Siddhāntas.

(e) In the case of the inferior planets the degrees should add upto zero. When the degrees are positive and greater than the days, the planet is gaining upon the sun, and the total gain is its elongation. When the degrees are less than the 'days', the planet is lagging behind. When the degrees are negative and numerically greater than the 'days', the planet is retrograde and comes at the middle of the cycle, if the cycle begins and ends at superior conjunction.

Example. Compute Mars at 1,20,553 days from epoch.

The solar days are, $1,20,553 \times 360 \div 365-15-30 = 118,817.5$, and the sun is $17^\circ.5$, taking the sun at epoch as zero, which it nearly is, as already shown.

$$118,817.5 - 6329 = 112,488.5$$

$$112,488.5 \times 4 \div 3075 \text{ leaves the remainder } 1004.$$

$$1004 \div 4 = 251, \text{ real remainder of 'days'}$$

During this period we get the movement: -15° in 56 'days', -60° in 188 'days', and -4° for the 7 'days' remaining, total -79° . Adding -79° to the sun, $17^\circ.5$, True Mars = $298^\circ.5$

Verses 70-72 deal with Mercury.

विं(वि)शशिवसुरसेन्द्र(न्द्रे)

नवनव(यम)गुणितेऽर्करा[म]गुणभक्ते ।

गुणकारहते लब्धा-

न्यहानि शीतांशुपुत्रस्य ॥ 70 ॥

दशभिर्द्वा(र्द्ध)दशहीनाः(न)

प्रागुदितो मनि(नु)भिरूनभश्चांशाः(नन्दांशाः) ।

धृतिभिः झ(स)नवोऽस्तमितः

त्रिंशद्भिरुदेत्ति(ति) सरसाश्वः(शराश्वः) ॥ 71 ॥

अष्टाद[श]भिः[स]नवः

षोड[श]भिश्चाष्ट(र्कं)वर्जितोऽस्तमितः ।

पञ्चाद्वसुभिर्नवव-

जितो निरंस(शं)बुधोऽपि(delete) याति ॥ 72 ॥

70. Subtract 14,681 from the 'days'. Multiply the remainder by 29, and divide out by 3312. Take the remainder and divide by 29. The 'days' for Mercury in the cycle is got.

71. In 10 'days' Mercury falls back by 12° , and rises in the east. In 14 'days' more he lags by 9° . Then in 18 'days' he gains 9° . Then he sets, and in 30 'days' gains 25° and rises heliacally.

72. Then in 18 'days' he gains 9° . In 16 'days' he lags 12° , and sets in the west. Then in 8 'days' he lags 9° , and gets into conjunction.

Note 1. In verses 71 and 72 my corrections are based on the need to conform as nearly as possible to reality, and they are also, as far as possible, kept close to the text. *śarāsvih*, may also be *jalāśvih*, i.e. 24. The values are :—

	1	2	3	4	5	6	7	Total
Given degrees	-12° <i>vakrāsta</i>	-9° <i>vakra</i>	$+9^\circ$	$+25^\circ$ (or $+24$) <i>astā</i> 30	$+9^\circ$	-12° <i>vakra</i>	-9° <i>vakrāsta</i>	0°
Given 'days'	10	14	18	30	18	16	8	114
Correct 'days'	8	16	18	30 (?28)	18	18	6 (?8)	114

In columns 6 and 7 it should be -9° and -12° , or at least -10° — 11° though the text letters are unmistakable.

Note 2. The constant for subtraction, 14,681 shows that the planet is in superior conjunction, since the epoch position of the planet must agree with that given by modern astronomy and other *Siddhāntas*, at least with in a few degrees. (See table appended). If so, the Table of cycle motions given should begin and close with superior conjunction. But in the Table given, the cycle begins and closes with the inferior conj. as can be seen from the retrograde motion with which the Table begins and ends, and the most rapid motion (*aticāra*) coming at the middle.

The astronomer of very inferior calibre, who has made this interpolation, has been misled by the two sets of heliacal rising and setting in the case of the inferior planets, Mercury and Venus. He has wanted to begin the motions with the rising in the east and setting in the west, to fall in line with others not realising that this occurs during its retrograde motion which falls at the inferior conjunction coming in the middle. This is another proof that VM cannot be the author of this set of dealing with the star-planets. (He has committed the same mistake in the similar case of Venus, where the mistake can be seen glaringly when the true Venus got is compared with that of the other siddhāntas or modern astronomy). If he does want to begin with rising in the east and end with setting in the west, he must begin and end his cycle table with the inferior conj. and to do this he must add to the days to be subtracted half the cycle days, equal to $57 \frac{3}{29}$ days. (The cycle days = $3312 \div 29 = 114 \frac{6}{29}$).

Next follows Jupiter, in three verses :

रहितेष्ट(delete)द्वियमशराष्टिभिः(ष्टयां)
 नाग(नगा)हते द्विविषयश्च(स्व)राश्विहते ।
 सप्तहते देवगुरौ(रोः)
 भवन्ति दिवसा तिरासंगम्याः (निराशस्य) ॥ 73 ॥

सर्वेऽका(र्का)त्संशोध्याः
 षोडशभिर्द्वादशोदितः प्राक्(च्याम) ।
 कृतविषयैः कृतवेदाः
 सप्तत्या सार्णवाः षष्टिः ॥ 74 ॥

नव दिग्भिः शून्यार्काः
 [सा]शान्या रसस्वराद्याभिः(श्चैव) ।
 शून्यकृति(तैर्द्वात्रिंशत्
 तोतुमस्तगा (ततोऽस्नगः) षोडशभिर्कर्कात् (र्का) ॥ 75 ॥

73. Deduct 16,522 from the 'days'. Multiply the remainder by 7 and divide by 2752. Divide the remainder here by 7. The 'days' from conjunction are got,

74-75. All degrees given are to be subtracted from the sun. In 16 'days' he moves 12° and rises in the east. Then in 54, 70, 49, 88, 40 days he moves 44°, 64°, 120°, 76°, 32°. Then he sets in the west, moves 12° in 16 days and joins the sun.

Note 1. The first foot of verse 73 is faulty containing 3 *mātrās* extra, and corrupt. So it has been corrected. The rest of verses 73 and 74 are with TS's emendations. In 75, all emendations are TS's, excepting those for grammar.

Note 2: The cycle is $2752 \div 7 = 393 \frac{1}{7}$ days. The days and degrees are :

	1	2	3	4	5	6	7	Total
Degrees	-12°	-44°	-64°	-120°	-76°	-32°	-12°	-360°
Given days	16	54	70	109	88	40	16	393
Near correct days	16	54	70	109	88	40	16	393

Venus is dealt with in 3 verses :

नयनार्कमितिदु(धृतीन्दू)ने
 द्विगुणे रु(रू)पेन्द्रियैः स्व(येश्व)रैर्भक्ते ।
 शेषः(षं) यत्तद(द)लितं
 भृगुतनयनिरंशदिवसाः स्युः ॥ 76 ॥

विषयैर्नवकविहीनः
 प्रागुदितस्तिथि[भि]रेकप(य)महीनः ।
 वसुकृत्या तिथ्युन(थ्यूनः)
 कृताष्टिभिः स[पञ्चकास्त्रिंशत्] ॥ 77 ॥

षष्टा(पञ्चा)[ष्ट]केन सदश(शः)
 निरंसतो(शनो) तो विलोमगः पञ्चात् ।
 उदय(दे)ति थिऽ(नि)रंशकालो(ले)
 नय(प्रया)ति धा(चा)रत्तं विनाथ(लोम)गतिः ॥ 78 ॥

76. Deduct 1,18,122 from the 'days'. Multiply by 2 and divide by 1151. Take the remainder and divide

by 2. We have the 'days' from the conjunction of Venus.

77-78. In 5 'days' Venus lags by 9°, and rises in the east. In 15 days he lags 21°. In 64 days he lags 15°. In 164 days he gains by 35°. Then he sets in the east. Then in 40 days he gains 10°, and joins the sun. Then, moving in accordance with the reversed order of the days for cycles given, he rises after the days given from setting to conjunction (*i.e.*, 40 days) in the west, and moves till he reaches the setting in retrograde (and getting into the inferior conjunction) section.

Note 1. I have corrected *mitindu* into *dhytindu* for agreement with the sun in superior conjunction which alone fits. TS's correction *mahindu* does not bring agreement with the sun either at superior conj. or inferior conj. There can be another possible correction *matindu* (*mati* is 8.). In the 64 days, flanking the retrograde, the days may be a little more or less, since a small error of observation can produce a difference of a large number of days. The lacunae is filled by me with *sa-paṇcakāstrimṣat*, meaning 35°, to fit the number of degrees wanted to make up the total zero, and fitting the number of days given. TS's emendation, *kytāṣṭabhiḥ* will be far from fitting the total. Moreover, their filling the lacuna by *seṣuḥ* meaning 5° is quite inadequate to make up zero. I have emended *ṣaṣṭāṣṭakena* into *pañcāṣṭakena*, meaning $5 \times 8 = 40$, which will fit the number of days. Also, 10° synodic motion there requires 40 days and it is also the period from setting to going into superior conjunction. *ṣaṣṭa* is patently wrong spelling, and *ṣaṣṭāṣṭakena* is meaningless. But TS keep it, which is wrong. That this is the segment of heliacal setting to conj. can be inferred from *udayati* given for the next segment, and 10° for heliacal setting and rising at superior conj. is given by many *siddhāntas*. The other minor emendations are TS's.

Note 2. 118,122 seems to be a very large subtractive constant, equal to more than 300 years, while all others are very near VM's time. But I cannot think of any other number to fit.

Note 3. The maximum elongation is seen to be 45° , correctly, (cf. Table).

Degrees	-9°	-21°	-15°	$+35^\circ$	$+10^\circ$	$+10^\circ$	$+35^\circ$	-15°	-21°	-9°	0
	<i>vak-rāsta</i>	<i>vakra</i>			<i>asta</i>	<i>asta</i>			<i>asta</i>	<i>vak-rāsta</i>	
Given	5	15	64	164	40	40	164	64	15	5	576
days											
Correct	5	15	64	164	40	40	164	64	15	5	576
days											

Note 4. The remark about Mercury, that the cycles begin and end with the superior conjunction according to the subtractive constant given, but the motions in the cycles begin and end with the inferior conjunction, holds in the case of Venus also, showing thereby that the author is an ignorant imposter, and cannot be VM. To correct the fault, $287\frac{3}{4}$ 'days' should be added or subtracted from the subtractive constant.

Example. Compute Venus at 1,20,553 days from epoch :

If the subtractive constant given in the text is used, 1,18,817.5 (already found in the example in Mars) — 1,18,122 = 695.5.

This $\times 2 \div 1151$ leaves the remainder 240.

This divided by 2 gives 120 'days' gone in the cycle. We have for the first 5 days -9° , and the next 15 days -21° and the next 64 days -15° and the remaining 36 days, $36 \times 35 \div 164 = 7^\circ 40'$, totally $-37^\circ 20'$. Adding the sun $17^\circ.5$ already found in the example for Mars, the true longitude of Venus is -20° , i.e. 340° . (The example in the *Saurasiddhānta* for the same date has given 46°).

The error in Venus, in using this method here, is 66°. On the other hand, let us use the cycle order re-arranged to begin from superior conjunction. It is 10° for 40 days, 35° for 164 days etc. We have 10° for 40 days, and the remaining 80 days, $80 \times 35^\circ \div 164 = 17^\circ$, total 27°. Adding the sun, 17°.5, we have, true Venus, 44°.5. This is close to the correct 46°. This exposes the ignorance of the impostor.

Saturn follows in the next three verses :

विधृतिशररसषट्कवर्कं(delete)शशाङ्के

त्रिघ्ने धृतिरुद्रभाजितेऽ ग्रहते ।

सौरस्प(स्य) धृति[भि]रष्टाभिर(ष्टिः)

सार्धाकिं(च) हानिरुदितः प्राक् ॥ 79 ॥

अष्टनवतिर्ज्या(त्या) नवति-

र्दलं च मनुभिस्त्रयोदशविहीनाः ।

गुणरुद्रेः शून्यार्काः

द्वयूनेन शतेन शशिनवकम् ॥ 80 ॥

अतिजग...रर्का[जगत्या सार्धार्का]-

र(न)स्तमेत्यं(त्य)तो नवति(कु)भिर्वि(र्नि)रंशम् ।

षोडश सार्धात्सौ(त्र सौ)र-

श्चरति रवेस्सर्वदा होनः ॥ 81 ॥

70. Subtract 16,518 from the 'days', multiply by 3 and divide by 1118. Take the remainder here and divide by 3. The days left over in the cycle are got. In 18 days Saturn lags behind by $16\frac{1}{2}^\circ$, and rises in the east.

80-81. In 98, 14, 113, 98, and 13 'days' he falls behind $90\frac{1}{2}^\circ$, 13° , 120° , 91° , and $12\frac{1}{2}^\circ$ respectively. Then Saturn sets in the west, and joins the sun passing $16\frac{1}{2}^\circ$ in 19 days.

Note 1. In verse 79, षट्कवर्कं is patently extra, forming syllables not required for the foot, and has been deleted by me as also by TS. अष्टाभिः is corrected into अष्टिः by me, as also by TS to conform to grammar and facts.

In the rest of the minor corrections there is no difference between our corrections.

In verse 80, I have retained the द्वि in द्वयूनेन, while TS have made it द्यु, meaning one day, which is not necessary, and which leads into trouble later, needing further correction. The minor corrections are common to both. In 81, I have filled up the lacuna by (रा साधं) while TS have made it (तिभिर्घं). The word is अतिजगती and not अतिजगतिः, which alone can justify TS's ...तिभिः. Also only साधर्कं can mean $12\frac{1}{2}$, but their अर्धर्कं can mean only 6. I have corrected नवति into नवकु keeping the. But TS have corrected it into अतिधृति, making unnecessary changes in the lettering, though both of us mean the same. The rest of the corrections are minor, and common to both of us.

Note 2. The following is the table of motions :

Degrees found	$-16\frac{1}{2}^{\circ}$ <i>asta</i>	}	$-90\frac{1}{2}^{\circ}$	-13°	}	-120° <i>vakra</i>	-91°	$-12\frac{1}{2}^{\circ}$	}	$-16\frac{1}{2}^{\circ}$ <i>asta</i>	-360°
Given days	18		98	14		113	98	13		19	373
Correct days	18		98	14		113	98	$13\frac{1}{2}$		$18\frac{1}{2}$	373

Note 3. The text ends abruptly without the usual verses giving details about the author, his parentage, date of writing etc.

Note 4. The colophon is simply "The star-planets of the *Paulīśa siddhānta* ends". But after this is found details about the scribe, his lineage, his time of writing, viz. 1673 Vikrama Samvat, and 1538 Śaka, equal to 1616 AD, and the purpose of his copying the work, ("for his own reading and helping others" i.e., other astronomers).

THE SAKA ERA OF VARĀHAMIHIRA (ŚĀLIVĀHANA ŚAKA)*

Introduction

With reference to chronology the word *Śaka* is used in two senses : (1) As a common noun meaning any era (as for e.g., in the terms *Yudhiṣṭhira Śaka*, *Vikrama Śaka*, *Mālava Śaka*, *Śālivāhana Śaka* etc.) and, (2) As a proper noun to mean a particular era called the *Śaka-kāla* or *Śaka Era*. Most Indologists believe that the *Śaka Era* is the same as what later is generally referred to as the *Śālivāhana Śaka* which commenced with the month of *Caitra* occurring in 78 A.D., i.e., at the end of 3179 years of the Kali Era, for it can be shown that all astronomical works and commentaries thereon, wherever they mention a *Śaka Era*, mean only the *Śālivāhana Era*, starting, as mentioned above, from 3179 Kali elapsed. But some like the late T. S. Narayana Sastri,¹ Gulshan Rai,² Kota Venkatachalam,³ and V. Thiruvengkatacharya⁴ (VT) take the word to mean a certain Cyrus Era or Andhra Era,

* Rep. from *Journal of Indian History* (Trivandrum), 36 (1958) 343-67.

1. Cf. his *Age of Sankara*, (Madras, 1918), Pt. I, pp, 224ff.
2. Cf. his article, 'The Persian Emperor Cyrus, the Great, and the Śaka Era', *Journal of the Panjab University Historical Society*, (JPUHS), 1 (1932) 61-73, 122-36.
3. Cf. his *Plot in Indian Chronology*, (Vijayawada, 1953), 49-51; 'Indian Eras', *Journal of the Andhra Historical Research Society*, (JAHS), 20 (1949-50), 43ff; 21 (1950-52), 61-73, 122-36.
4. Cf. his 'Ayanāṁśa and Indian chronology: The Age of Varāhamihira, Kalidāsa etc.', *Journal of Indian History* (JIH) 28 (1950) 103ff., and 'The Andhra Śaka' *JAHS*, 22 (1952-54) 161-8.

which they say, started from 550 B.C.⁵ Kane mentions two others of the group: Jagannatha Rao, *Age of Mahabharata war* (1931), C. V. Vaidya starting the *Śaka-kāla* from Buddha's *nirvāṇa*. We now find that T.S.N is the source for all these people, and almost every argument used by them is his. In his *Age of Śaṅkara* he has used a *Yudhiṣṭhira Era* of 3140-39 B.C., and a *Śaka Era* of 576 BC, which he later shifted to 550 B.C. Still another view is expressed by K. Rangarajan, who takes it to mean an era which commenced from 523/22 B.C. with the first Viceroy of India appointed by the Persian Emperor.⁶ They also try to show that it never means the *Śālivāhana Śaka*.⁷ What astounds us is that even where there is clear evidence that *Śālivāhana Śaka* is to be taken, (in the shape of statements that 3179 is to be added to the years gone in the *Śaka Era* to get the years gone in *Kali*)⁸ these scholars ignore it implicitly as in the case of the *Śaka-kāla* mentioned by Brahmagupta and Bhāskara II.⁹ When this is the fate of such clear evidence, we need not be surprised if they identify with their alleged Cyrus or Andhra Era, the *Śaka Era* mentioned in giving the epochs of *karaṇas* (astronomical manuals) as in the case of the *Pañcasiddhāntikā* (PS), the

5. In his *Popular Astronomy*, (Madras, 1958), pp. 135, 136, VT has changed to 551 B.C. without assigning reasons therefor.
6. Cf. Summary of his paper, 'On the Origin of Śaka-kāla', *Proceedings of the Indian Historical Congress*, Fifth Session, Hyderabad, 1941, p. 164.
7. In the case of Bhāskara II alone, VT concedes that the *Śaka Era* mentioned by him is the *Śālivāhana Śaka*.
8. E.g., Brahmagupta's *Brāhmasphuṭa-Siddhānta*, Madhayamādhikāra, I. 26; Bhāskara I's *Mahabhāskariya*, I. 4, and *Laghubhāskariya*, I. 4; Śrīpati's *Siddhāntasekhara*, I 25; Bhāskara II's *Siddhantaśiromaṇi*, Gaṇita., Madhyama., Kālamāna., 28; *Vaṛaṇasiddhānta*, Madhyamadhikāra, I. 10.
9. Cf. VT, *Popular Astronomy*, p. 137; Kota Venkatachalam, *The Plot in Indian Chronology*, Appendix, p. xxx.

Khaṇḍakhādya or the *Laghumāṇasa*, or in giving the date of a work given by the author, as for instance by Bhaṭṭotpala at the end of his commentary on the *Byhājātaka* or in inscriptions like the Aihole Inscription, or in sundry other places as in the *Byhatsaṃhitā* I.13, in all of which cases the identification has got to be made by examining the months and *tithis* and *kṣepas* mentioned therewith.

The reason why they want to identify the *Śakakāla* with the so-called Cyrus or Andhra Era is this: They believe that there was a "plot hatched by European Indologists" to post-date by several centuries the ancient events of Indian history, and that most Indian Indologists have become unconscious victims of that plot. They try to show that the *Yudhiṣṭhira* and *Saptarṣi Eras* are everywhere identical, and were actually started 25 years after the beginning of the Kali Era. Using this they try to show that it is Samudra Gupta of the Gupta dynasty that is to be identified with the Sandracottus of the Greeks, and not Candragupta Maurya, which latter identification has been taken by the European Indologists as the sheet-anchor of Indian chronology, and the chronology of the dynasties before and after that time is established therefrom. Now, the identification of *Śakakāla* with *Śālivāhana Śaka* stands in their way. Hence their attempt to identify it with the so-called Cyrus or Andhra Era whose very existence is a matter of dispute, there being no evidence for it.

Most historians have not taken these people seriously, thinking that the very extravagance of their claims would be a deterrent to the acceptance of their views. But attempts have been made by Professors Gulshan Rai and VT to give astronomical and mathematical proofs to show that Varāhamihira (VM) belongs to 123 B.C. and not to 505 A.D., (as he is generally believed to be), and thereby that the *Śakakāla* mentioned by VM

is the Cyrus or Andhra Era.¹⁰ They also attempt to show that the *Śakakāla* mentioned by Bhaṭṭotpala as stated above is the Cyrus or Andhra Era, and therefore the Śaka year 888 given by him corresponds to 338 or 339 A.D.;¹¹ which would mean that Brahmagupta, Āryabhaṭa, Bhāskara I etc. must precede this date. The present article is intended to expose the hollowness of the above theory and to show that the astronomical arguments adduced in support of it (which to the lay reader may look formidable) are erroneous, and thus knock the bottom out of the claims of this set of writers.

Varāhamihira uses the word *Śakakāla* in a few places in his works.

- (1) In the *Bṛhatsaṃhitā* he says :

āsan maghāsu munayaḥ śāsati pṛthvīm yudhiṣṭhira
ṣaḍ-dvika-pañca-dvi-yutaḥ Śakakālas tasya nṛpatau /
rājñas ca || XIII. 3

“The Sapta-ṛṣis were in the asterismal segment Maghā when Yudhiṣṭhira was ruling over the earth. Any date by the Śaka Era plus 2526 gives the time from that king, i.e., the date in the Yudhiṣṭhira Era.”

- (2) In his *Pañcasiddhāntikā* (PS) the following occurs :

sapta-aśvi-veda-saṅkhyam Śakakālam apāsya
caitra-śuklādu /

10. *JPUHS* I (1932) 124-27; and *JIH* 28 (1950) 103ff. and *JAHS* 22 (1952-54) 172. Following his change to 551 B.C. as the *Śaka Epoch*, in his *Popular Astronomy*, VT has changed VM to 124 B.C. from 123 B.C. But the arguments for the refutation of 123 B.C. are applicable *in toto* for the refutation of 124 B.C. also.
11. *JPUHS* I (1932) 73 (date given 338 A.D.), and *JIH* 28 (1950) 123 (date given 339 A.D.) In his *Popular Astronomy* VT has shifted this to 338 A.D. But in his ‘Andhra Śaka’ VT gives this date as 340 A.D. (*ib.*, p. 173).

ardhastamite bhānau yavanapure

*somadivasādye. // 1. 8 //*¹²

“Deducting 427 of the Śaka Era, (from the years in the Era) at the beginning of the light half of Caitra, which falls near sunset at Yavanapura, beginning a Monday...”

- (3) *Br. Sam.* VIII 20-21. This will be discussed, later.
 (4) In *Pañcasiddhāntikā*, XII. 2. but it is not used by these scholars.

In (1), a synchronism is found between the *Śaka Era* and the *Yudhiṣṭhira Era*. We shall not discuss this synchronism here but rest content with saying that whatever be the *Sakakāla* mentioned in (2), it is highly probable that the same is mentioned by (1). In (2), it is clear, the epoch of the *Pañcasiddhāntikā* is given as 427 *Śaka* elapsed, which means the date of the work must be *c.* 427 *Śaka*, and thus VM's time can be fixed. If as VT and others say the *Sakakāla* meant here is the Cyrus or Andhra Era of 550 B.C., then the date of VM must be 427 years after 550 B.C., *i.e.*, 123 B.C., which Gulshan Rai and VT have tried to establish by their special arguments. If it is the same as the *Śālivāhana Śaka*, then VM's date must be 427 years after 78 A.D., *i.e.*, 505 A.D.

Here we do not propose to go into the question whether there was a *Śaka Era* beginning from 550 B.C. or whether it is necessary to postulate such an era in

12. *Somadivasādye* is the reading as emended by the late Dr. Thibaut and MM. Sudhakara Dvivedi in their edition of the *PS*. From the two manuscripts of the work available from the edition and from quotations elsewhere, four readings are known: *saumya-divasādye*, *bhaumya-divasādyaḥ*, *bhauma-divasādyaḥ* and *bhauma-divasākhyah*. On the propriety or correctness of these readings see below.

view of the reference in the *Bṛhatsamhitā* śloka quoted above which is discussed in the next paper 'The untenability of the 'postulated Śaka era of 550 B.C.' We shall confine ourselves to showing that the *Śakakāla* of VM's PS is the *Śālivāhana Śaka*, and therefore 427 Śaka (elapsed) corresponds to 505 A.D. As we have stated before, we shall also show that the special arguments to the contrary advocated by Gulshan Rai and VT and their conclusion that VM's date is 123 B.C. cannot stand.

Internal Evidence for Śālivāhana Śaka

There is plenty of internal evidence to show that the date meant by VM is 505 A.D. and not 123 B.C. It consists of the many *kṣepas* (i.e. values of the Mean longitudes etc. at Epoch) found in the work, and the names of certain authors which it mentions. We shall take the *kṣepas* first.

In PS I.14, VM gives a *Saura* period of 1,80,000 years or revolutions of the Sun, in which there are 66,389 intercalary months and 10,45,095 suppressed *tithis*. From this we can get that there are in this period 2,406,389 revolutions of the Moon and 65,746,575 civil days. Comparing this with the *Yuga-elements* derivable from the *Khaṇḍakhādyaka*¹³ of Brahmagupta (which follows the *Ārdharātri* system of Āryabhaṭa and whose elements are identical with those of a *Paulīśa Siddhānta* quoted by Bhaṭṭotpala in his commentary of the *Bṛhat Samhitā*),¹⁴—not the *Paulīśa* of the PS—we find that this is only a *sub-yuga* forming a twentyfourth part of the *Yuga* given by them, and this suggests that the *Yuga-elements* of the original *Saura Siddhānta*, of which the *Saura* of the PS is a compendium, are identical with

13. *Khaṇḍakhādyaka*, (Tr. P. C. Sengupta, Calcutta, 1934), I. 3-5, 13, 14; II. 1-5.

14. Cf. *Bṛhat Samhitā*. Ed. Sudhakara Dvivedi, Banaras, 1895, Pt. I, pp. 28-30.

those of the *Khaṇḍakhādyaka* etc. mentioned above; these elements, therefore, may also be called hereafter, the *Saura* elements. Now, all these systems have arrived at 0° Mean longitude for the Sun, Moon, Mars etc., 3 *rāśis* for the Moon's *ucca* (Apogee), and 6 *rāśis* for *Rāhu's head* (Moon's Ascending Node), at the beginning of *Kali yuga*, viz., midnight at Ujjain, Thursday/Friday, 17/18, February, 3102 B.C. Taking that *Śaka* 427 mentioned in *PS* I. 8, refers to *Śālivāhana Saka* 427, (equivalent to 3606 Mean Solar years after the beginning of *Kāli*), we have 1,317,123 days, 3 *nāḍis* 9 *vināḍis* gone in *Kali*, and arrive at 3 *nāḍis*, 9 *vināḍis*, after the midnight at Ujjain, Sunday/Monday, 20/21 March 505 A.D.¹⁵ The *Saura* of the *PS* takes this midnight as the Epoch for the computation of its Star-planets (*Tārā-grahas*), viz., Mars etc. If we compute the Mean Mars etc. for this epoch, using the *Saura* elements, *the results agree with the respective kṣepas given in the PS* to the second in the case of Jupiter and Saturn, within 4" in the case of Mars and Venus, and 7" in the case of Mercury. Even this small difference is due to VM having arrived at the *kṣepas* using the shortcut given by him in the *karaṇa* and the number of days gone in *Kali* as the '*Ahargana*' (days from epoch).¹⁶ If we also do the same there is complete agreement in the case of Venus also, and the difference is reduced to 2" in the case of Mars. In the case of Mercury there is difference of a few seconds still, which may be due either to VM desiring to give its *kṣepa* correct to the minute only, or to some defect in the

15. 3606 after *Kali* is not 504 A.D. as some people may think. As there is no zero year B.C. or A.D., we apparently arrive at a date one year later as the correct date. For errors of this kind see Kota Venkatachelaṃ, *JAHRS* 21 (1950-52) 4, 7 etc.; Gulshan Rai, *JPUHS*, *ib.*, 73, 127.

16. *PS*, ch. XVI. (Thibaut and Sudhakara Dvivedi's Edn. Reprinted by Motilal Banarsi Dass, Lahore, 1930).

manuscript reading which has omitted the seconds; and one of the manuscripts has actually a reading '*vilipti*' here.¹⁷ For the Mean Sun and Moon, and the Moon's *Ucca* and *Rāhu*, the epoch taken is the Midday at Ujjain just preceding the epoch of the Star-planets, i.e., the midday of Sunday.¹⁸ Here too, checking the *kṣepas* in the manner given for the Star-planets, we find perfect agreement in the case of the Sun and Moon, and agreement within 4'' in the case of the *Ucca*. In the case *Rāhu* the available manuscripts are so vitiated that Thibaut and Sudhakara Dvivedi (T-S) have failed to give the *kṣepa* fully. Using the letters available in the manuscripts, the relevant verse may be read as :

*trighanaśataghe navakaikapakṣarāṁendu-
dahanaṣaṭ-sahite |
svarayamavasubhūtārṇavaguṇadhṛti-bhakte
kramād rāhoḥ || IX. 6 ||*

The *kṣepa* for *Rāhu* enunciated in this verse as reconstructed above, agrees within 1'' with its value according to the *Khaṇḍakhādyaka* elements.¹⁹

This perfect agreement is the reason why S. B. Dikshit has retained the date March 505 A.D. in spite of the difficulties he encountered in interpreting *PS* I. 8 with reference to the *Saura*.²⁰ For, *no date, within many thousand years before or after 505 A.D. will agree with the kṣepas in the manner shown above, not to speak of 123 B.C.* When such is the case, VT quoting from Dikshit,²¹ a passage, which to those that have not read

17. See under *PS*, XVI. 9, '*davo vilipti*'.

18. *PS*, IX. 1.

19. S. B. Dikshit has arrived at the same result independently. See his article, 'The Original *Sūrya-Siddhānta*', *Indian Antiquary (IA)*, 19 (1890) 49, 54.

20. *Dikshit, Ib.*, 45-54.

21. *Ib.*, 46-47.

Dikshit's article in full will appear to involve an irremediable contradiction, says that 505 A.D. should be abandoned in favour of 123 B.C. on account of this. As the manner of VT's quoting²² from the article may create an impression in the readers' minds which Dikshit did not intend, and as VT himself concludes from the quotation that the agreement in the *kṣepas* discovered by Dikshit is null and void, and as he does not realise (as seen from his remarks under the quotation) that if he gives 123 B.C. for VM, he still has the responsibility to point out that the *kṣepas* agree with his date, we intend making the discussion a little elaborate so that we may give Dikshit's views in full with some pertinent observations on them.

The *Saura Epoch* occurs before the *True Vaiśākha Śukla Pratipad*, ending on Tuesday. Dikshit wants to reconcile this with the statement in *PS* I. 8, *caitra-śuklādaḥ*. He considers the point that according to *Mean reckoning*, it is '*Adhika*'-*Caitra Śukla*, but dismisses it, giving two objections: i. Why does not VM use the term '*adhika-caitra-śuklādaḥ*'? ii. Why should he take the *Adhika-Caitra* instead of the *regular Caitra* for the Epoch?²³ Dikshit concludes by saying that '*caitra-śuklādaḥ*' might stand, because *Amānta-Vaiśākha-Śukla* is *Caitra-Śukla* according to *Pūrṇimānta* reckoning. So there is no trouble at all for Dikshit as far as this goes. Therefore there is no need for VT to abandon 505 A.D., go to 123 B.C., and show that the *Caitra Śukla Pratipad* of this year occurs on a Wednesday, which weekday also is admissible according to one manuscript reading.²⁴

22. *JIH* 28 (1950) 108.

23. It will be shown below that there is absolutely no weight in either of these objections.

24. If it is 124 B.C., to which VT has shifted in his *Popular Astronomy*, the weekday is Thursday, and this is an additional argument against his date.

(Cf. the readings given above). It should be remembered that VT can score a point only if the weekday, viz., Wednesday of the *Caitra Śukla Pratipad* of 123 B.C. alone can effect the reconciliation, and not the Tuesday of the *Caitra Śukla Pratipad* of 505 A.D.

But really speaking, there is no need to reconcile the *Saura* with any part of *PS* I. 8, because it has reference only to the *Romaka* and the *Pauliśa*. (If it can be applied to the *Saura* also, as indeed it can, it is good, but we have no right to demand it as Dikshit does.)²⁵ *PS* I. 8-10 give the computation of *Ahargana* according to the *Romaka*; and I. 8 and 11 (and perhaps also 12 and 13) according to the *Pauliśa*. I. 8 gives the Epoch, which is thus the same for both. The Epoch is the beginning of *Caitra Śukla* which ends 427 Śaka, and the exact time is sunset at Yavanapura²⁶ beginning Monday, i.e., 7 nāḍis, 20 vināḍis after sunset at Ujjain. This is equivalent to 37 nāḍis 20 vināḍis after Ujjain sunrise on Sunday, 20th March 505 A.D. The *Ahargana* with which to compute the Mean Sun etc. must be reckoned from this point for *Romaka* and the *Pauliśa*, and their *kṣepas* are for this point. The expression *caitra-śuklādau* is an indication that the months gone are to be counted from *Caitra* in computing the *Ahargana*, and the words

25. *Ib.* 1A 19 (1890) 45ff.

26. VT has mistaken (*JIH* 28, p. 108; *JAHRS* 22, pp. 172) Yavanapura for Romakapura, and giving it a longitude of 90° west of Ujjain implies a *deśāntara* of 15 nāḍis instead of 7 nāḍis 20 vināḍis which is specifically given as the *deśāntara* for it, in *PS* III. 13:

Yavanāntarajā nāḍyaḥ sapta Avantyaṁ tribhāga-saṁyuktāḥ !

For an explicit mention see *PS*, XV. 25:

anyad Romaka-viśayād deśāntaram anyad eva Yavanapurāt |

Yavanapura would correspond to Alexandria as calculated from the *deśāntara*.

“beginning Monday” is a check for the *Ahargana*, Monday being stated to be the first day of the *Ahargana*. For, the *Ahargana* got by computation may be a day more or less than the correct one (a fact well known to astronomers) because the ‘varying’ *True Tithi* has got to be used in the formula; and checking by Monday beginning from the Epoch, viz., 0 *Ahargana*, it may happen that one day has got to be added or subtracted. This can be made clear by an example.

Problem: What is the *Ahargana* for Saturday beginning, next to the Epoch?

By counting we see we must get 5 days for *Ahargana*. Let us now compute it. By the *Romaka* or *Pauliṣa* (or even *Saura*) almanac, the *tithi* gone at Saturday beginning is *Caitra Śukla Caturthi*. Using it in the above formulae enunciated in *PS I*. 8-11, 4 is got as *Ahargana*. But counting from Monday, 4 will give only Friday beginning. So we must add 1, and give 5 as the correct *Ahargana* if it should agree with Saturday beginning. We see here the use of the check. This is the purpose for which the weekday beginning the Epoch is given, and it is not merely to satisfy the curiosity of the reader. From this we can see that ‘Monday’ is necessary, and ‘Tuesday’ or ‘Wednesday’ will be wrong. So in *PS I*. 8, ‘*soma-divasādye*’ or ‘*soma-divasādyah*’ must have been VM’s original reading. ‘*Bhauma*’ must have been a scribal error, or the correction of some revisor who did not understand what was necessary, but thought that the weekday of the *True Śuklapratipada* gone must be given here, and this must have given rise to ‘*saumya*’, a mixing of the two. Here T-S have rightly given the emendation ‘*soma-divasādye*’. We may venture to give another suggestion, even if it may not appear very convincing to some. The emendation of T-S is not really essential and we can adopt the manuscript reading ‘*saumya*’ as such and take it in the *yaugika* (derivative) sense, meaning

'day pertaining to the Moon', i.e. Monday. Though there is the dictum '*Rūḍhir yogam apaharati*' ('the meaning obtained by usage is stronger than that got by derivation'), still at such an ancient period as VM's, when the weekdays must have come into use very recently, the word *Saumya-divasa* might not have become *rūḍha* in *budha-vāra* as it is now. Also, when other things require the derivative sense, we are permitted to abandon the *rūḍha* sense.

The above discussion has been necessitated here by a desire to remove any doubt created in the readers' minds by Dikshit's dissatisfaction, which may be interpreted as going against the case for 505 A.D.²⁷

We may now proceed to show that the *kṣepas* of the *Romaka* and the *Paulīśa* also as well as the *adhimāsa* and *avamāseṣa* of their rules for *ahargaṇa*, agree with 505 A.D. and not with 123 B.C. We have seen that the epoch for the *Romaka* and *Paulīśa* is 37 *nāḍis*, 20 *vināḍis* from sunrise at Ujjain on Sunday, 20 March 505 A.D. (It must be noted that Dikshit does not question this.) The *Romaka* Mean Sun at Epoch can be seen to be 359° 34½' by taking the *ahargaṇa* as zero in PS VIII. 1, and working with the *kṣepa* left. This means that 26 *nāḍis* after Epoch, the Mean Solar month *Meṣa* begins. In the same way we get the Mean Moon at Epoch from PS VIII. 4, to be 356° 12', using the emended reading '*kṛtāṣṭanavakaika*'; if the reading '*kṛtāṣṭanavakaika*' found in the manuscript and followed by TS and Dikshit

27. It is only dissatisfaction and nothing more, and it may be noted that Dikshit himself gives reasons for the adoption of 505 A.D. (*Ib.*, p. 46).

Also, Bhaṭṭotpala's reading is सोमदिवसाद्ये. We have since found in the editorial work with regard to the *Pañcasiddhāntikā* that his readings are generally correct. We have also found during this work that PS I 17-20 gives Monday alone.

is used, it is $359^{\circ} 19'$ at Epoch for TS, and $2^{\circ} 24'$ for Dikshit who taken that the Moon is given for sunset at Ujjain. From this we see that the Mean New Moon according to our interpretation will take place at 16 *nādis*, 36 *vinādis* after Epoch, i.e., 9 *nādis*, 24 *vinādis* before the Mean Sun comes to *Meṣa*. According to TS's value for the Moon, it is 24 *nādis*, 42 *vinādis* before the Mean Sun at *Meṣa*; and according to Dikshit, 32 *nādis*. It must be noted that according to all the three interpretations, the Mean New Moon just *precedes* the Mean Solar year, i.e., the New Moon end begins the Mean *Caitra* and is very near the Epoch.²⁸ The corresponding *kṣepas* of the *Paulīṣa* also will be found to give the same result. Thus the word *Caitra* in PS I. 8 presents no difficulty, as it is mentioned only in relation to the *Romaka* and the *Paulīṣa*. Also, the *avama* and the *adhimāsa śeṣas* of the *Romaka* and the *Paulīṣa* found in PS I. 9-11, agree, within the limits of accuracy, with the time of the day when the respective New Moon occurs, and its distance from the beginning of the Mean Solar year as found from PS VIII and III. From the foregoing facts we see that the beginning of *Caitra* should fall very near the beginning of the Mean Solar year, which it does if we take 505 A.D. If 123 B.C. is taken, it is about 20 days away, and so there is disagreement with the *kṣepas* of PS I. 8-11, and those of the Sun and the Moon in PS III, VIII and IX.

In the case of the *Saura*, an examination of the Sun's and Moon's *kṣepas* given in PS IX. 1-2 will show that the

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28. It may be asked why only the Mean *Caitra* and the Mean Solar year are taken into consideration in this explanation. It is because the word *Caitra* is given in the context of the *Ahargana* rule, whose constants, multipliers and divisors depend upon the Mean Sun and Moon. If we use the *True month* and *Tithi* in the computation, it is because we have no other go and that is why there has to be the check by the weekday of the Epoch.

Mean Solar year ends at 3 *nāḍis*, 9 *vināḍis* after midnight, Sunday/Monday, 20/21 March, (as we have already shown), and the Mean New Moon falls about 12 *nāḍis*, 30 *vināḍis* after the Mean Solar year. Thus there is a Mean *Adhimāsa* following the Epoch of the Star planets, i.e., midnight, which can be called *Caitra*, as Dikshit himself has accepted.⁹⁹ As it is very close to (i.e. only 12 *nāḍis*, 30 *vināḍis* from) the beginning of the Mean Solar year, and as in computing the *Ahargana* the practice is to see whether an *Adhimāsa* has taken place or not in the months gone used for reckoning, and adjust the number of *adhimāsas* got by adding one or reducing the *adhimāsas* by one, by treating a large *adhimāsa-śeṣa* as unity or not counting one just got by computation, no harm will ensue if this *Adhika-Caitra* is treated as *regular Caitra*, taking the previous *regular Caitra* as *Adhika-Phālguna*. And there is the advantage of dispensing with a *kṣepa* for months gone at Epoch. So even if *PS I. 8* applies to the *Saura* as Dikshit thinks, the objection which he has to using the term '*caitrāda*' for this vanishes, and there is no need to explain it in the manner he has done.⁹⁹ Thus all the difficulties raised by Dikshit are answered, and not a trace of any objection for 505 A.D. is left.

We may now proceed to give another piece of evidence to show that the date cannot be 123 B.C. In *PS XV* we find the following śloka :

Laṅkārdharātrasamaye dinapravṛttim jagāda
cāryabhaṭaḥ ||
bhūyas sa eva sūryodayāt prāha Laṅkāyām || 20 ||

29. The rule giving this is this :

ravisāṅkramaṇād ūrdhvam yo yo māsaḥ prapūryate cāndraḥ |
caitrādiḥ sa jñeyaḥ pūrtidvīte 'dhimāso 'ntyah ||

30. Dikshit, *Ib.*, p. 51.

Here is a reference made by VM to Āryabhaṭa and his two works, the well-known *Āryabhaṭīya*, and his less known work referred to by later authors as his *Ārdha-rātrika System*, manuscripts of which are yet to be discovered, but whose nature is fully given by Bhāskara I (6th-7th cent.) in his *Mahābhāskarīya*.³¹ In the *Kāla-kṛiyāpāda* of his *Āryabhaṭīya*, Āryabhaṭa says :

ṣaṣṭyabdānām ṣaṣṭir yadā vyatītās trayas ca
yugapādāḥ /
tryadhikā vimśatir abdās tadeha mama janmano
'tītāḥ // 10 //³²

31. *Mahābhāskarīya*, (Ed. T. S. Kuppanna Sastri, Madras, 1957), VII. 21-35. See the Editor's Introduction to this edition, pp. xlv-vi.
32. There is a suggestion by some to emend *ṣaṣṭih* into *ṣaḍbhiḥ* and take Āryabhaṭa to 360 Kali (e.g., Kota Venkatachalam, *The Plot in Indian Chronology*, Ap. III, p. xxi). But with *ṣaḍbhiḥ* the word *ca* in the verse becomes meaningless. It is T.S.N. that started this too. He says in his work that he has a copy with the reading *ṣaḍbhiḥ*. (Knowing him, we have to doubt his veracity.) We can add some more arguments: (1) T.S.N.'s reading would mean that Āryabhaṭa wrote c. 2741 B.C. Who will swallow this! What about the language. (2) Āryabhaṭa's first point, as of all other astronomers, is an insignificant position, is *Aśvini* near a very faint star (V. Piscium). But the vernal equinox c. 2741 B.C. was at nearly 45° off, in *Rohiṇī*. If his work was to be of any use in the matter of *Ahas*, *Lagna*, declination, shadow, solar ecilpse, etc., he should have instructed an addition of about 45° to the longitude got by his work, or else these items would be got very very wrong. T.S.N. has not seen the self contradiction here, being a layman, and his trick has failed. Also manuscripts of the work give only *ṣaṣṭih* and all known ancient commentaries explain the verse only with *ṣaṣṭih*; Cf. *Bhāskara I's Ārya bhaṭīyabhāṣya* : *ṣaṣṭyabdānām ṣaṣṭih, ṣaṣṭir abdāḥ ṣaṣṭiguṇāḥ ityārthaḥ*; Sūryadeva Yajvan's *Āryabhaṭaparakāśa* : *ṣaṣṭatādhika-trisahasramiteṣu* (3600) *sūryābdeṣu gateṣu*; Paramesvara's *Bhaṭadipikā* : *ṣaṣṭatādhika-sahasratraya* (3600) (Edn. Kern.

This says that at 3600 *Kali* (expired) Āryabhaṭa had completed twenty-three years of age, and 3600 *Kali* is 499 A.D. VM's reference is certainly to this Āryabhaṭa as can be gathered from the mention of him as the author of both the *Ārdharātri* and the *Audayika* systems. It follows, from this that VM must be later or at least a contemporary of this Āryabhaṭa. So VM can belong to 505 A.D. and not to 123 B.C. Thus all internal evidence—and we have seen plenty of it—points to 505 A.D. as the time of VM.

The *Ayanāṁśa* argument examined

Now Profs. Rai and VT have advanced an argument based on *Ayanāṁśa* to show that VM must be as early as 123 B.C.³³ Being interspersed with mathematics, this argument may seem unassailable to some, unless its hollowness is exposed.

Being more full, we may discuss VT first. What VT says may be put succinctly as follows: (i) At the time of VM the Summer Solstice was at the *end* of the asterismal segment *Punarvasu*, (or what comes to the same thing, the Vernal Equinox had a longitude of 3° 20' reckoned from the zero point of the Ecliptic), as gathered from VM's own statements in the *PS* and the *Bṛhat Saṁhitā*. (ii) Taking the *Ayanāṁśa* (i.e. the total precession) to be zero at VM's time, there is an *Ayanāṁśa* of 28° 15' in April 1909 A.D. (It comes to this: The Vernal Equinox has receded 28° 15' from the original position of 3° 20', and its position in 1909 is 335° 5' from the zero point of the Ecliptic). (iii) Using the correct rate of precession (*ayana-calana*) per annum, 50".2585— $n \times 0''.000225$, where n is the number of years *before* 1909,

Leiden, 1874, p. 58); Gārgyakerala Nīlakaṇṭha Somayāji's *Bhāṣya...śaṣṭyabdanām śaṣṭeḥ...kaler ārabhya śaṣṭyabdanām śaṣṭir gatā....idānim prakṛtiṣṭham ayanam* (TSS No. 101, pp. 12-13).

33. Rai, *JPUHS* I (1932) 124-27; VT, *JIH* 28 (1950) 104-06.

for a precession of $28^{\circ} 15'$ to take place, n must be 2031 years. (iv) This means 2031 years before 1909, i.e. in 123 B.C., the *Ayanāṃśa* was zero, and therefore 123 B.C. is the date of *PS*.

We admit that if (i) and (ii) are correct, (iii) and (iv) follow automatically. But (i) and (ii) are not correct, as we shall show. Relating (i) there are the following three ślokas of VM, which are quoted by VT also :³⁴

āśleṣārdhād dakṣiṇam uttaram ayanam raver
dhaniṣṭhādyam |
nūnam kadācid āsīd yenoktam pūrvaśāstreṣu ||
sāmpratam ayanam savituḥ karkaṭakādyam
mṛgādiś cānyat |
uktābhāvo vikṛtiḥ pratyakṣaparīkṣanaiḥ vyaktiḥ ||
Br. Sam. III. 1-2 ||
āśleṣārdhād āsīd yadā nivṛttiḥ kiloṣṇakiraṇasya |
yuktam ayanam tadā 'sīt, sāmpratam ayanam
Punarvasutaḥ || PS III. 21 ||

“Certainly at one time the turning of the Sun towards the south was from the middle of the *Āśleṣā* segment, and the turning north was from the beginning of the *Dhaniṣṭhā* segment, because this is mentioned in ancient works.

“But now the turnings are from the beginning of the *Karkaṭaka* and *Makara rāśi* segments, respectively. If this does not happen (in future, on account of precession), the amount of deviation is to be determined by observation.” (*Br. Sam. III. 1-2*).

“When the Sun turned away south from the middle of *Āśleṣā*, it was proper for that time. But now the turning away is from *Punarvasu*.” (*PS III. 21*).

34. *Ib.*, p. 104.

Now in the śloka from the *PS*, “from the middle of *Āśleṣā*” corresponds to the same phrase in the quotation from the *Bṛ. Saṁ.* III. 1; and “from *Punarvasu*” corresponds to “the beginning of *Karkaṭaka*” in *Bṛ. Saṁ.* III. 2 above, the same phenomenon of precession being described in both. So “from *Punarvasu*” must be taken to mean a point three quarters from the beginning of the segment, for that is the point corresponding to the beginning of *Karkaṭaka*. But VT who wants the end of *Punarvasu* to be the turning point, wants us to shut our eyes to the specific reference to the “beginning” of *Karkaṭaka*, and take it to mean “somewhere” in *Karkaṭaka*, giving the reason that the word is found in a mere *Samhitā* and not in a *karāṇa* like the *PS*. It seems he has not taken note of the many passages in the *PS* itself that specifies the ‘beginning’ of *Karkaṭaka* as the point. For instance, in the śloka next but one, i.e., *PS* III. 23, we find “*meṣa-tulādaṁ viṣuvad*”, “at the beginning of *Meṣa* and *Tulā* are the Equinoxes”. One śloka later we have again :

udagayanam makarādaṁ ṛtavaḥ śiśirādayas ca
sūryavaśāt |
dvibhavanakālasamānam, dakṣiṇam ayanam ca
karkaṭakāt || PS III. 25 ||

In XIII. 10, we have, “At the end of *Mithuna* the Sun revolves at an altitude of 24° at the N. Pole”.

Also VT says that *Punarvasutaḥ* can mean only from the “end of *Punarvasu*”. This interpretation is wrong. It only means “from *Punarvasu*”, and can mean any point in it. *Grāmataḥ pattanam pratiṣṭhate* does not only mean ‘he starts from the border of the village’. It can mean any point in the village.

Further, the context in which *āśleṣārdhāt* etc. is found, itself specifies a point $1\frac{3}{4}$ segments from the middle of *Āśleṣā* and this point is three quarters of *Punarvasu*,

In the immediately preceding śloka, VM states that *Vyatiṭpāta-puṇyakāla* occurs when Sun *plus* Moon equals 17 asterismal spaces, i.e., $17 \times 13^\circ 20'$, or $226^\circ 40'$, as opposed to our expectation that it should occur at the middle of the 14th (i.e., at 180°) according to the definition given in the Śāstras.³⁵ There is a difference of $46^\circ 40'$ or $3\frac{1}{2}$ spaces that has to be explained. As *Yoga* is obtained from the combined longitudes of the Sun and the Moon, a change of $1\frac{3}{4}$ asterismal segments in the longitude of each, caused by the shifting of the origin of reference will explain the difference of the $3\frac{1}{2}$ spaces. This shifting of the origin, by the precession of the equinoxes, is mentioned in *āśleṣārdhāt* etc., and this must be $1\frac{3}{4}$ segments as required, and the point at $\frac{3}{4}$ *Punarvasu* follows, for it is this point that is $1\frac{3}{4}$ segments behind the middle of *Āśleṣā*.³⁶

Still another proof can be adduced to show that $\frac{3}{4}$ *Punarvasu* is to be considered as the point in question. If it is the end of *Punarvasu*, the Vernal Equinox will be, as we have already stated, at $+3^\circ 20'$ from the zero point from which the longitudes of the Sun, the Moon etc. are reckoned. So, to get the declination of the Sun etc., to compute the daylight, the shadow and other things, in short, for all work usually given in the *Tripraśnādhikāra* of a *siddhānta*, we must be instructed to deduct $3^\circ 20'$

35. See for instance, *Sūrya Siddhānta*, XI. 1-2:

ekāyanagatau syātām sūryācandramasau yadā |
tadyutau maṇḍale krāntyos tulyatve vaidhṛtābhidhaḥ ||
vīparitāyanagatau candrārkau krāntilīptikāḥ |
samās tadvad vyatiṭpāto bhagaṇārdhe tayoṛ yutau ||

36. For a fuller discussion, see the writer's Introduction to his edition of *Mahābhāskariya*, Madras, 1957, pp. xxv-xxxv. Further, since VT has not brought into the argument either the *Caitra* or the *Dhanīṣṭhā Pakṣa* of the zero-point of the Ecliptic, we have avoided dragging them in and confusing the issue.

from the longitudes got by computation, and use this for the calculation, as the longitudes from the Vernal Equinox are to be used here. In as much as such an instruction has not been given anywhere in the text, we must take it that the zero point and the Vernal Equinox were coincident, which means that the Summer Solstice was at $\frac{3}{4}$ *Punarvasu*. Now in (ii), VT has budgetted for an *Ayanāṁśa* of $28^{\circ} 15'$. But the above fact will result in a cut of $3^{\circ} 20'$, and VM will be lifted 240 years from the intended 123 B.C. towards the true place, 505 A.D.

Now we may pass on to consider (ii), viz., VT's statement that in April 1909 A.D. there is an *Ayanāṁśa* of $28^{\circ} 15'$, taking it to be zero at VM's time, when according to VT the Vernal Equinox was $+3^{\circ} 20'$ from the zero point, i.e. there is a total *ayanacalana* of $28^{\circ} 15'$, from VM's time to 1909 A.D. VT makes up the $28^{\circ} 15'$ necessary for him, by piecing together four different quantities: (a) the distance between the Vernal Equinox and the zero point, both referring to VM's time, equal to $3^{\circ} 20'$; (b) the late L. D. Swamikannu Pillai's (LDS) calculation of the *Ayanāṁśa* in 1909 to be $22^{\circ} 25'$ which is the equivalent in degrees of the time from the Sun at the Vernal Equinox of 1909 to its entering the Sign *Meṣa* in the same year according to *Sūrya Siddhānta*; (c) what VT calls a *Bīja* (i.e. correction) of 2.18 days, equivalent to $2^{\circ} 9'$; and (d) an error of observation equal to $16'$. Of these four quantities, we have already seen that VT cannot have (a), by the fact that the Summer Solstice was at $\frac{3}{4}$ *Punarvasu* and not at the end of *Punarvasu* in VM's time. So $3^{\circ} 20'$ is cut off from the $28^{\circ} 15'$. We shall not discuss (d), for we except to point out below what mischief even this can do. That leaves us (b) and (c) to deal with.

We shall take (b) first. VT uses the *Ayanāṁśa* $22^{\circ} 25'$ calculated by LDS in a manner not intended by him. To understand how it is so, it is necessary to make clear

the principle involved in the calculation.³⁷ LDS found from the *Nautical Almanac* that at 0·2143-day on the 21st March 1909, the True Sun was at the Vernal Equinox. He found that according to the *Sūrya Siddhānta* (Modern, not the *Saura* of PS), the True Sun reached the *First Point of Meṣa* at 0·9492 day on 12th April 1909. From the difference between the two moments, equal to 22·7349 days, using the rate of motion of the sun at that interval, he calculated the *Ayanāṁśa* to be 22° 25'. Suppose LDS had used the time of the True Sun at *Meṣa* (*Meṣa Saṅkramaṇa* as it is called) of some other Almanac like the *Dṛk Almanac* or *Vākya Almanac*, he would have got different *Ayanāṁśas*, for it is a well-known fact that Almanacs vary in their times of *Saṅkramaṇa*. Which *ayanāṁśa* are we to adopt? Which is the 'correct', *Ayanāṁśa*? By 'correct' *Ayanāṁśa* is meant the total precession in degrees, of the Vernal Equinox, from a specific point on the Ecliptic, which we call the zero point, during the interval 1909 and the time when we take the Vernal Equinox to coincide with the zero point, in our case the time of VM. According to this criterion none of the present-day Almanacs gives the 'correct' *Ayanāṁśa*. The following is the reason: If the length of the year adopted by an almanac is the correct Sidereal year, viz., 365 days 15 *nāḍis*, 22·9 *vināḍis*, so that at the end of every year the Sun returns to the specified zero point, then this way of finding the *Ayanāṁśa* will yield the 'correct' *Ayanāṁśa*. But the old system Indian Almanacs use, instead of the above correct Sidereal year, the Sidereal year of the *Āryabhaṭīya* (365-15-31-15, adopted by the *Vākya Almanacs*), or of the new *Sūrya Siddhānta* (365-15-31-31, adopted generally by LDS in his *Ephemeris*) and the like, which though called Sidereal,

37. See LDS, *An Indian Ephemeris*, Madras, 1922, Vol. I, Pt. i, pp. 457-58: "Appendix (ii) Luni-Solar Precession as applied to Indian Astronomy—The year of *Sūnya-ayanāṁśa*, A.D. 533, how determined".

are very nearly Anomalistic, being about 8.5 or 8.6 *vināḍis* longer than the correct Sidereal year. As a result, the *First Point of Meṣa* moves forward leaving the zero point behind at the rate of 8.5'' per annum. So if we adopt LDS's method of using the time of the True Sun at the *First Point of Meṣa* according to a particular Almanac to get the *Ayanāṁśa*, we must deduct from the *gross Ayanāṁśa* got, the accumulated interval between the zero point and the *First Point of Meṣa* of that Almanac, to get the correct *Ayanāṁśa*. (This accumulated interval may be called the '*Procession*' of the *First Point of Meṣa* for that Almanac). It is this *correct Ayanāṁśa* that should be divided by the *correct rate of precession* of 50'' .2585 etc. to get the year when the Vernal Equinox was at the zero point. If, on the other hand, we use the *gross Ayanāṁśa* got by LDS's method, we should divide it by the *gross rate of precession* (which is the correct rate of precession '*plus*' about 8.5''), to get the year, for the *gross Ayanāṁśa* increases not by the correct rate of precession but by the *gross rate of precession*, viz. 50'' .2585 etc. etc. increased by about 8.5''. This is the reason why most Indian systems give nearly 1' as the rate of precession. The reader will find our statement corroborated by sections 64 and 277 of LDS's *Indian Chronology* (Madras, 1911).

This is the reason why LDS divided his *gross Ayanāṁśa* by 58'' .78 and got 536 A.D. for VM as a first approximation.^{37a} The gross rate of precession 58'' .78 is got from the rate of *procession* (viz. 8'' .52) *plus the rate of correct precession* 50'' .26, for it is at this combined rate of 58'' .78 that the Vernal Equinox recedes with reference to the *First Point of Meṣa*, per annum. From this we see that it is wrong to use for this purpose the actual rate of precession given, even by the system, if any, as for eg. 54'' per annum in the case of the new *Sūrya*

37a. Using this date LDS gets 533 A.D. as the correct date.

Siddhanta or 1' per annum in the case of certain other *Siddhāntas*, and so on; for these *Siddhāntas* have found the rates of precession by actual observation of the Sun at the Vernal Equinox, and there is likely to be an error in the observation. According to the error the rates may vary. The nearer their rates are to 58''·78, the better are their observations.

It is incumbent on our part, in the present context, to answer certain remarks made by VT on the above procedure of LDS. VT remarks:³⁸ "There are the following drawbacks in the whole argument (of LDS):

"(a) It was considered that Dakshinayana began when the Sun reached the beginning of Karkataka instead of the end of Punarvasu.

"(b) The fact that the modern tropical year goes on decreasing at the rate of 0·53 seconds per century was not taken into consideration.

"(c) At least at the time of Varahamihira, the Indian Siderial year—so designated at present—was really a tropical year and the value for the precession of the equinoxes must be taken as 50''·2585— $n \times \cdot 000225''$ and not as 54''·7505 as assumed by Swamikannu Pillai."

Of these (a) has already been answered. As for (b), in the 14 centuries considered by LDS, the time neglected by him is about 56 seconds, equivalent in 2" of the Sun's motion. Is this not negligible in the context? As for (c), this is against the internal evidence of the *PS*. Excepting *Vāsiṣṭha*^{38a} and the *Romaka*, all the other *Siddhāntas* in it give Sidereal years. The *Paulīṣa* gives 365 days, 15 *nāḍikās*, and 30 *viṇāḍikās*, the *Saura*, 365-15-31-30, and the *Paitāmaha* 366 days. By what stretch of imagination

38. His article, *JIH* 28 (1950) 105.

38a. *Vasiṣṭha's* is 365-15-0, midway.

can these be called Tropical years, these years that are so far greater than the correct Sidereal year that they border on the Anomalistic? As for LDS not taking $50''\cdot2585$ for division, we have answered it by saying that this would be proper only if the correct Sidereal year, $365\cdot15\cdot22\cdot9$, had been used throughout the period of which we are considering the *Ayanāṁśa*. Secondly, where has LDS assumed a rate of precession of $54''\cdot7505$, and in which context?

It should not be thought that because the modern *Dṛk Almanacs* use the correct Sidereal year equal to $365\cdot15\cdot22\cdot9$, the time of the True Sun at their *First point of Meṣa* will give the correct *ayanāṁśa*. These almanacs were started recently and they arbitrarily fixed for themselves such *ayanāṁśas* as would keep their *saṅkramaṇas* within reasonable distance from those of the old almanacs. The very fact that the *saṅkramaṇas* vary only within a matter of *nāḍīs*, shows this, for considering the difference between the correct Sidereal year and the so-called Sidereal years of our siddhāntas, even within a period of 420 years there will be a difference of one day, and for the period we are considering, viz. 1400 years or more, there should be a difference of more than three days, the *Dṛk Saṅkramaṇas* occurring earlier. To avoid the hue and cry that would be raised if the *saṅkramaṇas* in their almanacs are found to occur thus, more than three days earlier, the *Dṛk Almanac* makers fixed for themselves *ayanāṁśas* that would keep their *saṅkramaṇas* near enough to those of the old system almanacs.³⁹ The

39. In 1925 the Almanac makers met in Conference at Poona and adopted an *ayanāṁśa* of $22^\circ 40' 39''$ for 1925 proposed by R. N. Apte, M.A., Professor of Mathematics, Rajaram College, Kolhapur, arrived at by taking the True Sun's position at Meṣa-saṅkramaṇa in that year according to the *Sūrya Siddhānta* as the zero point. On this see C. G. Rājan, *Rāja Jyotiṣa Gaṇitam*, (Madras, 1933), Section—Conversion of Heliocentric etc., ch. VI, p. 56.

Caitra or the *Dhaniṣṭhā pakṣa* has come in handy for them to fix their *ayanāṁśa* in this manner, but these *pakṣas* are contradictory to all schools of traditional astronomy which have adopted the *Raivata-pakṣa* alone.⁴⁰

To continue the main argument. As, in the manner already stated, the number of years got to be deducted from 1909 to find the Zero Point should be the same, whether we divide the *gross ayanāṁśa* by the *gross rate* of precession, or the *correct ayanāṁśa* by the *correct rate* of precession, and as the rates are in the ratio 7:6 approximately, the *gross ayanāṁśa* found by LDS (by using the time of *saṅkramaṇa* of the new *Sūrya Siddhānta*) should be reduced by one seventh of itself to get the equivalent *correct ayanāṁśa*, which we find to be 19° 12'. So VT can have only 19° 12' and not 22° 23' by (b).

Now, we pass on to (c). (This is VT's special.) What VT says amounts to this: Kali began at midnight 17/18, February 3102 B.C. But most Ephemerides give 0.579 days after sunrise on 15th February as the Epoch of *Kaliyuga*. So there is a difference of 2.18 (?2.17) days which must be a *bīja* correction. So we must add 2.18 days to the time of the True Sun reaching the *First Point of Meṣa* in (b), viz. 0.9492 on 12th April. Thus the interval in days is increased by 2.18 days; which means 2° 9' more in *ayanāṁśa*, which will make up the 28° 15' required.

Now, what VT thinks to be a *bīja* is really the interval between the times of the True and the Mean Suns reaching the *First Point of Meṣa*. According to Indian astronomy the Sun's Equation of the Centre is about 2° 9' at the time of *Meṣa Saṅkramaṇa*. So the True Sun is in advance of the Mean Sun by 2° 9' and

40. C. G. Rajan gives the actual *ayanāṁśa* according to the *Raivatapakṣa* to be 18° 56' 45".7 in 1925; see *Ib.*, p. 58,

reaches the *First Point of Meṣa* earlier by about 2.18 days. As the Apogee of the Sun has an extremely slow motion according to Hindu astronomy, the 2 18 days practically continue through the ages to be the same. In (b), LDS took for calculation the interval between the True Sun at the Vernal Equinox and the True Sun at the First Point of Meṣa which is quite proper. If he had taken the interval between the times of the Mean Sun at Vernal Equinox and the True Sun at the First Point, then indeed we shall be justified in adding 2.18 days; for then the interval first got would have been less by 2.18 days, on account of the Mean Sun reaching the Vernal Equinox later by 2.18 days than the True Sun. If we add 2.18 days, as we ought to now, we get the same interval of 22.735 days. Thus VT cannot have (c).

The error of observation, (d), is possible and may be allowed if required; but it must be remembered that it is arbitrary, indefinite and may be plus or minus. VT has taken (d) as error of observation, not from *apriori* considerations, but *aposteriori*, because this alone will give him, when added to the other quantities and divided by 50'' 2585 etc., 2031 years to be deducted from 1909 and get 123 B.C. So the reader is warned against getting predisposed in favour of 123 B.C., simply because 1909 A.D. minus 2031 is exactly equal to 123 B.C., for this particular amount of error of observation has been arbitrarily presumed to get this very result.

In conclusion, we find that in VT's *ayanāṁśa* of 28° 15', (a) is cut off, (c) is cut off, (d) may be ignored, and (b) is reduced to about 19° 12'. If we divide this by the correct rate of precession, 50''.2585 etc., we get c. 534 A.D. as VM's time. It may be noted how far away this is from 123 B.C., and how near it is to 505 A.D. (epoch).

The *ayanāṁśa* argument of Prof. Rai (JPUHS I.124-27) is the same as VT's (a) and (b), with the

difference that he takes (b) for 1931 instead of 1909. This amounts to $26^{\circ} 3' 40''$ according to him, and committing the same mistake as VT of dividing this *gross ayanāṁśa* by the *actual rate of precession*, he says VM lived 1866 years before 1931, i.e., in 65 A.D. Since this does not take him to the desired 123 B.C. Prof. Rai thinks that this discrepancy may be overcome by assuming an appropriate error of observation, which in this case has to be as large as 3 degrees or so !

With showing that 427 Śaka of VM is 505 A.D. and that the *ayanāṁśa* argument is fallacious, the main object of this paper is over. It is unnecessary to discuss the ślokas from *Jyotirvidābharaṇa* quoted by these scholars enumerating the nine gems of Vikramāditya's court (*dhanvantari-kṣapaṇaka* etc.) and the year given therein; for in the light of the foregoing discussion these must be taken as part of a romance, or an attempt at imposture by the author of the work.^{40a} Nobody will take seriously this śloka jumbling men of different ages together, as no one will take seriously the other romance, the *Bhoja Prabandha*, for matters of history.

The Date of Bhaṭṭotpala

Both Rai and VT seek additional evidence for VM's earlier date by making his commentator Bhaṭṭotpala himself earlier than 505 A.D.⁴¹ We shall examine this now. Bhaṭṭotpala says at the end of his commentary on VM's *Byhājātaka* that he finished writing it in Śaka 888 (elapsed) on *Caitra Śukla Pañcamī*, which was a Thursday :

40a. For the unreliability of this work, see below, Sn. V of the paper on 'The untenability of the postulated Śaka era of 550 B.C.'

41. Rai, *JPUHS* I (1932) 73; VT, *JIH* 28 (1950) 103, *JAHS* 22 (1952-54) 173,

caitramāsasya pañcamyām sitāyām guruvāsare |
vasvaṣṭāṣṭamite śāke kṛteyam vivṛtir mayā ||

Here VT says that “the weekday does not come out correctly if we take either the Śālivāhana Śaka or the Vikrama Śaka. *So the Śaka mentioned by.....Bhaṭṭotpala refers only to the Śaka with 550 B.C. as epoch*”.⁴² This means that if Bhaṭṭotpala’s Śaka is taken as given in the Śaka of 550 B.C., the weekday agrees; and so the date referred to is 888 years after 550 B.C., i.e., 339 A.D., (but in his ‘Andhra Śaka’ he gives 340 A.D., cf. fn. 11 above) and so VM must be earlier still. But we have made the calculations, and we find that it is 339 A.D. that does not give the agreement; in that year the Caitra Pañcamī falls on Friday, ending at about 35 nāḍis. In his *Popular Astronomy*, pp. 136–37, VT has changed the date of Bhaṭṭotpala to 338 A.D., in accordance with his changing the Śaka Epoch to 551 B.C. Strangely enough, here too VT asserts that he finds agreement with the weekday, viz. Thursday.^{42a} My calculation here gives Sunday, i.e. three days off, on this date. On the other hand there is perfect agreement with Śālivāhana Śaka 888 (corresponding to 966 A.D.) if Caitra is in the Pūrṇimānta reckoning which was prevalent in Bhaṭṭotpala’s time⁴³ and place. If Śaka 888 is elapsed year, Caitraśuddha-pañcamī falls on Thursday, at 25 nāḍis, February 28, 966 A.D. So we get the time of Bhaṭṭotpala’s finishing

42. JIH 28 (1950) 109.

42a. It is an obvious fact that for a particular Tithi in a particular month the weekday cannot be the same in two consecutive years.

43. In certain editions of Bhaṭṭotpala’s commentary on the *Bṛhajjātaka* we find instead of the śloka quoted above, another saying that he finished the commentary on a Thursday in Śaka 888 on Phālguna-kṛṣṇa-dvitiyā. This too gives agreement with the weekday only if 888 is in Śālivāhana Śaka. If in the Śaka starting from 551 or 550 B.C. alleged, there is disagreement.

the work correctly as we expressed. Because we took the current *Śaka* instead of the elapsed (elapsed is the more usual practice of the Hindus), we had recourse *Pūrṇimānta* reckoning, where too it is the previous *Phālguna* that agrees.

And there is also positive evidence to show that Bhaṭṭotpala has meant only the *Śālivāhana Śaka*. He has commented on the *Khaṇḍakhādyaka* of Brahmagupta, who says that it is a re-presentation of the (*Ārdharātri*) system of Āryabhaṭa.⁴⁴ This means that Brahmagupta is later than Āryabhaṭa (3600 Kali, corresponding to 499 A.D.) and that Bhaṭṭotpala must be later still. It is not possible to drag down, as VT and others do, both Āryabhaṭa and Brahmagupta together into the earlier centuries, for the following reasons: The date of Āryabhaṭa is definitely 3600 Kali, as already shown. Brahmagupta gives 587 *Śakakāla* as the epoch of his *Khaṇḍakhādyaka* (I. 3). Brahmagupta elsewhere states that 3179 is to be added to the *Śaka* year to get the corresponding Kali year (Cf. *Brāhmasphuṭasiddhānta*, I. 26). Āmarāja commenting on the above verse of *Khaṇḍakhādyaka* (I. 3) gives the Kali year corresponding to the epoch of the work (*Śaka* 587) to be 3766 by adding 3179 to 587; and also calculates and verifies the *kṣepas* and the weekday of the epoch taking the Kali year 3766, which is A.D. 665,⁴⁵ which therefore must be the time of Brahmagupta. Further, Brahmagupta is linked to Bhāskara II (who VT at least admits wrote his *Siddhānta-siromaṇi* in 1150 A.D.) by an observed *ayanāṁśa* of about 11°. Bhāskara II also says that in Brahmagupta's time the *ayanāṁśa* was so little that it was "unobservable even

44. See *Khaṇḍakhādyaka*, I. 1.

45. So VT's statement in his *Popular Astronomy*, p. 137, that only 36 A.D., which date he gives for Brahmagupta, would agree with the weekday and not any other date, is wrong.

to that expert astronomer".⁴⁶ So Brahmagupta cannot be dragged too far away from Bhāskara, and this condition will be fulfilled only if his epoch is in the *Śālivāhana Śaka*. (TSN says in this connection that Brahmagupta, whom Bhaskara II eulogises as his learned ancient teacher, could not detect an observational error of 5' !!) And so his commentator, Utpala's date, *Śaka* 888, has also to be in the *Śālivāhana Śaka*.

Prof. Rai advances another argument,⁴⁷ which is his own and not given by anybody else. It is this: *Bṛhat Samhitā*, VIII. 20-21, gives a rule from which, by using the *Śaka* year, the corresponding *Jovian* year in the 60 year cycle *Prabhava* etc., can be got. He works it out for 1932 using the *Śakā* starting from 550 B.C., and gets 52 years gone in the *Prabhava* series. Using the years gone in the *Śālivāhana Śaka* of 78 A.D., he gets the 18th year in the series, viz. *Tāraṇa*. He finds this *Tāraṇa* given in North Indian almanacs. But he says this proves nothing beyond showing that the North Indian almanac-makers have adopted the *Śālivāhana Śaka* for this rule. But the point at issue is which is the correct *Śaka* to take. This can be found by working out the year from the *Kali* years gone till 1932, and seeing which of the two (52 or 18) it agrees with. Prof. Rai works out the *Jovian* years gone from the beginning of *Kali*, using the elements of the *Sūrya Siddhānta*,⁴⁸ and dividing the result by 60 gets the remainder 52. Lo! this is the same as the

46. Cf. his *Vāsanā-Bhāṣya* on his *Siddhāntaśiromaṇi*, under *Golādhyāya*, *Golabandhādhikāra*, 17-19.

47. *JPUHS* I (1932) 123-24.

48. The *Sūrya Siddhānta*, which Prof. Rai uses, wants actually the years from 'Creation' (i.e., a point 17,064,000 years from the beginning of the *Kalpa*) to be used for this. But as a whole number of 60 year cycles have gone at the beginning of *Kali*, no harm will ensue if the *Kali* year is used as he does.

remainder got by using the Śaka of 550 B.C. So, that is the Śaka intended by VM in his *Bṛhat Saṁhitā*, he says. The argument seems to be perfect.

But this is the fallacy in it: If the Jovian year is to be worked out *a priori* using the *Kali* years gone, the years should be counted *from Vijaya and not from Prabhava*. This condition is specified in the very *Sūrya Siddhānta* whose elements Prof. Rai uses for computation, and this has been missed by him.⁴⁹ Now counting 52 from Vijaya, we get only the 18th year of the *Prabhava* series, viz. *Tāraṇa*, and this agrees with Śaka of 78 A.D. and not the Śaka of 550 B.C. Thus Prof. Rai's argument fails. In the result, it is only a proof for taking VM's *Śakakāla* to be the *Śālivāhana Śaka*, and discarding the Śaka of 550 B.C.

Prof. Rai seeks further support to his theory by stating (*ib.*, p. 71) that, "Albiruni writing in 1030 A.D. not only talks of Bhaskaracharya, but also mentions his book 'Karana Kutuhala'", that the date of composition of Karanakutuhala given in the work itself, viz. Śaka 1105, if taken in the *Śālivāhana Śaka* would be 1183 A.D., i.e. 150 years after Albiruni, which is patently impossible, that "Weber in his Book on Sanskrit Literature (p. 262) notices this anomaly, but is unable to offer any explanation" (Weber, *History of Indian Literature*, English Translation, London, 1914), and that "if we take this Śaka commencing from 550 B.C., the riddle is solved", for this would take Bhāskara to the 6th cent. A.D., long prior to Albiruni.

The answer to Prof. Rai is given by Bhāskara himself who indicates that he uses only the *Śālivāhana Śaka*, for he says that 3179 is to be added to the Śaka year to get

49. Cf. *Dvādaśaghnā guror yātā bhagaṇā vārtamānakaiḥ |*
rāṣibhiḥ sahitāḥ, suddhāḥ śaṣṭyā syur "Vijayādayaḥ" || I.55 ||

the *Kali* years gone (Cf. *Siddh. Śiromaṇi*, Gaṇita, Madhyama, Kālamāna, 28). Moreover Albiruni's words in the context do not warrant the name Bhāskara at all, nor does he mention anywhere a work *Karaṇa-‘Kutūhala’* (the work named being a *Karaṇa-‘Sāra’*). It has also to be added that *Prof. Rai is not speaking the facts* when he says that Weber “is unable to offer any explanation”. As a matter of fact, in contradiction to what Prof. Rai says, Weber offers on the very page that Rai refers to (page 262) several explanations: Weber says that “we have scarcely any alternative save to separate Albiruni's ‘*Bashkar*’ son of ‘*Mahdeb*’, and the author of ‘*Karaṇa-sāra*’ from the *Bhāskara*, son of *Mahādeva*, and author of *Karaṇakutūhala*’”. (Note that none of the three names, neither that of the author, nor of his father, which is really *Maheśvara* and which Weber himself draws attention to in a footnote, nor of the work, tallies). Weber again suggests that his translation of the Arabic words of Albiruni might be wrong, for “Albiruni usually represents the Indian *bh* by *b-h*, and for the most part faithfully preserves the length of the vowels, neither of these is here done in the case of *Bashkar*, where, moreover, the *s* is changed into *sh*”, and adds in a footnote that in the passage under discussion “there lurks *not a Bhāskara at all*, but perhaps a Pushkara”.⁵⁰ Even if the

50. Weber's doubts about the translation of the Arabic passage are only too well founded, for we find Sachau translating the passage as: “Further there is an astronomical hand-book...by *Vitteśvara*, the son of *Bhadatta* (? *Mihdatta*) of the city of *Nāgarapura*, called *Karaṇasāra*.” Cf. *Alberuni's India*, E. C. Sachau, London, 1910, vol. I, p. 156. The *वटेश्वरसिद्धान्त* since published by the Astronomical and Sanskrit Research Society, New Delhi, gives “महदत्तसुतः, वटेश्वरः अहम्” (मध्यमाधिकार, अध्याय I.1). मध्य I.21 gives the date of work as 826 शक, and the author's birth as 802 शक. I. 10 gives that 3179 years are to be added to the Śaka years to get *Kali* years.

passage refers to a Bhāskara, Weber suggests that “we may have to think of that elder Bhāskara, ‘who was at the head of the commentators of Āryabhata, and is repeatedly cited by Pṛthūdakasvāmin, who was himself anterior to the author of the *Śiromaṇi*’”. It is in the face of these facts that Prof. Rai coolly asserts that Weber “is unable to offer any explanation!” (Here Rai only follows T.S.N’s remarks.)

We may add here that the epoch of *Karaṇasāra*, which is mistaken for *Karaṇakutūhala*, is given by Albiruni as Śaka 821 (A.D. 899) (*Alberuni’s India*, Sachau, I. 392), and obviously Prof. Rai’s Bhāskara of the 6th cent. cannot write a work 300 years later! So Prof. Rai’s argument only goes against his own theory.

Thus nothing can shake the evidence showing that the Śaka mentioned by VM is the Śālivāhana Śaka and that the date Śaka 427 given by him in his *PS* is 505 A.D. Incidentally it has also been shown that the Śaka era used by Brahmagupta, Bhāṭṭotpala and Bhāskara II is the Śālivāhana Śaka of 78 A.D.

We propose to show in a subsequent article the untenability of certain other claims of these scholars referred to in the Introduction and that everywhere when the word Śaka occurs as the *name* of an era, it is only the Śālivāhana Śaka that is meant, and therefore or otherwise there is no case for postulating a Cyrus or Andhra Era of 550 B.C.

THE UNTENABILITY OF THE POSTULATED ŚAKA OF 550 B.C.*

I. Introduction

It has been shown in the preceding study that the Śaka Era used or alluded to by astronomers like Varāhamihira (VM), Brahmagupta, Bhaṭṭotpala, Śrīpati, Bhāskaras I and II, etc. is the era starting from 78 A.D., later known as the *Śālivāhana Śaka*, and not the era of 550 B.C. postulated by the late T. S. Narayana Sastri (TSN) or V. Thiruvenkatacharya (VT) and called by them the Cyrus Era or the Andhra Era, respectively. Incidentally we have shown to be untenable their statements that Āryabhaṭa belonged near to 2742 B.C., VM to 123 B.C., Brahmagupta to 36 A.D., Bhaṭṭotpala to 339 A.D. and Bhāskara II to 522 A.D., and thereby we have proved that VM belongs to 505 A.D. and Bhaṭṭotpala to 966 A.D. and indicated that the real date of Āryabhaṭa is 499 A.D. and of Brahmagupta 654 A.D.¹

In the same way it can be shown that wherever other astronomers or writers like Kalhaṇa and Albiruni mention a Śaka Era, it is this Śaka of 78 A.D. they mean. The tradition of almanac-makers also supports this, for they all give in their almanacs only this Śaka Era and not the alleged other one. In inscriptions and documents also, in short, in every case where a date in Śaka Era is given, it

* Rep. from *JIH* (Trivandrum), 37 (1959) 201-24.

1. In the same way we can easily see that the date of Bhāskara II's work, the *Siddhānta Śiromaṇi*, is 1150 A.D., from his statement :

rasa-guṇa-pūrṇa-mahī (1036) *sama Śaka-nṛpa-samaye*

'bhavan mamotpatih/

rasa-guṇa (36) *varṣeṇa mayā Siddhānta Śiromaṇi racitaḥ//*

is this Śaka alone, though this is disputed by TSN and (till recently) by Sri Kota Venkatachelaṃ (KV) in the case of the Aihole Inscription (to which we shall revert later).

II. VM's Brhat-Samhita XIII. 3 considered

We shall now take up for discussion *Brhat-Samhita* of VM (*Br. Sam.*) XIII. 3 referred to by us in the previous paper, which TSN and others consider as their stronghold, and which we left over for detailed consideration later :

āsan maghāsu munayaḥ śāsati pṛthvīm yudhiṣṭhire
nṛpatau /
ṣad-dvika-pañca-dvi-yutaḥ Śakakālas tasya
rājñas ca ||

This stanza occurs in the context of the *Śaptarṣi-cāra* or the alleged 'Motion of the Seven Sages', (i.e., the group of stars Ursa Major or the Great Bear), among the twenty-seven asterisms, given for use in astrological prediction. To find the position of the group at any time, three things are necessary : (i) its position at a given time ; (ii) the time elapsed from the given time to the time for which the position is required, and (iii) its rate of motion. The above stanza gives (i) and (ii), viz., that at the time of Yudhiṣṭhira's rule the Sages were at the asterismal segment *Maghā*, and the time elapsed from this time to any year in the *Śaka Era* is the number of the year in the *Śaka Era* plus 2526. (Requirement iii is given in the next stanza, XIII. 4, as one asterismal segment for 100 years).

Now TSN and KV argue thus : (a) This stanza is a quotation from Vṛddha Garga (VG), and so VG knows a Śaka Era which he mentions here. It is accepted by all that VG lived long prior to 78 A.D., the starting point of the Śāliivāhana Śaka. So this Śaka must be an earlier Śaka, viz., that of 550 B.C. postulated by them. (b) The first half of this stanza says the Sages were in *Maghā* during

Yudhiṣṭhira's time. The Purāṇas and VG etc. say that at the junction of Dvāpara and Kali yugas, the Sages were at *Maghā* and Yudhiṣṭhira was ruling. 25 years after the advent of Kali, the Sages moved to the next asterism from *Maghā* and in that year Yudhiṣṭhira left this world for heaven. The second half of the stanza states that the *Śakakāla* mentioned therein started 2526 years after *Yudhiṣṭhirakāla*. If we take *Yudhiṣṭhirakāla* to have started from the time he went to heaven, i.e. after 25 Kali equivalent to 3076 B.C., this Śaka must have started 2526 years after this, i.e. from 550 B.C., and is evidently quite different from the Śaka starting from 78 A.D.

It is in the light of this conclusion, and in support of it, that TSN etc. say (as we have discussed already in the previous paper), that the *Śakakāla* mentioned by VM in other places in his works, and also by other astronomers like Brahmagupta, is this Śaka of 550 B.C. But we have proved conclusively in the previous study that in those places it is the Śaka of 78 A.D. that is referred to.² Therefore the conclusion here arrived at by TSN etc. must stand on its own legs. We shall proceed to examine this now. Even at the outset we can say that it is extremely unlikely that VM means here alone a Śaka different from what he means by the same word elsewhere in his works; and therefore he must mean the Śaka of 78 A.D. here also. All the same we shall examine their arguments.

(a) The alleged quotation of Vṛddha Garga

The reasoning (a) is based on the assumption that the stanza is a quotation from VG, which it is not. The

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2. There is one other place where VM mentions the *Śakakāla* viz. *dvyūnam Śakendrakālam* etc. (*Pañcasiddhāntikā*, XII. 1), which we have not taken for discussion. This mention is in connection with the rough *Paitāmaha Siddhānta*, and as no useful purpose will be served by discussing it, and as it is not taken into consideration by TSN etc. also, we have left it out.

actual words of VG are quoted by the commentator Bhaṭṭotpala in his commentary on this stanza: *cf. tathā ca Vyddha-Gargaḥ*: “*Kali-Dvāparayoḥ sandhau sthitās te pīṭydaivatam*” (At the junction of Kali and Dvāpara, they—the sages—were at Magha). It is to be noted that thus would be redundant if the stanza in question also were VG’s, both giving the same idea, *viz.* the situation of the Sages. It may also be noted that this is in a different metre. What VM means by his statement in the introductory stanza, *kathayiṣye Vyddha-Gargamatāt* (*Bṛ. Saṃ. XIII. 2*) is only that he is giving the astrological predictions due to the motion of the Sages *as based on the work of VG*, as indicated by the word *matāt* (‘opinion’) used here. Also in all the other *cāras* given in the other chapters of *Bṛ. Saṃ*, like *Adityacāra*, *Candracāra*, *Rāhucāra* etc., what VM means by *cāra* is *the prediction based upon the motion and not the actual motion*, and so must it be here also, (the actual motions being given in a *gaṇita* work like the *Pañcasiddhāntikā*). If in the case of the Sages the motion also is given, it is because it is simple, has not been given elsewhere, and is necessary for the main purpose, *viz.*, the prediction according to the motion. Thus it is the prediction that VM says he gives according to VG. So this stanza which serves to find the position of the Sages need not necessarily be, and as we have shown, is not, VG’s.³ This

3 This is the reason why this stanza has not been taken as VG’s by other scholars also. For *e.g.* Colebrooke writes: “The commentator, Bhaṭṭotpala, supports the text of his author (*viz.* VM) by quotations from VG and Kāśyapa: ‘At the junction of the Kali and Dvāpara ages’, says Garga, ‘the virtuous Sages.....stood...’ at Maghā” (*Miscellaneous Essays*, London, 1873, vol. III, p. 313). Cunningham writes: “His (VM’s) words are,.....‘The Seven Seers etc.,’ But unluckily for VM, his commentator Bhaṭṭa Utpala has given us the very words of Garga, who simply says, ‘At the junction of the Kali and Dvāpara ages the virtuous Sagesstood at the asterism over which the Pitṛs preside,

being the case, it cannot be argued that Garga who came long prior to 78 A.D. knows a *Śakakāla* and therefore this *Śakakāla* must be the earlier postulated one of 550 B.C.

(b) The Time of Yudhiṣṭhira

We now pass on to consider (b), the second and more important reasoning of TSN etc., viz., that VM in this verse refers to Yudhiṣṭhira who lived at the beginning of Kali and rose to heaven 25 years after Kali set in (i.e. in 3076 B.C.) and so the Śaka Era beginning 2526 years after that must be the postulated Śaka of 550 B.C. But we answer, there is nothing in this verse to show that in VM's opinion Yudhiṣṭhira lived at the beginning of Kali. On the other hand, it can be shown that VM might have meant a time about 650 years after Kali, or even that he *did mean* this later period for the time of Yudhiṣṭhira, and therefore the Śaka Era following 2526 years after, cannot be the postulated Śaka of 550 B.C., but can only be the well-known Śaka of 78 A.D. It is a fact well known to scholars (inclusive of TSN etc.) that the junction of Dvāpara and Kali (3102 B.C.) is not the only period with which Yudhiṣṭhira is associated. This is according to one school ; but there is at least one other

that is Maghā'. On comparing this quotation with Varāha's statement (in the stanza in question) we see at once that he (VM) has suppressed Garga's mention of the Kaliyuga....." (*Book of Indian Eras* Culcutta, 1883, pp. 9-10). This shows, that Cunningham considers that VM is *not quoting the stanza*, but that it is VM's own. P. V. Kane says: 'In the preceding verse VM says that he will declare the motion of the seven sages by *deriving* it from the doctrine of VG. The first mistake of the writer (It is KV that he refers to here) is to hold that verse XIII.3 came originally from Garga *Saṁhitā*. Really it is VM's own verse. Utpala quotes the verse of VG, on this point, which is in a different metre, though the meaning is the same as the first half of XIII.3" (*JAHRS* XXI (1950-52) 41).

school (e.g. that of the Jain and Buddhist writers) who take it that Yudhiṣṭhira lived about 500 years later. They use a *Yudhiṣṭhira Era* which began in 468 Kali (corresponding to 2634 B.C.).⁴ Even of the first school mentioned, not all associate the same event of Yudhiṣṭhira's life with the beginning of Kali, 3102 B.C. There are four sub-schools here (Fleet says three, but mentions all four, *JRAS* (1911) 676-78, 'The Kaliyuga Era of B.C. 3102'). One sub-school believes that the first coronation of Yudhiṣṭhira at Indraprastha was the beginning of Kali and the commencement of the *Yudhiṣṭhira Era*.⁵ Another makes the Bhārata war and the beginning of Kali

4. *Vide*: i. Jinavijaya "ṛṣir vāras tathā pūrṇam martyākṣau (2077) vāmamelanāt" "nandāḥ pūrṇam bhūṣca netre maujānām (2109) ca vāmataḥ" quoted by TSN in his *Age of Śaṅkara*, (Madras, 1916 ff.), Pt. I. ch. iii, pp. 139 and 141, and also his adding 468 to get the year in the Kali Era. It has been added that TSN's alleged quotations from this work are his concoctions. We have shown this in the 'Date of Śaṅkara' (see supra). ii. Kota Bhaviah Chowdary's statement: 'According to Jain authorities Yudhiṣṭhira was crowned in 2634 B.C. only..... From Purāṇic Kaliyuga (of 3102 B.C.).....468 years passed upto Yudhiṣṭhira'. (*JAHRS* XXII (1952-54) 53 Cf. also Cunningham, *Book of Indian Eras*, p. 7, where he speaks of Abul Fazal giving in his *Ain-i-Akbari* three views on the subject, of which one is the reign of Kāṁsa, (uncle of Kṛṣṇa and so contemporary of Yudhiṣṭhira) "above 4000 years before the fortieth of Akbar", (i.e. 1595 A.D.), that is between 2400 and 2500 B.C.' This would give Yudhiṣṭhira a date about 2407 B.C. or the 7th cent. in Kali,

5 The inscription in the temple of Hanumān at Jasalmer, Rajaputana, gives a date in this era. The speech of Hanuman in the *Mahābhārata*, *Vanaparva*, ch. 151, verse 39 (Kumbakonam edn.) containing the words *etat kaliyugam nāma acirād yat pravartate*, and Kṛṣṇa's excuse for the unfair fight with the words *prāptam Kaliy ugam viddhi*, (ib. *Salyparvan*, ch. 61, verse 27) support this. Abdul Fazl expresses another view that the Mahābhārata War was fought 4801 years before the 40th year of Akbar's rule and 105 years before the end of the Dvāpara age, (see *ib.*)

synchronous.⁶ A third says that Kali began at the death of Kṛṣṇa and his ascent to heaven.⁷ The fourth sub-school says that Yudhiṣṭhira's abdication and starting on the *Mahāprasthāna* was at the beginning of Kali.⁸ The reason why there are so many views must be explained by the fact that the traditional idea of the ages like *Kṛta*, *Tretā*, *Dvāpara* and *Kali* with their specific characteristics, was earlier than the integration of the beginning of the traditional Kali with that of the astronomical Kali answering to 3102 B.C., which was computed later by astronomers like Āryabhaṭa so as to form a convenient point of reference for the Mean Planets. Thus the *Kali Era*, said to begin with 3102 B.C., is an extrapolated era, and in examining any date mentioned in this Kali Era, this fact should be borne in mind.

Now, in this multiplicity of schools on this point, which is a fact accepted by all, resulting from the integration of the traditional Kali with the astronomical Kali, there is the possibility of VM's statement representing one other school or at least a variant of the Jain school, differing as it does, from it only by about two centuries. Kalhaṇa, the Kashmirian chronicler of the 12th cent. A.D., is one of those that subscribe to this school; for not only does he quote in his *Rājataranginī* this verse of

6. Eg. the Aihole Inscription. See discussion *infra* for details. This view is mentioned also by Abul Fazl, which is 4696 year before the 40th year of Akbar's rule. (see *ib*).

7. The *Purāṇas* express this view. Cf:
yasmin Kṛṣṇo divam yātaḥ tasmin eva tadāhani |
pratipannaḥ Kaliyugaḥ tasya saṅkhyāṁ nibodhata ||

Brahmāṇḍa Purāṇa, ch. 74, verse 241 (Venk. Press edn.).
Vāyu has the same reading. *Viṣṇu*, *Matsya* and the *Bhāgavata* have almost the same reading.

8. The words in the *Āryabhaṭīya*, *Gurudivasāc ca bhāratāt pūrvam*, (*Gitikā*, 5), which is explained by Parameśvara as 'the day of the *Mahāprasthāna*', support this.

VM. but also expresses his own concurrence with it in so many words :

*Bhāratam Dvāparānte 'bhūd vārtayeti vimohitāḥ |
kecid evam mṛṣā teṣām kālasaṅkhyām pracakrire ||*
I.49 //

*śateṣu ṣaṭṣu sārdheṣu tryadhikeṣu ca bhūtale |
Kaler gateṣu varṣāṇām abhūvan Kuru-Pāṇḍavāḥ ||*
I.51 //

ṣat-dvika-pañca-dvi-yutaḥ Śakakālas tasy rājñas ca |
I.56b //

“Some people have been misled by the statement that the Bhārata (War) was at the end of Dvāpara, and have given a wrong chronology to the kings (the Pāṇḍavas, Gonanda etc.)...The Kurus and the Pāṇḍavas came when 653 years had passed in Kali..... The time in the Śaka Era plus 2526 is the time of his rule, i.e. the time in the epoch beginning from his (Yudhiṣṭhira's) rule.”

It may noted that 653 plus 2526 (the numbers here given) equal 3179, the well-known converter of Śaka into Kali and *vice versa*. Not only is Kalhaṇa a believer in this school, but he is also certain that VM belongs to this school, as seen by his statement ‘*Samhitākāraiḥ*’ (*Rājatarāṅginī*, I. 55) and his quotation of VM following immediately (I. 56). Cunningham also thinks the same as seen from his statement, “As VM places the Great War 653 years after the beginning of the Kali Age.....” (*op. cit.* p. 11). Again, Prof. P. C. Sengupta, who in his *Ancient Indian Chronology* (Univ. of Calcutta, 1947) in seeking to determine the date of the Bhārata War astronomically (see chs. I-III) favours this school, and comes to the conclusion that: “The date of the

Bhārata Battle is thus astronomically established as the year 2449 B.C. (Kali 653), which is supported by the Vriddha Garga tradition recorded by Varaha Mihira., (see p. 19). Now it must be noted that the mere possibility of following this school is sufficient for our purpose, as we have stated above,

Nor, can the objection be raised that VG and the Purāṇas associate the Sages with *Maghā* at the beginning of Kali, and that in this verse too, as the Sages are declared to be at *Maghā* in Yudhiṣṭhira's reign, the time here should be taken as the beginning of Kali, and so the time given for Yudhiṣṭhira's rule must be the beginning of Kali, and not 653 Kali. For, the beginning of Kali is not associated with *Maghā* alone. The *Matsya Purāṇa* says (ch. 271, st. 41) that according to the Śrutar-sis the Sages were at *Kṛttikā* at the beginning of Kali, and TSN and KV are aware of it (TSN quotes it and explains it, see *The Age of Sankara*, Madras, 1916 ff., App., pp. 166-67; so also KV, *Plot in Indian Chronology*, 34-36). They themselves say that in VM's opinion also the sages were at *Kṛttika* at the beginning of Kali (TSN. *ib.*, p. 171; KV, *ib.* p. 36, and 'Indian Eras', *JAHRS* XX. 77).⁹ According to Āryabhaṭa II and Parāśara too

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9. In fact, nowhere does VM say that the sages were at *Kṛttikā* at the beginning of Kali. This must be an inference of Cunningham when he gives (cf. Table on p. 17 of his *Book of Indian Eras*) 3177 B.C. for the beginning of *Kṛttikā* according to VM, inference from the fact that according to him the Sages passed to *Maghā* in the 7th cent. after Kali (cf. same Table). TSN and KV seem to have simply taken Cunningham's statement as true without question. They only object to Cunningham's treating the motion as direct, while according to them it is retrograde. But they fail to see that if the motion is retrograde, the Sages should be at *Anurādhā* (and not at *Kṛttikā*) at the beginning of Kali if they are to come to *Maghā* seven centuries later. The Cunningham's inference would be wrong and their acceptance of it would also be wrong.

(for details see below), the Sages were at *Kṛttikā* at the beginning of Kali. They were at *Śravaṇa* according to Śakalya and Munīśvara, and at *Rohiṇī* according to Lalla (for details, see below). So the objection raised above does not stand. Now, according to Āryabhaṭa II and Parāśara, who give *Kṛttikā*, it is easily seen that the Sages will be in *Maghā* in the 7th cent. Kali, for *Maghā* is the seventh asterism from *Kṛttikā* and the motion is about one century per asterism. Thus, there can be no objection to the Sages being in *Maghā* in the 7th cent. Kali. It is only if the motion of the Sages is taken to be retrograde (as TSN and certain others think) according to Āryabhaṭa II, Parāśara and VM, that the Sages cannot be in *Maghā* in the 7th cent. Kali, but would be far away from it. But *it is not retrograde* according to Āryabhaṭa II, Parāśara and VM, as also according to other astronomers who give rules for the motion, which we shall show.

III. (a) 'Motion of the Sages'—Direct, not Retrograde

This requires a knowledge of the motion of the Seven Sages¹⁰ which we shall give in some detail because there is a lot of misconception among scholars (including TSN, KV and VT) about this, which in turn vitiates the results

10. The following are the names of the Sages, with their Right Ascension and Declination for c. 1900 A.D.: (i) *Kratu* (Alpha Ursa Major) 10^h 58^m, +62° 17'; (ii) *Pulaha* (Beta Ursa Major) 10^h 56^m, +54° 55'; (iii) *Pulastya* (Gamma Ursa Major) 11^h 49^m, +54° 15'; (iv) *Atri* (Delta Ursa Major) 12^h 10^m, +57°; (v) *Angiras* (Epsilon Ursa Major) 12^h 50^m, +56° 30'; (vi) *Vasiṣṭha* (Zeta Ursa Major) 12^h 50^m, +56°; (vii) *Marici* (Eta Ursa Major) 13^h 44^m, +49° 49'. For comparison we shall give the asterisms belonging to the corresponding ecliptic segment: *Maghā* (Alpha Lenois) 10^h 3^m, +12° 27'; *Pūr. Phal.* (Delta Lenoise) 11^h 9^m, +21° 4', *Ut. Phal.* (Beta Lenois) 11^h 44^m, +15° 8'; *Hasta* (Beta Corvi) 12^h 29^m, -25° 5'; *Citrā* (Alpha Virgo) 13^h 20^m, -10° 38', *Svāti* (Alpha Bootes) 14^h 11^m, +19° 42'.

of their researches. It was believed by the authors of the ancient *Jyotiṣa Samhitā*, like VG, that the Sages had a motion of their own among the other stars, just like the planets, the rate of motion being given as 100, or nearly 100, years per asterism ($13^{\circ} 20'$). It may be said, even at the outset, that there is no such motion as claimed to exist by the authors of these *Samhitās* and followed up the *Purāṇas* and some of the later astronomers; that the Sages are always to be associated only with the *Phalgunīs*, *Hasta* and *Citrā* asterisms (see fn. 10); and that the theory of their motion is wrong, howsoever it might have originated.¹¹ That is why many standard astronomers and

11. (i) A number of explanations can be given as to how this wrong theory of motion could have originated, but as this is beside the point, we stop with saying this much. Some people believe in the reality of this motion and try to explain it accordingly, using the theory of the Precession of the Equinoxes. For e.g. (i) VT says that because the celestial Pole moves in a small circle once round the Pole of the Ecliptic in about 27,000 years, the point of inter-section of the Ecliptic with the line joining it and the mid-point of the first two stars constituting the Sages also moves. As the Sages are said to be at this point of intersection, they are also considered to move (see his *Popular Astronomy*, Madras, 1958, pp. 138-40). But this simulated motion can be only a small fraction of of the value of the motion according to the *Samhitās* which is as great as $13^{\circ} 20'$ per century. Also, while the *Samhitās* say that the motion is uniform and traverses the ecliptic completely, this simulated motion will not be uniform, and will be oscillatory and restricted to a small segment of the ecliptic. Thus it will be forward and backward, the former during the past 7,000 years and more, against the opinion of VT (cf. *ib.*, p. 139) who says that the motion is retrograde like that of the First point of Aries.
- (ii) Prof. R. Krishnamurti (according to VT, *ib.* 139-40) holds the view that the extent of the Sages in longitude is about a tenth of the ecliptic. This extent is divided into 27 equal parts, each part 'symbolically' forming a *Nakṣatra*. The traversing of these 27 symbolic *Nakṣatras* will take

astronomical works like Āryabhaṭa I, Brahmagupta, Śrīpati, Bhāskaras I and II, the *Sūryasiddhānta*, etc., do not deal with the subject at all as being outside the pale of real astronomy. That is also why Kamalākara is constrained to say in his *Siddhānta Tattvaviveka* (Banaras, 1880-85), *Bhagrahayutyadhikāra* :

Śākalyasamjñā-muninā kathitās sabāṇāḥ
saptarṣitārakabhavā dhruvakās calās ca/25a/
 * * * * *
yair golatattvam vivṛtam hi taiś ca
sūryādibhir naiva viśeṣa eṣaḥ/

2700 years. If it is taken that the other 9 segments also are so divided, it will take $10 \times 2700 = 27,000$ years for traversing the whole ecliptic. According to him, the authority for this symbolic division is the following in the *Rgveda* :

śatm te rājan bhiṣajas sahasram urvi gabhirā sumatiṣ ṭe
astu |
bādhasava dūre nirṛtim parācaiḥ kṛtam cidenāḥ pramumug-
dhyāsmat ||
ami yo ṛkṣā nihitāsa uccā naktam dadṛṣe kuhacid dīveyuḥ |
adabdhāni varuṇasya vratāni vicākaśaccandramā naktam eti ||
Rgveda 1.24.9-10

But what have these *ṛks* in prayer of Varuṇa to do with the alleged symbolic division? What flight of imagination can create the idea of the symbolic division of the ecliptic from the two words *śatam* and *sahasram* in one sentence, and the word *ṛkṣā* in quite a different sentence? And then Prof. Krishnamurti seems to suggest that the motion of the Sages is only another name for the phenomenon of Precession, artificialised for the purpose of chronology.

- (iii) Dr. D. S. Triveda (*JIH* XIX (1940) 9-12) confuses the motion of the Sages with Precession itself and says that the ancient *ṛṣis*, far older than *Samhitākāras*, had discovered a cycle of 27,000 years for the motion (the same as for Precession), but by the time of the *Samhitākāras* one cypher was lost, and the period was mistaken as 2700 years! He does not mention how Precession simulates the motion of the Sages.

*proktas svaśāstre 'sti gatir munīnām
ato na yuktā divi golarītya||30||*

*adyāpi kair api narair gatir āryavaryaiḥ
dṛṣṭā na yātra kathitā kila Śamhitāsu|32a|*

*prāyo 'tha te ca munayaḥ kila devatāṁśā
dṛggocarā nahi nṛṇām iha satphalāptyai|36a|*

“Sage Śākalya has given the motion of the Sages with their positions in his time...Sūrya and others who explain the nature of the celestial sphere in their works do not give it and therefore the theory cannot be sustained astronomically...Even today this motion mentioned in the *Śamhitās* is not observed by knowing astronomers...Therefore the seven real Sages who are (only) *the presiding deities (of these stars) are to be considered to be moving, unobserved by men, for the prediction of the fruits thereof.*”

But the motion has been accepted as a fact by the common people and the authors of the *Purāṇas*, and an era called *Laukika Era* (by the people belonging to the Kashmir region) and the *Saptarṣi Era* (by the *Purāṇas*), has been founded upon this theory.¹² As already mentioned, there are also astronomers like VM, Āryabhaṭa II (cf. his *Āryasiddhānta* or *Mahāsiddhānta*, *Madhyamādhya*, 11), Parāśara (cf. *Āryasiddhānta*, *Parāśaramatādhya*, 9), Lalla (quoted by Munīśvara in his commentary

12. We do not know when these eras were founded. The *Purāṇas* say that 25 years after Kali set in, the Sages who had been at *Maghā* for 100 years left it for the next asterism. The dates of the dynasties of kings are given in terms of the situation of the Sages in the different asterisms. The *Laukika Era* is the same as the *Saptarṣi Era* with the number of the year in each century being generally used and the centuries or the reference to the asterisms omitted. This is used in the *Rājatarāṅgiṇī* to give the dates of dynasties and kings, as also the date of the work.

on the *Siddhānta Śiromaṇi*), *Śākalya* (quoted by Muniśvara, *ib.*, and by Kamalākara in his *Siddhānta Tattvaviveka*, *Bhagrahayutyadhikāra* 25 and under), Vateśvara, and Muniśvara (see his *Siddhānta Sārvabhauma*), who have accepted the motion as real on the authority of the ancients and given rules for the motion, which necessarily must agree with their own observation, or else they would be meaningless even for them.¹³ This means that whatever be the rule, if applied to the time of the author, the position of the Sages must be got as between *Maghā* and *Svātī*.¹⁴ In giving the rules, the authors all consider

13. *vide* Colebrooke, *ib.*, p. 316-17: "...a probable inference may be thence drawn as to the period when these authors lived, provided one position be conceded; namely, that the rules, stated by them, gave a result not grossly wrong at the respective periods when they wrote. Indeed, it can scarcely be supposed that authors, who, like the celebrated astronomers in question, were not mere compilers and transcribers, should have exhibited rules of computation, which did not approach to the truth, at the very period when they were proposed."
14. Because the Sages are always to be seen within this limit. Though strictly speaking the position of the first two stars are to be taken into consideration, still, in practice, the situation of the whole group must have been vaguely taken as the position. The Sages are said to be at the asterism where the declination circle passing through the mid-point of the two front stars (*Pulaha* and *Kratu*) that rise first, cut the ecliptic. This would give a position beyond *Maghā* and near *Pūrva Phalguni*, at present. (But in ancient times *Maghā* might well have been the position an account of the Celestial Pole having been a little more to the East of its present position.) But it seems that later on it came to be considered that the asterism against which the Sages are seen generally is the position. Thus we can get *Pūrva Phalguni*, *Uttara Phalguni*, *Hasta* or *Citrā*. The rule may also be interpreted as the segment which is seen to rise together with the two front stars. If this interpretation is accepted, we can get, on account of *cara* (oblique ascension), any asterism from *Kṛttikā* to *Maghā* as

the motion as *direct* and *never as retrograde* as fancied by some scholars like TSN, KV, VT etc, for which fancy there is no support anywhere.¹⁵ Let us take the rules one by one and examine them for the facts mentioned.

VM's rule is as follows (*Bṛ. Sam.*, XIII. 3-4): The number of years gone from the time of Yudhiṣṭhira is to be found by adding 2526 to the Śaka years gone at the time for which the position of the Sages is wanted. This is to be divided by 100, which gives the number of asterismal segments gone, and these are to be counted from *Maghā* to get the position. In the context there is no mention of any retrograde motion, nor is it mentioned that the number got is to be counted backwards as in the case of the *pātas* like *Rāhu*. In the absence of any such specific indication we do what is normally done, i.e., count the segments forward, as in the case of all other *grahas* like the Sun etc. Working for VM's time, i.e., 427

the position for observers in North India according to their latitude.

15. The idea of the retrograde motion must have originated very late, the reason being that it can, at that period, serve to reconcile faith in the theory of the motion with observation. (Giving various positions like *Maghā*, *Kṛttikā*, *Rohiṇi*, *Śravaṇa*, to the Sages at the beginning of Kali, giving different rates of motion, and being satisfied with rough positions, are evidently only different means of effecting this reconciliation). The Pandits of Banaras who informed Col. Wilford in c. 1804, believed in the retrograde motion. The *Kaliyugarājavṛttānta*, (Bhāga. III, ch. iii), as quoted by TSN, KV and VT, undoubtedly believes that the motion is retrograde, by stating that 25 years after Kali set in, the Sages left *Maghā* for *Āśleṣā* (See *Age of Sankarā*, Pt. I, *App.*, 139ff; *JAHS* XX. 62ff; *JAHS* XXII. 169-70). But as can be seen from the dynasties it deals with, it is a recent work, and cannot command authority like the *Purāṇas*, for the author of this work is not known and may be only like one of us trying to reconstruct ancient history from Purāṇic evidence.

Śaka, we get the middle of *Uttara Phalgunī* as the position of the Sages, which is well within the limit for agreement with observation. If we count backwards taking the motion as retrograde, we get the middle of *Puṣya*, which is far outside the limit, and this also shows that the rule implies only forward motion, as we have already determined.

Āryabhaṭa II gives the rule in the form of cycles per Kalpa, even as the Siddhāntas do for the planets. He says there are 15,99,998 cycles in the *Kalpa* and the cycles commence 30,24,000 years from the beginning of the *Kalpa*. Here too there is no indication of retrograde motion. Calculating from the above date we find that at the beginning of the present Kali the Sages are at 2·38 segments (counting from *Aśvinī*), i.e., they have passed *Bharaṇī* and been in *Kṛttikā* for 38 years. It is easily seen that at 662 Kali the Sages will cross to *Maghā* according to this Siddhānta. Compare this with VM's rule that would give the crossing to *Maghā* in the 7th century Kali (exactly speaking 653 Kali; for going back 2526 years from zero Śaka, equal to 3179 Kali, we arrive at this date).¹⁶

We now pass on to Parāśara. His rule is the same as that of Āryabhaṭa II, with the difference that in Parāśara's case the cycles begin at the commencement of the *Kalpa* itself. This would give for the commencement of Kali the position 2·34 *nakṣatras*, counting, of course, from *Aśvinī*, i.e., after crossing *Bharaṇī*, the Sages have been in *Kṛttikā* for 34 years, and at 666 Kali the Sages pass on to

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16. For c. 700 A.D. the position of the Sages would be *Citrā*, which, being within the limit of observation, we can conclude that the date of Āryabhaṭa II is c. 700 A.D. Though a date within three centuries earlier is possible, it is not likely; a date as late as can be within the possible period has to be fixed for Āryabhaṭa II on other grounds,

Maghā.¹⁷ See how close this is to 662 and 653, the dates according to Āryabhaṭa II and VM, derived above.¹⁸

Now for Lalla's rule: As said before, the rule is quoted by Muniśvara in his commentary on the *Siddhānta Śiromaṇi*. It is this: Deduct 14 from the Kali years gone and divide the remainder by 100. Asterisms are got, which are to be counted from *Rohiṇī*.¹⁹ Here too it is to be noted that no backward counting is enjoined, and the rule must normally mean forward counting as in all rules where nothing is said. It is to be noted that according to this rule, the Sages pass to *Rohiṇī* from *Kṛttikā* 14 years after Kali set in, *i.e.*, they have been in *Kṛttikā* for 86 years before the beginning of Kali, (agreeing with VM within half a segment). Taking the date of Lalla to be 650 A.D.,²⁰ the Sages would be at *Citrā* in his time,

17. For details see *Mahāsiddhānta*, Ed. Sudhakara Dvivedi, Banāras, 1910, Contents in English, pp. 1-4. See also Cunningham, *ib.*, p. 8.
18. Obviously, the probable date of Parāśara would also be c. 700 A.D. or within some three centuries earlier. This would give sufficient time for Āryabhaṭa II to refer to Parāśara's views in his work.
19. Lalla uses the word *virin̄ci* (a synonym for Brahmā or Prajāpati). All but Muniśvara take this as *Rohiṇī*, and this would give the Sages a position agreeing with observation in Lalla's time, c. 650 A.D. But Muniśvara, in order to make it agree with his own observation, takes it as *Abhijit* (whose deity is Vidhi, another synonym for Brahmā), which is almost the same as *Sravaṇa*. Naturally, *Muniśvara* is unaware that for Lalla's observation it is *Rohiṇī* that would give the agreement. He seeks support for his interpretation from *S'ākalya Samhitā*, Praśna II, ch. ii, the statement, "Kratu was at Viṣṇu's star at the beginning of Kali."
20. It has been shown in the Introduction to the *Mahābhāskariyam* (Ed. T. S. Kuppanna Sastri, Madras, 1957), p. xviii, that Lalla is later than Brahmagapta, having commented upon his *Khaṇḍakhādyaṅka*, and so he could not have been a disciple of Āryabhaṭa I, and so not much earlier than 650 A.D.

According to *Śākalya Samhitā*, “Kratu, one of the Sages, enters *Śravaṇa* at the commencement of Kali and the Sages have *direct motion* every year at the rate of 8' (which rate is equivalent to one asterism per century);” cf.

*yugādaṁ viṣṇutārāyaṇ kratur bhādye vyavasthitāḥ |
pratyabdam “prāggatis” teṣāṁ aṣṭau liptā Munīśvara||*

(Quoted in Kamalākara's Com. on his own *Siddh. Tat. Viv.*, *Bhagrahayutyadhikāra*, under stanza 25)

According to Vāteśvara, Madh. 15,

कमलविष्टरवक्त्रसरोरुहस्फुटगिरिभिहिता मुनिपर्ययाः ।

य इह तानपि वच्मि युगोद्भवान् द्युचरलब्धवरो मुजगोष्ठयः ॥ 1692 ॥

So the rule would be to multiply the Kali years by 8 and divide by 800, to get the asterisms. These are to be counted forward from *Śravaṇa*. It may be noted that here *eastward motion*, i.e., *direct motion*, is *specifically stated*. Kamalākara, too, explaining the motion as really that of the presiding sages, says in the same context that the *motion is eastward*, i.e., *direct*; cf., *sā prāggatir munivarair bhagatā muninām* (ib., 36). According to this rule, after 1100 A.D. the Sages would move to *Maghā*, and we can place this work at the earliest in c. 1100 A.D.

Lastly, for the rule of Muniśvara given in his *Siddhānta Sārvabhauma*. Deduct 600 from the Kali years, Double the remainder and divide by 15. The position of the Sages in degrees in got. This divided by 30 gives the position in the *rāśis*. This rule again clearly takes the motion as direct. According to Muniśvara the Sages cross to *Aśvinī* at 600 years Kali (which is equivalent to the statement of Śākalya, for according to Śākalya's rule too the Sages enter *Aśvinī* at 600 Kali). At the time of Muniśvara, according to his own rule, the Sages would have crossed from *Citrā* to *Svātī* which is just outside the limit, and which position Muniśvara should have accepted as agreeing with observation because the difference was

not much. (It would have satisfied him better if some astronomer had said or if he could obtain, by quibbling, the Sages were at *Śraviṣṭhā* or *Śatabhiṣak* at the commencement of Kali. No such thing was available, and the best he could have was *Śākalya*, and he had to be satisfied with that).

Thus all authorities either state or imply only direct motion, and there is no authority for retrograde motion. That is why scholars like Cunningham (as already mentioned), “Śrīyut Śrīś Chandra Vidyārṇava, Dr. Jayaswal and many others” (in the words of Dr. Triveda, *JIH* XIX, 11) have considered the motion direct. There may be some like “the most famous astrologers of Banares who informed Col. Wilford”, (cf. Triveda, *ib.*, p. 10), and the author of *Kaliyugarājavarṇānta* who believe the motion to be retrograde. But in the light of what we have said, they must be wrong (see Fn. 15).

III (b). The Purāṇas on the motion of the Sages

Also, the Purāṇas do not say whether the motion is direct or retrograde. We cannot get any indications regarding the direction of the motion of the Sages from the Purāṇas themselves, as they are vitiated by emendations and interpolations, made to affect the very point which we are trying to decide. Still, some scholars resort to them for support, and it is not surprising that they fail to establish anything. Dr. Triveda is one such:²¹ he not only does not prove his point, but proves the contrary of what he desires to prove, as also the lack of clearness in his mind. For e.g.: He says: (i) “But in fact their (the Sages’) motion is retrograde as from the word *Precession*, *pre*=pūrva or east, and *cession* from Fr. *cedare*=go” (*ib.*, p. 11). (ii) “Kamalākara Bhaṭṭa also says in his *Siddhānta Tattvaviveka*, ‘*pratyabdam prāggatis teṣām*’; that is, in

21. Cf., his article, “The Intervening age between Parikshit and Nanda”, *JIH* XIX (1940) 1-16.

every year their motion is from West to East.” (*ib.* p. 11). (iii) “C. A. Young in his *A Text Book of General Astronomy* published in 1904 says on page 141, ‘The Equinox moves westward on the ecliptic, as if it advanced to meet the Sun on each annual return’. So it is certain that *their* motion is contrary to that of the Sun, and it is retrograde.” (*ib.*, p. 11)

Let us examine his statements here. (i) If Precession is retrograde, why should the motion of the Sages be retrograde also? Are they the same phenomenon? If he means that the motion is derivable from Precession, he has not shown it, and cannot show it, because it is not so (see Fn. 11 (i) above). Even if some connection be established between the two, in the period we are considering the simulated motion would be directed only opposite to Precession (see Fn. 11 (i)). He is not aware that the derivation he gives for the word ‘Precession’ would mean *direct motion* even for Precession, and *not retrograde motion*, for ‘going east’ means ‘direct motion’. (ii) Dr. Triveda’s quotation from Kamalākara is plainly against himself, for ‘from west to east’ is ‘direct motion’, and not otherwise as Dr. Triveda thinks. (iii) Young rightly says that the Equinox moves westward, *i.e.* it has retrograde motion. But what has that to do with the Sages? Triveda does not perceive that from here it can be understood that it is westward motion that is retrograde, and not eastward motion, as he thinks.

Under the delusion that he has proved the motion of the Sages to be retrograde, Dr. Triveda proceeds to apply this to the following statement in the Purāṇas in order to establish his thesis that the interval between Parīkṣit and Nanda is 1500 years (given by one reading) and not 1015 or 1050 years (given in certain other readings (*cf.* Triveda, *ib.*, pp. 1-3, 12-15). We shall briefly examine this in order to expose the errors in his reasoning, for if he establishes his point by using his theory of retrograde

motion, that might be taken by some as a point in favour of the theory of retrograde motion itself, even after all that has been said by us to establish that the motion is direct.

The Puranic statement is as follows :^{21a}

Mahāpadmābhiṣekāt tu yāvaj janma Parīkṣitah |
(OR *yāvat Parīkṣito janma yāvan Nandābhiṣecanam |*)
evam varṣasahasram tu jñeyam pañcāśaduttaram
(1050) //

The last foot has the variants : *śatam pañca daśottaram* (i.e., 1510) or *pañcadaśottaram* (i.e., 1015) or *pañcaśatottaram* (i.e., 1500) (*Viṣṇu Purāṇa*, IV. xxiv. 104; *Bhāgavata*, XII.ii 26; etc.). Triveda's thesis is to establish the interval to be 1500 or 1510 (according to two readings given) using the Saptarṣi Era given in the following Puranic statement:²²

prayāsyanti yadā caite pūrvāṣāḍhām maharṣayaḥ |
(OR *yadā maghābhyo yāsyanti pūrvāṣāḍhām maharṣayaḥ |*)
tadā Nandāt prabhṛtyeṣa Kalir vṛddhim gamiṣyati |

(*Viṣṇu Purāṇa*, IV. xxiv. 112; *Bhāgavata*, XII.ii.32; etc.)

It is said here that when the Sages pass from *Maghā*, (their position at the beginning of Kali when Parīkṣit was ruling), to *Pūrvāṣāḍhā* at the time of Nanda, the Kali will worsen. From *Maghā* to *Pūrvāṣāḍhā* the Sages pass 10 asterisms in their course, taking the motion to be direct, (as we have established), i.e., about 1000 years from

21a. See the text and the variants recorded by Pargiter, *The Purāṇa Text of the Dynasties of the Kali Age*, O.U.P., 1913, p. 58.

22. See Pargiter, *ib.*, p. 62. In the place of this line mentioning *Pūrvāṣāḍhā*, the *Kaliyugarājavṛttānta* gives : *śravaṇe te bhaviṣyanti kālē nandasya bhūpateḥ*. This is not supported by any Purāṇic source and hence not fit to be taken as authority.

Parikṣit to Nanda (or 3700 years if one cycle has gone), and this is supported by two readings. But Triveda suggests 1500 years for this interval, supported by the other two readings. Counting backwards from *Maghā* to *Pūrvāṣāḍhā* (in accordance with his theory of retrograde motion) he should get 17 asterisms, not counting either *Maghā* or *Pūrvāṣāḍhā* and at least 16, not counting both. Thus he should get at least 1600 years as the interval. But this will not suit his theory, and so he omits to count *Śravaṇa*, and gets the 15 asterisms he wants, to give him the required interval of 1500 years! (see *ib.*, p. 12, lines 10-11). This is proof that the author of the Purāṇas, who employed the Saptarṣi Era for purposes of chronology, has taken the motion only to be direct and used the Era; and not retrograde, for if taken as such, at least 1600 years will be got as the interval, which is not supported by any reading of the text.²³

One thing clearly emerges from this discussion, *viz.* that the motion of the Sages as given by the astronomers

23. On page 13 of his article, Triveda gives a tabular statement of the chronology. There he counts *Śravaṇa*, but to compensate for the extra 100 years that would occur, gives the period 3233 to 3133 B.C. for *Maghā* and 3133 to 3076 B.C. for *Āśleṣā* (this giving only 57 years for *Āśleṣā* instead of 100) against the Purāṇas that give 3176 to 3076 B.C. to *Maghā*, 3076 to 2976 B.C. for the next star and so on. Triveda's scheme is supported by no Purāṇa.

Incidentally we may mention another mistake he employs to achieve his purpose. He wishes to give 1724 A.D. to 1824 A.D. for *Svāti* (see p. 15), so that his table might agree with the statement of Wilford's Pandits of Banaras who have told that the Sages were at *Svāti* in 1804 A.D. So he includes *Abhijit* among the Nakṣatras counted and gives for it the century 1024 to 1124 A.D. (see p. 14), apparently unaware of the fact that this trick would make the cycle last 2800 years instead of the usual 2700 years, and that he himself has not counted it in the previous cycle (see p. 13). If this kind of trick is resorted to one can prove anything!

and the Purāṇas is direct and not retrograde. So VM can be right in saying that the Sages were in *Maghā* in the 7th century Kali and in this he is supported by Āryabhaṭa II, and Parāśara, as well as the Śrutarṣis. Therefore Yudhiṣṭhira's reign associated with the Sages at *Maghā* can well be in the 7th cent. Kali, also supported as it is by a whole school of chronologists. As the Śaka Era mentioned is to come 2526 years after this period, it is the Śaka of 78 A.D. that must have been meant by VM in the śloka, *Bṛ. Sam.* XIII.3, and not the one postulated by TSN etc., concurring with what we have established already in the previous study from an astronomical point of view.

IV. The Aihole Inscription

Now we shall take up the Aihole Inscription and show that the Śaka used in it is only that of 78 A.D. and not the other one alleged by TSN and echoed by certain other scholars. Discussing the age of VM in his *Age of Sankara*, Pt. I-D, pp. 224ff., TSN takes up the Aihole Inscription for consideration,²⁴ and tries to show that the Śaka Era mentioned therein is his own Śaka of 550 B.C. from the synchronism found in it between the Śaka Era and the Bhārata War. The portion of the inscription relevant to our discussion is the following :

*triṁśatsu trisahasreṣu bhāratād āhavāditaḥ/
saptābdaśatayukteṣu śa (?ga)teṣvabdeṣu pañcasu/
pañcāśatsu Kalau kāle ṣaṭsu pañcaśatāsu ca/
samāsu samatītāsu Śakānām api bhābhujām ||*

In trying to interpret this passage, Dr. Fleet at first (*Indian Antiquary*, V (1876) 67-73) made the mistake of thinking that the time of the inscription is given in three eras, viz. Bhārata War, the Kali and the Śaka. Perhaps

24. We have not had access to this section of TSN's work. Our authority is the long quotation in KV's *Plot in Indian Chronology*, ch. X. 185-90.

he was led into this mistake by the word 'śata' occurring thrice (*saptābaśatayukteṣu*, *śateṣvabdeṣu*, and *pancaśatāsu*) and the statement in the Purāṇas that the Kali epoch is different from the Bhārata War. But subsequently, in IA VIII (1879) 240-41, Dr. Fleet acknowledged his mistake and gave the correct reading by emending *śateṣu* into *gateṣu* (for, in the Kanarese-Telugu script in which the inscription is engraved on rock, *ga*, with a horizontal stroke across would become *śa* and the engraver might have been misled into adding the stroke here by the large number of *śa* letters occurring the context ; or it might have been caused by weathering) and interpreting the passage as 3735 years from the Kali epoch, after the Bhārata War, and 556 years after Śaka kings, i.e. 556 years in (*Śālivāhana*) Śaka Era.²⁵ This interpretation is accepted by all scholars (see for instance, Kielhorn, *Ep. Ind.*, VI (1900-01) 1-12), except TSN and KV.²⁶ But the emendation of *śateṣu* into *gateṣu* is accepted by TSN. He also accepts the fact that only two dates are given, of which one is Śaka Era. This necessitates the two expressions 'after the Bhārata War' and 'from the

25. This Era is variously given as *śakānām nṛpāṇām* (or *bhūbhujām*) *kāla*, *śakanṛpati* (or *bhūpa*) *kāla*, *śakendrakāla*, in the earlier centuries and as *Śālivāhana Śaka* later. The origin is also attributed to various causes: e.g. as named after the good rule of the Śaka kings (Govindasvāmin's *Bhāṣya* on *Mahābhāskariya*), the destruction of the Śakas by Vikrama (Bhaṭṭotpala commenting on *Bṛ. Sam.* VIII. 20), and after Śālivāhana who established his rule (later commentators and tradition).

26. After siding with TSN in *The Plot in Indian Chronology*, ch. X, and speaking with some variation in *JAHRS* XXI (1950-52) 52-53, K.V, has changed over to the correct interpretation in his note, 'The Aihole Inscription of Pulikesin', (*JAHRS* XXII (1952-54) 210-12, wrongly numbered 206-08) without a word of regret for having talked lightly and questioned the bonafides of the very persons to whose opinion he has now been converted. On p. 212, he still seems to be unaware of the fact that Dr. Fleet has corrected himself long ago.

Kali epoch' to be taken together, as giving one date. If the Kali epoch is meant as important and the Bhārata War is mentioned here simply to describe it, without any more trouble we get the interpretation, '3735 years from the Kali epoch', which beautifully synchronises with the Śalivāhana Śaka year 556 given, (about this number there is no dispute), for if we deduct from 3735 the wellknown converter 3179 we get 556, which itself proves that this must be the Śalivāhana Śaka of 78 A.D. If, on the other hand, the Bhārata War is taken as important, and also that the War was fought 36 years earlier (TSN makes it 38 to suit his calculations) according to one sub-school²⁷ taken advantage of by TSN, then there is trouble, for the War took place in 3140 B.C. according to TSN. 3735 years from this date there is no Śaka epoch to synchronise with. But TSN sorely wants it to synchronise with the Śaka of 550 B.C. postulated by him. He clutches at an error committed in a collection of old records published for literary study, the *Prācīnalekhamālā*, (N. S. Press, Bombay, *Kāvya-mālā Series* 16), thinking that it will help him. In the *Prācīnalekhamālā*, *saptābdaśata* is printed as *sahābdaśata*. Whether this is a misprint or an intended emendation, we do not know. But this much we can say, that the letter is certainly *pta* and not *ha*, as anyone can verify from the photo-print of the inscription reproduced in *IA V* (1876) op. p. 69, *ib. VIII* (1879) op. p. 241, *Ep. Ind. VI* (1900-01) op. p. 7, etc.) and comparing the letters. Not only this; the word *saha* will be a repetition, because there is the word *yukta* giving the same meaning; also *saha* requires an instrumental to govern, which is not available in the verse. In spite of all this, TSN takes this *saha* instead of *sapta* and gets the number 3135, of course, as we have pointed out, with a duplicate *saha*

27. See fn. 7 *supra*. Note that there is still another sub-school that takes the war synchronous with the Kali epoch. Obviously according to this school also, the interpretation is what we have already mentioned as the correct one.

serving no purpose in the interpretation) and begins to effect the synchronisation thus (see p. 189, *Plot in Indian Chronology*): The Aihole Inscription is 3135 years from the War, viz. 3140 B.C. So the date of the inscription is 5 B.C. And then the inscription is 556 years from the Śaka epoch (of TSN), viz. 550 B.C. 556 years from 550 B.C. is 6 B.C. (so says TSN, for he wants it, and wish is father to thought). 6 B.C. is only one year off 5 B.C. (obtained above), which can be easily accounted for, and the synchronism established; which shows that the Śaka mentioned in the inscription is his Śaka of 550 B.C. But TSN and KV who quotes him seem to be unaware of the blunder in the calculation, and that 556 years from 550 B.C., is not 6 B.C., but 7 A.D.; and this date is 11 years off 5 B.C., and no amount of jugglery can spirit this period of 11 years off and the synchronism is far from being established. What is more, having failed to prove the 550 B.C. Śaka, but thinking that it has been proved, TSN indulges in a tirade against Orientalists and their ways (see p. 190, *ibid.*), unconscious all the while, that it all applies to TSN himself!: “Alas! it is a great pity that these Orientalists should at first conceive a theory of their own, and then actively set themselves to work out the same by hook or by crook, by changing every authority to suit their own favourite hypotheses, and by hoisting up the fabricated text as the only true version, while they perfectly know all the while in their own heart of hearts that they have been able to achieve their objects only by fabricating evidence and meddling with the original authorities.....The Orientalists simply beg the question, and beat about the bush in discussing such matters (*here*, explanation of the word Śaka), blowing hot and cold at the same time, misjudging themselves, and misleading others, and thereby keeping back the Truth as far away as possible from the ken of ordinary public.” How aptly these words apply to TSN himself!

V. The Evidence of the Jyotirvidābharaṇa

Even though we have stated that the evidence of the *Jyotirvidābharaṇa* does not merit any consideration (see previous paper), still because it is made much of by TSN, KV and VT (see for e.g., KV: *JAHRS* XXI (1950-52) 28-32, *Chronology of Nepal History*, Vijayawada, 1953, pp. 14-19; VT: *JIH* XXVIII (1950) 107-08), we shall consider that too. Their contention is that the author of the *Jyotirvidābharaṇa* is the famous Kālidāsa himself as claimed by the work, that he with VM and several other great scholars lived at Vikramāditya's court,²⁸ that he wrote the work in Kali 3068 (34 B.C.),²⁹ and that therefore VM cannot belong to 427 in the Śaka of 78 A.D. (corresponding to 505 A.D.), but only in the postulated Cyrus (or Andhra) Śaka of 550 B.C. (corresponding to 123 B.C.), and that thus the Cyrus (or Andhra) Śaka is proved. But the work could not have been written before 78 A.D., (though it says it was written in 34 B.C.), for in that work Śālivāhana is mentioned as a *śaka-kāra* (founder of an era), and that he founded the Śaka Era 135 years after Vikrama founded his own Śaka 3044 years after Kali (*i.e.* 57 B.C.).³⁰ How could Kālidāsa, the alleged author of the work, be the contemporary (however junior it might be) of VM (said to have lived in 123 B.C.) and at the same time know the starting of the Śālivāhana Śaka in 78 A.D.?

28. Cf. the verse *Dhanvantari* etc. *Jyotir*, XXII. 10.

29. Cf. *varṣe sindhuradarśanāmbaraguṇair* (3068) *yāte Kales sammite/ māse Mādhavasaṃjñike ca vihito granthakriyopakramaḥ* || XXII. 21.

30. Cf. *Jyotir*. X. 110-11, giving the several Eras of the Kali Age: (i) *Yudhiṣṭhira Era* for the first 3044 years, (ii) *Vikrama Era* for the next 135 years, (iii) *Śaka Era* for the next 18000 years, (iv) *Vijayābhinandana Era* for the next 10,000 years, (v) *Nāgārjuna Era* for the next 400,000 years, and *Bali Era* for the following 821 years.

The late date of the *Jyotirvidābharaṇa* can be established also by other internal evidence in that work. Thus in giving the rule for the calculation of *ayanāṁśa*, it is stated that 445 is to be deducted from the years in the Śaka Era and the remainder divided by 60. Cf.

Śākaḥ śarāmbhodhiyugo (445) nito hṛto
mānam khatarkair (60) ayanāṁśakās syuḥ/ (1.18a)

This means that in 445 Śaka the *ayanāṁśa* is zero. This can be only the Śālivāhana Śaka, for Indian astronomical works give zero *ayanāṁśa* for c. 421 Śāli. Śaka (Kali 3600), (some give 444). It cannot be argued that the author means the postulated Cyrus Era here, because firstly among the six *śakas* given by him he does not mention this *śaka* at all, and secondly nobody gives zero *ayanāṁśa* for this time (445 Cyrus Era would be 105 B.C.) not even VT, who, as we have seen, implies $-3^{\circ} 20'$ *ayanāṁśa* for 123 B.C. (though he takes it as the starting point for calculation) (see *JIH* XXVIII. 106) and our discussion on it in the previous paper). Thus, having seen that it is the Śālivāhana Śaka that the author uses, we can say that he is later than 445 of this Śaka, (523 A.D.), for this rule can be applied only later than 445 Śaka, no instruction being given as to what to do if the time taken is before 445 Śaka.

Again, the rule given in the *Jyotirvidābharaṇa* for finding the year in the 60-year cycle of Jupiter corrobor-

31. Cf. the rule: *nagair nakhais sannihato dvidhā śākaḥ*
sakhatrīśakro 'kṣayamāṅgabhājitaḥ/
gatāptatallabhaśako 'bhraṣaḍhṛto-
'vaśeṣake syuḥ prabhavādivatsarāḥ// (I. 15)

It means: 'Do the operation, $\{(7x + 20x/60 + 1430) \div 625 + x\} \div 60$, where x is the Śaka year gone. Get the remainder. Count years from *Prabhava* equal to the remainder and the *prabhavādi*-year is got.'

ates this.³¹ If it is applied to the current year, 1881 Śaka (1959-60), we get the year *Virodhikṛt*, which we also get if we work it out according to the methods in the *Siddhāntas*. If the year reckoned in the Cyrus Era or if the Vikrama Era is used in the rule, there is disagreement. So it is the Śālivāhana Śaka that is required to be used in this rule and not the Cyrus Śaka nor the Vikrama Śaka which reigned in his time (for he says he is a contemporary of Vikrama). Thus, again, the conclusion that the author is later than the starting of the Śālivāhana Śaka follows, and his use of that śaka.

In answer to these objections KV seems to have argued that Kālidāsa could actually have lived earlier than the Śālivāhana Śaka epoch and have mentioned that epoch as a future historical event on the basis of the śāstras³² (evidently meaning the *Bhaviṣya Purāṇa* etc.). But then how did the śāstras know? Does KV want us to believe that they actually predicted future events? Clearly the śāstras themselves should have been written after the Śālivāhana Śaka epoch, and the *Jyotirvidābharaṇa* should be later still. And the jumbling of people of various ages already alluded to! We are asked to take this bundle of lies as sober history!

In the same manner other romances, like the *Kathāsaritsāgara*, *Bhojaprabandha*, *Vikramārka-carita* etc., (there is no dearth of them) based on popular stories should be dismissed wherever they contradict what may be judged as solid evidence, for we do not know who their authors were, nor what equipment they had for giving historical facts.

32. This kind of work we have already done, when dealing with Prof. Gulshan Rai. (See previous paper).

33. *Vide* his letter to Kottah Bhavaiah Chowdary referred to in *JAHS* XXII, 55.

VI. Conclusion

Thus in all places where the word *Śaka* is used for the name of an era, it is the *Śaka* of 78 A.D. (what latterly came to be called *Śālivāhana Śaka*) that is meant. Further, there is no evidence to show that an era was started in 550 or 551 B.C. in Persia or in India as postulated by TSN and accepted by KV, which he calls the Cyrus Era, or as postulated by VT, which he calls the Andhra Era. It may be that Cyrus founded the Persian Empire in 550 B.C., but what evidence is there to show that he started an era then? No such era was in use in Persia itself, not to speak of India. Many great events happen in the reigns of great kings. But they are not necessarily the starting points of eras. (VT does not even mention a great event in 550-51 B.C. for the starting of his Andhra Era). Now, these people have taken all this trouble in order to prove the antiquity of the Indian dynasties and, in so doing, to reconcile texts of varied historical worth. Let them by all means attempt it, for it is only too true that unconscious prejudice has had some hand in the writing of the history of our land. But what we wish to show here is that their stand on the interpretation of the term *Śaka* Era, with all its ramifications, is wrong, and will not help them, as also the various other ideas of theirs which we have shown to be wrong. Also we wish to point out that attributing base motives and questioning the *bona fides* of people (the writings of TSN and KV are replete with these) will not only not help, but may also be "paid back with interest", as Dr. P. V. Kane says.

DETERMINATION OF THE DATE OF THE *MAHĀBHĀRATA*: THE POSSIBILITY THEREOF*

Hindus generally believe that the story of the *Mahābhārata* (*MBh.*)¹ is a narrative of events that actually happened, and that they all took place near the end of the *Dvāparayuga* and the beginning of the *Kaliyuga*. Some hold that the War ended with the *yuga* and many, supported by the *Purāṇas*, say that Kṛṣṇa passed away at the end of the *yuga* and so the War took place a few years earlier. About the question of the time when the *Dvāpara* ended, there is difference of opinion. The popular view is that *Dvāpara* ended and *Kali* began at the time fixed for it by the astronomical *siddhāntas*, 3179 years before the Śaka era of 78 A.D., which corresponds to Friday, 18th February, 3102 B.C., sunrise, or the previous midnight according to some schools.² We do not know the exact grounds on which the *siddhāntins* fixed the date as 3179 years before the Śaka era of 78 A.D. Most probably, the first *siddhāntins*, like the author of the 'Old' *Sūryasiddhānta* and Āryabhaṭa, fixed the point of time as a convenient epoch, when the mean planets, according to them, coincided with the zero-point of the zodiac, and the later astronomers accepted it, and adjusted their own planetary cycles to agree with the epoch exactly, or nearly there, finding the difference to be small.

* Reprinted from *Vishveshvaranand Indological Journal*, Vol. XIV (1976) pp. 48-56.

1. The references to *MBh.* given below are to its edition issued by Gopal Narayan and Co., Bombay, Śaka 1823.
2. Āryabhaṭa does not give the exact time but merely states that 3600 years of *Kali* had ended when he was twenty-three years of age. But the followers of his school concur with the general astronomical tradition noticed above,

But there are other dates fixed for the end of Dvāpara by people like Varāhamihira, on the authority of the astronomical *Samhitās* and tradition current in their times. Varāhamihira fixes the date as c. 2449 B.C., which can be known from his statement in his *Byhatsamhitā* that the Saptarṣis stood at Maghā when Yudhiṣṭhira was ruling and that the year in his era can be got by adding 2526 to the years of the Śaka era. The authority for this is Vṛddha-Garga's statement :

कलिद्वारयोः सन्धौ स्थितास्ते पितृदैवतम् ।

Kalhana, in his *Rājataranginī*, giving the chronology of the Kashmir kings in the Saptarṣi or Laukika era current in Kashmir and the Himalayan regions, accepts Varāhamihira's view in toto, saying that people who fixed other dates were misguided :

भारतं द्वापरान्तेऽभूद्वर्तयेति विमोहिताः ।

केचिदेषां मृषा तेषां कालसंख्या प्रचक्रिरे ॥

The Jain tradition, giving 2634 B.C. for the Yudhiṣṭhira era is only a variation of Varāhamihira's view. The *Saptarṣicāra* of Parāśara and the *Āryasiddhānta* of Āryabhaṭa II, giving Maghā for the sages in the seventh century of astronomical Kali, and the *Matsyapurāṇa*, giving Kṛttikā for the beginning of Kali, support Varāhamihira.

But many think that both c. 3100 B.C. and c. 2450 or c. 2600 B.C. for the Bhārata events are periods too early, considering the state of society and the political conditions depicted in the *MBh*. They try to fix the Kaliyuga epoch coupled with Yudhiṣṭhira's rule, by reckoning backwards from the time of the Nanda dynasty, which historians have fixed at c. 400 B.C. onwards. The *Viṣṇupurāṇa* and the *Bhāgavata* state :

महापद्मऽभिषेकात्तु यावज्जन्म परीक्षितः ।

(or यावत्परीक्षितो जन्म यावन्नन्दाभिषेचनम् ।)

एवं वर्षसहस्रं तु क्षेत्रे पञ्चाशदुत्तरम् ॥

Variants: (1) शतं पञ्च दशोत्तरम् (1510), (2) क्षेयं पञ्चदशोत्तरम् (1015), (3) क्षेयं पञ्चशतोत्तरम् (1500).

The above would mean that between Parikṣit's (grandson of the Pāṇḍavas) birth and Nanda's coronation, the interval is 1053 years (*variants*: 1510, 1015, 1500). From this, they fix Parikṣit's time as *c.* 1500 B.C. (or 2000 B.C.) and thence the time of the Bhārata story.

Besides these four main periods, several other periods are fixed based on various hypotheses, some plausible, some grotesque. For *e.g.*, some scholars take the *yuga* measure of 12000 years as human years instead of divine, and fix a date accordingly. One interprets the word *samā* and *varṣa* used in the Purāṇas as half-years and brings down the story to *c.* 1200 B.C. But few scholars make any clear distinction between the period of the 'events' and the period when they were written down in the form of the epic *Mahābhārata*, while the orthodox traditional belief is that Vyāsa, grandfather of the Pāṇḍavas and Kauravas, wrote the work, and his pupil Vaiśampāyana narrated it to King Janamejaya, grandson of the Pāṇḍavas.

Determining the period thus, each in his own way, these scholars try to fix the year and exact date of the war from the calendrical details and various astronomical phenomena mentioned in the context of the War, like certain planetary combinations, occurrences of eclipses etc. This is not an easy matter, because there is a lot of contradiction between various sets of planetary combinations themselves and among the other phenomena mentioned. Some of these passages may be set out here :

1. विशाखाया समीपस्थौ बृहस्पतिशनैश्चरौ ॥

(*MBh.*, *Bhīṣma*, 3. 27)

2. वक्राऽनुवक्त्रं कृत्वा च श्रवणं पावकप्रभः ।

ब्राह्मरशिं समावृत्य लोहिताङ्गो व्यवस्थितः ॥ (*Bhīṣma*, 3. 18)

3. भृगुसुनुधरापुत्रौ शशिजेन समन्वितौ ।
चरमं पाण्डुपुत्राणां पुरस्तात्सर्वं (कुरु) भूभुजाम् ॥ (*S'alya*, 11.17)
4. शुक्रः प्रोष्ठपदे पूर्वे समारुह्य विरोचते ।
उत्तरे तु परिक्रम्य सहितः समुदीक्षते ॥ (*Bhīṣma*, 3. 15)

Another gives :

5. बृहस्पतिस्संपरिवार्य रोहिणीं बभूव चन्द्रार्कसमो विशांपते ॥
(*Karṇa*, 100. 17)
6. मघास्वङ्गारको वक्रः श्रवणे च बृहस्पतिः ।
भगं नक्षत्रमाक्रम्य सूर्यपुत्रेण पीडयते ॥ (*Bhīṣma*, 3. 14)
7. प्राजापत्यं हि नक्षत्रं ग्रहस्तीक्ष्णो महाद्युतिः ।
शनैश्चरः पीडयति पीडयन् प्राणिनोऽधिकम् ॥
कृत्वा चाङ्गारको वक्रं ज्येष्ठायां मधुसूदन ।
अनूराधां प्रार्थयते मैत्रं संगमयन्निव ॥ (*Udyoga*, 143. 8-9)

In the first set cited above (*i.e.*, 1-4), we are told that Jupiter and Saturn are near the asterism Viśākhā. Mars is near Uttarāṣāḍha, Abhijit (*Brahmarāṣi*) and Śravaṇa. In the second set, (5-7), Jupiter is said to be near Rohiṇī. Mars is retrograde in Maghā. Jupiter is in Śravaṇa. (This contradicts two other statements.) Saturn is said to be in Pūrvaphalgunī. Saturn afflicts (?) Rohiṇī. Mars is retrograde in Jyēṣṭha and is about to go to Anūrādhā. To add to the confusion, many people interpret the comets of different colours mentioned in *Bhīṣmaparva*, chapter 3, as planets and, that too, each one differently.

Among the contradictory phenomena we can give the eclipses mentioned :

8. चन्द्रसूर्याबुभौ ग्रस्तौ एकमासीं त्रयोदशीम् । (*Bhīṣma*, 3. 33)

Here a lunar eclipse, and next a solar eclipse are mentioned as having occurred before the war. Then, at the time of Duryodhana's death the statement occurs :

राहुरग्रसदादित्यं अपर्वणि विशांपते । (*S'alya*, 27. 10)

mentioning another solar eclipse so near, when a lunar eclipse had occurred before the first solar eclipse.

Again, several impossible and some very rare phenomena, mentioned merely to indicate that these phenomena presage evil, are taken by many as having actually occurred, adding to the difficulty:

9. त्रयोदश्याममावास्यां तां दृष्ट्वा प्राऽब्रवीदिदम् ।
 चतुर्दशी पञ्चदशी कृतेयं राहुणा पुनः ॥ 8 ॥
 प्राप्ते वै भारते युद्धे प्राप्ता चाद्यक्षयायनः ॥ 9 ॥
 चन्द्रसूर्याबुधौ ग्रस्तौ एकाह्वा हि त्रयोदशीम् ।
 अपर्वणि ग्रहेणैतो प्रजासंक्षयमिच्छतः ॥ 28 ॥
 इमां तु नाऽभिजानेऽहं अमावास्यां त्रयोदशीम् ॥ 32 ॥
 (Mausala, Ch. 2)
 चन्द्रसूर्याबुधौ ग्रस्तौ एकाह्वा हि (एकमासीं) त्रयोदशीम् ॥
 (Bhīṣma, 3. 3)
 सोमस्य लक्ष्म व्यावृत्तं राहुरर्कमुपैति च ॥ (Udyoga, 143. 11)
 हते कर्णे..... सोमस्य पुत्रोऽभ्युदियाय तिर्यक् ॥ (Karna, 94. 51)
 अलक्ष्यः प्रभया हीनः पौर्णमासीं च कार्तिकीम् ।
 चन्द्रोऽभूद्ग्निवर्णश्च सम(पद्म)वर्णं नभस्स्थले ॥ (Bhīṣma, 2. 2)
 चतुर्दशीं पञ्चदशीं भूतपूर्वां तु षोडशीम् ।
 इमां तु नाभिजानेऽहं अमावास्यां त्रयोदशीम् ॥ (Bhīṣma, 3. 32)
 राहुरग्रसदादित्यं अपर्वणि विशांपते ॥ (Salya, 27. 10)
 त्रिषु सर्वेषु नक्षत्रनक्षत्रेषु विशांपते ।
 युधः संपतते शीर्षं जनयन् भयमुत्तमम् ॥ (Bhīṣma, 3.31)

Scholars trying to establish their conclusions interpret these verses differently, some neglecting one set and some another, some giving acceptable meanings and some far-fetched and extremely strained ones. A few examples will show to what extent these people go.³

3. See K. L. Daftari, *The astronomical method and its application to the chronology of ancient India*, Nagpur University, 1941. See Lecture II. 'The date of the Mahābhārata war', pp. 13-129,

Passage 2, cited above, is interpreted thus: The planet Mars moved retrograde again and again, towards the constellation Śravaṇa, and occupied the constellation of Brahmā, i.e., Jupiter.⁴ The interpreter is unaware that Brahmarāśi must mean 'the group presided over by Brahmā', viz., Abhijit. He is unaware that *anuvakra* is a technical term used in astronomy and not 'again and again'. Passage 3 is interpreted thus: 'The planets Mars, Venus and Mercury were in front or to the east of the eldest of the sons of Pāṇḍu who were the masters of the whole land.'⁵ To the interpreter, *caramam Pāṇḍuputrāṇām* means Yudhiṣṭhira, being the last counted from the last of the sons of Pāṇḍu, while it means, simply, 'behind the sons of Pāṇḍu and in front of the Kuru kings'. Line 9 of passage 9 is interpreted: The planet Mercury arose concealed, (invisibly).⁶ The meaning 'invisibly' is given to *tiras*, not realising that *añc* with *tiras* means, only 'across or obliquely'. Passage 5 is interpreted: 'Jupiter, having made Rohiṇī to conceal herself (i.e., set), became like the sun or moon'.⁷ The passage means only that, Jupiter by his lustre hid Rohiṇī. Passage 6 is interpreted: Mars is retrograde in Maghā, and Jupiter in Śravaṇa. Saturn is afflicting Pūrvaphalgunī.⁸ In the next verse (not quoted here) there is the word *sahita* which this interpreter takes to mean 'waiting', and cites as an example the *Raghuvamśa* verse, *dvitrāṇy ahāny arhasi soḍhum arhan*. In 9, line 8 is said to mean: 'The lunar eclipse has already happened (in Kārttikā Pūrṇimā) and a solar eclipse is going to happen in the next Amāvasyā'.⁹ Actually, the first part means that the dark patch on the

4. *Op. cit.*, p. 27-28, art. 68.

5. *Op. cit.*, p. 28, art. 70.

6. *Op. cit.*, p. 29, art. 72, 74.

7. *Op. cit.*, p. 30, art. 76.

8. *Op. cit.*, p. 31, art. 78.

9. *Op. cit.*, p. 32, art. 81.

moon¹⁰ is inverted (*vyāvṛttam*, not *nivṛttam*). In line 16 of passage 9, *gṛdhra* is interpreted as “an evil planet”, instead of ‘eagle’ which itself indicates an evil omen.

Thus, different years are fixed by different persons as follows :

N. Jagannatha Rao	3139 B.C.
T.S. Narayana Sastri	c. 3126 B.C.
K.V. Abhayankar	c. 3101 B.C.
C.V. Vaidya	Do.
P.C. Sengupta	2449 B.C.
Karandikar	1931 B.C.
P.V. Kane	c. 1900 B.C.
S.B. Dikshit	c. 1500 B.C.
K.G. Sankara Aiyar	1198 B.C.
K.L. Daftari	1191 B.C.
V. Gopala Aiyar	1194 B.C.

Within the year, the dates are fixed for the different occurrences by the day's *nakṣatra* or *tithi*, and the interval in days between one occurrence and another, given. Here, too, there are discrepancies and misinterpretations, leading to different dates. Though many have concluded that the war began on Kārttikā New Moon day, some say that it began on Mārgaśīrṣa Śukla Ekādaśī day. The day of Bhīṣma's death is stated at places as Māgha Śukla Aṣṭamī, while at others as Ekādaśī. Cf. :

कौमुदे मासि रेवत्यां शरदन्ते हिमागमे ।
स्फीतसस्यसुखे काले... .. ॥ (*Udyoga*, 80. 7)
सर्वौषधिवनस्फीतः फलवानल्पमक्षिकः ।
निष्पङ्को... .. ॥

10. For the meaning of *lakṣma* as ‘dark patch on the moon’ cf. Kālidāsa, *malinam api himāmṣoḥ ‘lakṣma’ lakṣmim tanot* (*Abhijñāna Śākuntala*, I. 20).

सप्तमाञ्चाऽपि दिवसात् अमावास्या भविष्यति ।
संग्रामो युज्यतां तस्यां तामाहुः शक्रदेवताम् ॥

(Udyoga, 143. 18)

चत्वारिंशदहान्यद्य द्वे मे निस्सृतस्य वै ।

पुष्येण सम्प्रयातोऽस्मि श्रवणे पुनरागतः ॥ (S'alya, 5. 6)

न कुर्वन्ति वचो मह्यं कुरत्रः कालचोदिताः ।

निर्गच्छध्वं पाण्डवेयाः पुष्येण सहिता मया ॥ (S'alya, 35. 10)

शेष्येऽहमस्यां शय्यायां यावदावर्तनं रवेः ।.....

दिशं वैश्रवणाक्रान्तां यदा गन्ता दिवाकरः ।

विमोक्ष्येऽहं तदा प्राणान् सुहृदः सुप्रियानिव ॥ (Bhishma, 20.51-53)

तत्र ते सुमहात्मानो न्यवसन् कुरुनन्दनाः ।

शौचं निर्वर्तयिष्यन्तो मासमेकं बहिः पुरात् ॥ S'anti, 1. 2)

आवृत्ते भगवत्यके स हि लोकान् गमिष्यति ॥ (Ib., 46. 29)

निवृत्तमात्रे त्वयने उत्तरे वै दिवाकरे ।

समावेश्यदात्मानः... .. ॥ (Ib., 47. 3)

पञ्चाशतं षट् च कुरुप्रवीर शेषं दिनानां तव जीवितस्य ॥

(Ib., 51. 10)

उषित्वा शर्वरीः श्रीमान् पञ्चाशन्नगरोत्तमे ॥

(Anusāsana, 167. 5)

अष्टपञ्चाशतं राज्यः शयानस्याद्य मे गताः ।

शरेषु निशिताऽग्रेषु यथा वर्षशतं तथा ॥

माघोऽयं समनुप्राप्तः मासस्सौम्यो युधिष्ठिर ।

त्रिभागशेषः पक्षोऽयं शुक्लो भवितुमर्हति ॥

(Anusāsana, 107. 5)

But most of the scholars do not seem to have gone to the heart of the matter, placing before themselves clearly the two things that have got to be investigated, viz. : (1) How much of the Bhārata story is true history, and when could it have happened. (2) When was it actually written down. Scholars who have studied the problem critically are of opinion that there is a historical core in the story, but much fictitious matter has been added to it in course of time. The Bhārata war must be true history, and the personages taking part in it, together with the line of the Bhāratas and Yadus, whose

names occur frequently in Vedic literature, even as early as the *Rgveda*, not to speak of the Brāhmaṇas like the *Śatapatha*. The state of society and the political conditions point to a time earlier than the *Chāndogya*, one of the earliest of the upaniṣads, as can be seen from two statements in the work :

“कृष्णाय देवकीपुत्रायोक्त्वोवाच—यद्यप्येनच्छुष्काय स्थानवे ब्रूयात् जायेरन्नस्य शाखाः, प्रोद्देयुः पलाशनी इति ॥” “श्वेतकेतुर्ह आरुणेयः कुरुपाञ्चालानां समितिमिषाय ।...—”

The latter of the above statements shows that, at the time of the Upaniṣad, the Kuru and the Pāñcāla country had coalesced, while at the time of the Bhārata war they were different, the Pāñcālas being the allies of the Pāṇḍavas. It is quite natural for stories to gather accretions when they are repeated generation after generation. Most of the superhuman and obviously exaggerated portions must have been added later. The core is generally placed between the eleventh and the ninth centuries B.C. Other story matter could have been added during a few subsequent centuries, when Kṛṣṇa came to be deified. The lot of Dharmaśāstra matter with the illustrative stories must have been added last, in the course of several generations. Anyhow, by the first or second century B.C. or A.D., the *Mahābhārata* must have arrived at its present form, with a few bits of interpolations here and there, made later.

As for its writing, the language is that of the early classical period, for it is clearly later than that of the genuine upaniṣads. The addition of the later matter and the development of the classical language must have, naturally enough, gone on together. By the first or second century A.D. most of the whole *Mahābhārata* must have attained the present form.

It is natural for story writers to incorporate into their stories ideas current in their own time. A lot of

the astronomical facts found in the work, especially in the context of the war, must have been cooked up by these later writers in the light of their own knowledge, and added by different people at different times. That explains the contradictions. It must be clearly noted that the astrological ideas mentioned in the *Samhitās* which developed from the 2nd century B.C. could not have been current as early as the 11th to 9th century B.C. and, even if current, are not likely to be remembered after so many generations. By the first century A.D. or B.C., the astronomical *Samhitās* had mostly been written, and naturally the ideas in them find a place in the work. The calendric system of the *Vedāṅga Jyotiṣa* continued to be current in this *Samhitā* period, as can be seen from the *Garga Samhitā*, and ideas showing Śraviṣṭhā as being the first star (beginning the winter solstice) are in evidence, together with its shifting to Śravaṇa, (c. third century B.C.) as can be gathered from the Viśvāmitra episode. The *MBh.* in its *Virāṭaparva* ch. 52 contains the *Vedāṅga* calendric system.

Again, in the context of the war, it is natural for writers, especially of epics, to describe portents as happening to presage evil. The *Samhitās* devote chapters to describe these portents. The *Ketucāra*, on the appearance of comets, is full of portents, as also separate chapters devoted to portents like rare or unnatural, impossible or terrible phenomena. These have been included in the work.¹¹ But most investigators have not interpreted these portions properly, for which a detailed study of the chapters on *Ketucāra* and *Utpātas* in the *Bṛhatsamhitā* of Varāhamihira would be advantageous. For example, the mention of the new moon together with solar eclipse occurring on Trayodaśī, the sun and the moon being eclipsed on the same day (the same month),

11. See, e.g., *Udyoga*, 143; *Bhīṣma*, 2, 3; *Karṇa*, 94, 100; *Sālya*, 11, 27; *Mausala*, 2.

and that on Trayodaśī, Mercury moving across the sky, (*i.e.*, north-south), the dark patch on the moon being inverted, the lunar eclipse at Kārttikā full moon, the solar eclipse at Kārttikā new moon, and again the solar eclipse at the time of the mace-fight, are all intended by the writer to be impossible things occurring. The mention of the red moon indistinguishable from the red sky (*digdāha*), eagles falling on the flag, appearances of comets of different colours and in groups are all portents. Ignorance of the fact that the 'grahas' of different colours mentioned in Bhīṣmaparva, chapter 3, are not planets but comets, has added to the confusion, because these scholars do not realise that, in the Saṃhitās, the word 'graha' means primarily comets, (vide the chapter on *Ketucāra* in the *Bṛhatsaṃhitā*).

It would be clear from the above, that all the skill shown in distorting the meanings of words and trying to show when these impossible or rare phenomena and contradictory planetary combinations would actually occur, has been wasted. Excepting the time of the year when the war might have happened, there is nothing in the *Mahābhārata* to fix the year definitely. We do not have adequate data to fix either the happenings or when the work, even part by part, was written.



A BRIEF HISTORY OF TAMIL ASTRONOMY*

I. Prefatory

By Tamil astronomy, I mean astronomy written by a Tamilian, whether the language is Tamil or Sanskrit. But, as far as I know, few outstanding works on astronomy have been written in Tamil Nadu, unlike in Kerala, where there has been a succession of scholars writing original astronomical works or commentaries, making their own extensive contributions. Two Tamil works on Mundane and Electional Astrology, *Cūḍāmaṇi Uḷlamuḍaiyān* (13th century), and *Vīmeśvara Uḷlamuḍaiyān* (17th century), contain small portions of astronomical computation of the planets, taken chiefly from the Parahita system of Haridatta of Kerala, as it is, or with the *Śakābda* or *Parahita Bīja* corrections carried out. Excluding two Sanskrit commentaries, one of which is by Sūryadeva-yajvan, only one work, called the *Vākya-karaṇa* (c. 1300), is wholly devoted to astronomy, and somewhat co-extensive with its content. Its source is mentioned to be the *Bhāskarīya* as studied in Kerala, with the *Parahita* correction, and with Haridatta's *Parahita* method used. There is no original material in it. But being a *karaṇa* or manual, it has invented devices for ease of computation, especially by almanac-makers. It is in these devices that it is original, in so much as perpetually repeating planetary tables are given, with mnemonic phrases (*vākyas*), being used for numbers. It is in Sanskrit. All later almanac-makers and computers of the Tamil country use this or Tamil adaptations of this.

About 1800 AD, first Le Gentil, and then Warren, in order to learn how the Tamils computed, asked some

* Reprint from Madras University Journal, Sec C 41(2) 120-133,

natives of Pondicherry to demonstrate to them. The methods and constants were almost exactly those of the *Vākyakaraṇa*. Le Gentil and Warren¹ reported the demonstration in their works, not knowing the source. Two periods, 248 days and 3,031 days, occur in the moon's tables. Any table-maker must get these as full days closely approximating to 9 and 110 anomalistic revolutions of the moon, on which the values must depend. But not considering this point, and since these periods occur in the *Vāsiṣṭha Siddhānta* of the *Pañca-siddhāntikā*, exhibiting Babylonian connections, Assyriologists and Indologists like Neugebauer began to declare a Babylonian derivation for the method, a mere surmise. Neugebauer published a paper examining the methods of the Pondicherry informant of Warren. The paper exhibits ignorance of the fact that the source for the informant was *Vākyakaraṇa*, since it was unknown to scholars at that time, not having been printed yet, and that this work itself is based on the works of Bhāskara I and Haridatta. Seeing Neugebauer's paper recently, I wrote to him explaining the position and incidentally clarifying certain difficulties expressed by him. But I thought I must write a history of Tamil astronomy, however meagre or insignificant it may be to lay the Babylonian ghost to rest. This short paper is the result. I consider this only as a beginning, so that others may follow up and make their own contributions.

II. Introduction

By the word Jyotiṣa, Hindus mean both astronomy and astrology combined. It is in three divisions, (1) *Siddhānta-skandha*: This consists of two parts, *Gaṇita*, giving the positions of heavenly bodies, and *Gola*, dealing with general astronomy like cosmogony. (2) *Samhitā-skandha*: This too consists of two parts. In

1. John Warren, *Kāla Sankalita*, Madras, 1825; Le Gentil, *Memoirs sur l'Indie*, Paris, 1776,

one, called Mundane Astrology, predictions for the whole world or regions of the world are made, based on planetary positions. Several other things considered useful to man, like knowledge of omens and indications of weather, are also given. In the other part called Electional Astrology auspicious moments for religious rites and ceremonies and journeys are given, together with rules for compatibility in marriage etc. (3) *Horā-skandha* gives life predictions for individuals, based on the planetary positions at conception or birth or the moment when the astrologer is approached and requested to make the predictions. The origin of the first two divisions can be traced to the Vedas and Vedic times. The last is declared by scholars to be foreign in origin, as can be inferred from the large number of Greek and Babylonian words used, and the period it appeared in India, namely the first or second century A.D.

From words like *Nakṣatradarśa* and *Gaṇaka* occurring in the *Yajurveda*, and a *Nakṣatra-vidyā* being mentioned in the *Chāndogya-upaniṣad*, and the fact that a knowledge of the positions of the sun and the moon is required for fixing various Vedic rites and rituals, we can infer that the *Siddhānta* division must have originated very early. The *Vedāṅga-Jyotiṣa*, by one Lagadha, whose content points to the 12th century B.C. though re-written later, is the earliest astronomical work extant. The *Samhitā* and the *Horā Skandhas* require planetary positions as a pre-requisite for prediction. So, by the period of the *Samhitās* which are earlier and flourished in the first few centuries B.C., astronomy proper must have been tolerably well developed. Varāhamihira (first part of the 6th century A.D.) has condensed five *Siddhāntas* current during his time or earlier, in his *Pañcasiddhāntikā*. Of these the *Paitāmaha* is the system of the *Vedāṅga-Jyotiṣa* itself. The *Vāsisṭha* and the *Pauliṣa* show clear connections with the Babylonian astronomy of the Seleucid period, and the *Romaka* with

that of Alexandria. The *Saura* is indigenous, and a model of the Hindu Siddhāntic astronomy. Being necessary to fix the times of the hundreds of Hindu religious rituals and the numerous fasts, feasts and festivals, as also for the use of the horary astrologers, scores of *Siddhāntas* and *Karaṇas* or manuals were written by astronomers like Āryabhata, Varāha and Brahmagupta, together, with commentaries or expansions of these, which are themselves astronomical works. Together with the Sanskrit cultural migration to South India beginning with the last centuries B.C., astronomy too spread to the South.

As far as astronomical works are concerned, it seems that the Kerala country was the seat of its development in the South. It is all based on the *Āryabhaṭīya*, with or without corrections called *bijas*, though several later astronomers like Parameśvara, Nilakaṇṭha etc., made their own original contributions. How Āryabhaṭa came to be connected with the Kerala country is yet to be explained. He is called Āśmaka (i.e., one born in the Āśmaka region) and some say that an early name of the erstwhile princely state of Travancore was Āśmaka (Apte's Dictionary). But many say that the region near the Vindhyas was called the Āśmaka country (i.e., the region of the Āśmaka people), and Āryabhaṭa was a native of this country. Bhāskara I, (c. A.D. 600), who was an exponent of his school, seems to have belonged to the Valabhi region in Gujarat. The Āryabhaṭa school must have migrated to Kerala from this region, through some Keralite who had learnt astronomy at Valabhi or some Valabhi astronomer settling in Kerala. Anyhow, before the end of the 7th century the first well-known astronomer of Kerala, Haridatta, had appeared. He has written two works, *Mahāmārganibandhana*, and *Grahacāranibandhana*. The former must be a full treatise, but manuscripts of this are yet to be found. But the latter is well known as the *Parahitagaṇitam*. It gives an easy method for

computers to use. Tables are given for each planet to find the equation of the centre and the equation of conjunction, at intervals of $3\frac{3}{4}^{\circ}$ of the respective anomalies. Haridatta uses the *Āryabhaṭīya* constants in this without any corrections, but it is he that is credited with the corrections on the *Āryabhaṭīya*, called the *Vāgbhāva* or *Śakābada*, or the *Parahita* corrections. It is said that he promulgated his *Parahita* system of computation together with his corrections in 683 A.D. at Tirunāvāy, to the astronomers who had assembled there for the “Māmāṅkam” festival occurring once in twelve years.²

From Kerala the *Parahita* system and the *Āryabhaṭa* school of astronomy spread to the Tamil part, and the Tamil astrologers have been using the *Parahita* when they desired to compute the sun, moon and planets for predictions.

II. Tamil Astronomy

The first astronomer of the Tamil country, known to us at present, was Sūryadevayajvan. He has written many fine commentaries in astronomy and astrology in Sanskrit, but has not produced any independent work.³ In these he says he was born in Gaṅgāpura in the Cola country, which can be identified with Gangaikondacholapuram (N. Lat $11^{\circ} 13'$, E. Long $79^{\circ} 30'$), about 40 miles north of Tanjore. He gives its equinoctial shadow to be 2.4 aṅgulas, which corresponds to N. Lat $11^{\circ} 17'$, and its distance, east of Kharanagara to be 11 yojanās, which is 1.2° according to the *Āryabhaṭa* measure. Kharanagara is said to be on the Ujjain

2. See *Grahacāranibandhana*, Edited by K. V. Sarma, K. S. Research Institute, Madras, 1954, Intro. p. vii.

3. See *Āryabhaṭīya* of *Āryabhaṭa* with the commentary of Sūryadevayajvan, ed. by K. V. Sarma, Indian National Science Academy, New Delhi, 1961, Introduction, pp. xxv-xxx,

meridian, as given by Bhāskara I in *Mahābhāskarīya*, ch. II. So, the east longitude comes to about 77°, but actually it is 70° 39'. Either the eleven *yojanas* given is a scribal error, or Sūryadevayajvan's calculation of the number of *yojanas* is different. He says he was born in 1113 Śaka (1191 A.D.), and learnt astronomy from his maternal uncle Sūryadeva, whose protege he was. He was a devotee of Krishna, whose grace he invokes throughout. The following are his works: (1) The *Āryabhaṭīya-Bhāṣya*, (2) *Laghumānasa-vyākhyā*, a commentary on Muñjāla's *Laghumānasa*, (3) *Jātakapaddhati-vyākhyā*, a commentary on the astrological work, *Jātakapaddhati*, of Śrīpati, (4) *Govindasvāmibhāṣya vyākhyā*, a supercommentary on the *Govindasvāmi-bhāṣya* on the *Mahābhāskarīya*, (5) A commentary on the *Yogayātrā* of Varāhamihira, (6) *Khaṇḍakhādyaka-vyākhyā* on the *Khaṇḍakhādyaka* of Brahmagupta. The last three works are now known only from reference.

Astronomy proper (next) appears in the Tamil region as a small part of a mainly astrological work dealing with mundane and electional astrology, by name *Cūḍāmaṇi Uḷlamudaiyān*,⁴ the name appearing in verse 7. The name and time of the author is found in the last verse : விளம்பிய சகரணுண்டாயிரத்தொரு நூற்றில்மிக்க வளம்புகழ் பாண்டமங்கை மாமுனியரியவன்சேய்/இளங்கொடி மடவார்காமனிசைத் திருக்கோட்டி நம்பி/யுளம்புகக் கனிதநூலை யுரைத்தனளுலகுக் கெல்லாம்

The name of the author is Tirukkoṭṭinambi son of 'Māmuniari', he lived in Pāṇḍamaṅgalam, near Uraiyur in Tiruccirāppalli. The period mentioned is *after* 1100 Śaka. But the *Khaṇḍa* for the moon giving c. 1107 Śaka,

4. Reprinted in 1872 in Kalvikkadal Press as edited by Kandiyanna-jōtisa-Sārvabhaumar, after an earlier edition by Jodidam Subrāya Mudaliar, together with its old commentary. Re-edited by Gunalinga Deśīkar in 1935, and printed at Vidya Ratnakaram Press, Madras, carelessly omitting many verses.

and the *Parahita Khaṇḍa* of all planets c. 1167 Śaka shows that the work must be later than 1245 A.D. (From this it can be inferred that the author was a junior contemporary of Sūryadevayajvan). It says that the source of all its content is from a Sanskrit text, and its computation of eclipses is according to a Sanskrit work called *Jayantamālā*, of which manuscripts are yet to be discovered.

The meagre astronomical matter of *Cūḍāmaṇi Uḷlamuḍaiyān* consists of: (1) Days from Kali epoch using the Śaka year reckoned in Solar years. The multiplier and divisor to get this gives 365-15 29 29 days which is less by about 2 vināḍis compared with the value given by all Hindu Siddhāntas. Perhaps the multiplier given by —“னலொன்றன் நூ தலிசன் சீர்த்தது” (verse 381) is “.....னலொன்றன்.....”. The sun’s motion in *rāśis* are to be found by using monthly periods, and the motion within the month, by the true motion per day given roughly for each month. (2) The sixty year cycle and the division of each into three parts are given. (3) A method to find the true moon using the two anomolistic cycles of 3031 and 248 days, together with the so-called *Vararuci-vākyas* giving the true anomaly for each day of the 248 days period, is given. No correction is given for the error in the two periods, with the result that the error will accumulate in course of time. (4) Haridatta’s *Parahita* method is given to compute all bodies, i.e., the sun, moon, and star planets. There is a very small difference in the constants, from those used by Haridatta, but the *Parahita* correction used by later computers, is not used. (5) As mentioned already, there is a section devoted to eclipses, taken from an earlier Sanskrit work, *Jayantamālā*. It is seen from the method for Days from Kali, given in this work, that the solar sidereal year, with the dates of the solar month, had already come to be used in the Tamil country for calenderical purposes. It is more advantageous than the luni-solar calendar, used everywhere in earlier times, and still used in all

parts of India except Kerala. Tamil nadu, Bengal and Orissa. Its ease to get the Kali-days is obvious as against the complicated luni-solar method of using the tithi and lunar month. When did it come to be used? Haridatta in his *Parahita-gaṇita* uses only the luni-solar method, as also earlier people like Bhāskara I. With the solar year is associated reckoning the Śaka years and the 60 year cycle years, which are Jovian, in solar years. The cycle years have fallen back by now in the south by 12 years. From this we can reckon that the practice has originated with the Kollam era of 825 AD, which is a landmark in Kerala astronomical history. Perhaps the astronomers used it first, for its obvious advantages, and the laymen and civil administration fell in line. The origin of its use in inscriptions will be revealing.

The next work coming to view is the *Vākyakaraṇam* or *Vākyapañcādhyāyī*, by Sundararāja.*

This is the one work that can really be called astronomical, though it is only a manual, using rough methods for ease of computation. It is in Sanskrit. It states that it follows the *Bhāskarīya* of Bhāskara I mentioned above, and the *Parahita-gaṇita* of Haridatta for its methods and constants, but we can see that the *Parahita* corrections have also been used.⁶ From the subtractive days given for commencing computation it can be inferred that its date is near the end of the 13th century A.D. The author Sundararāja must have hailed from the region of Kānchīpuram in Tamil Nadu, as can be inferred from his invoking the grace of Varadarāja of Kānchi in the first verse, and using words and phrases reminiscent of Kānchi and the Cholas, as mnemonics of tabular values.

5. This has been critically edited for the first time by T. S. Kuppanna Sastry and K. V. Sarma with a Sanskrit commentary by Sundararāja, K. S. Research Institute, Madras, 1962.

6. See Introduction to *Vākyakaraṇam* above,

This is the basis for the so-called *Vākya almanacs* extensively used in Tamilnadu. It can be seen that the informants of Warren and Le Gentil, c. 1800, at Pondicherry, were using the methods and constants of this work for computational demonstration.

Since the true longitudes of bodies repeat in periods, short or long, as the case may be, the actual values of convenient segments of the periods are computed and given by groups of words or *vākyas* being used for numbers. Since the sun's apogee has no motion according to Āryabhaṭa's system, the sun's true positions repeat in the year, and are easily computed. Since there are 9 anomalistic revolutions of the moon in 248 days, with an error of only 8', true anomalies for each day has been given as sufficiently accurate, and a correction given for each period of 248 days. In a period of 3,031 days of 110 revolutions, the error would be only 2' and in 12,372 days of 449 revolutions, it is taken as zero. Thus the error can be prevented from accumulating. Adding the mean moon at the beginning of the period to the true anomaly and corrections, the true moon is got. These 248 *vākyas* called *Candravākyas* or *Vararuci-vākyas*, are said to have been first formed by an ancient astronomer of Kerala called Vararuci.

In the case of the five planets, the true longitudes repeat when a number of synodic periods are also whole solar years, here too there being no motion of apogees as taken. These are very long periods. But sufficient accuracy can be secured by using a smaller number of synodic revolutions, using corrections for the small error, which can be prevented from accumulating as in the case of the moon. These are the *maṇḍala-vākyas* and *samudra-vākyas* of the planets. The explanations of their formation and use is found in Chapter II, while Chapter I is devoted to the sun, the moon, and Rāhu (the nodes).

The third Chapter is devoted to matters depending on the solution of spherical triangles, like daylight, rising ecliptic point, time and shadow. The fourth chapter deals with eclipses, and the fifth with heliacal rising and setting as also the *Mahāpātas*. Thus the astronomical matter given is practically full. The results are also sufficiently accurate. The much-spoken-of error in the *Vākya-almanacs* now-a-days is due to certain conventions followed to conform to the *Dharmaśāstras* (which is therefore not error at all) and the error in the astronomical constants used accumulating over the course of so many centuries now. [See the printed edition (loc. cit.) for details. Being well suited for almanac making, this or Tamil adaptations of this are used by computers.

The Sanskrit commentary mentioned gives some interesting details about the subsequent history of the *Vākyakaraṇa* in the colophon ending the commentary. A king by name Tipparāja is mentioned here who can be identified with Gopendra Tipparāya of the Sāluva dynasty known from his inscription of 1475 A.D. to have ruled the southern districts of the Vijayanagar empire, as a feudatory.

Tipparāja.⁷ An astronomical work of Tipparāja is mentioned in the above mentioned colophon, bearing the name *Tippa-rājīyam*. He has written three works in astronomy, the *Tantrarātna*, a set of *Candra-vākyas*, and *Uparāgadarpaṇa*. It is the *Tantrarātna* that must have been referred to as the *Tipparājīyam*. Besides these, he has written a commentary called *Kāmadhenu* on Vāmana's *Vākyālaṅkāra-sūtra-ṣṭi*, and the *Tāladīpika*, on the *Tāla* branch of music. All are in Sanskrit.

The *Tantrarātna* is a brief work, covering the whole field of astronomy. It follows the later *Sūryasiddhānta*

7. I am indebted to Dr. K. V. Sarma for the information set out here,

in its Yuga elements and other constants. It is in eight chapters. The first chapter deals with the computation of the chief items of the Hindu almanac, *Vāra*, *Nakṣatra* of the sun and moon, *Tithi*, *Yoga*, *Karaṇa* and *Tyājyam*, following the method of the *Vākyakaraṇa*. In the place of the *Bhūpādi-vākyas* for the sun, it gives the *Yogyādi-vākyas* which are easier to use, and appear here for the first time and later used by *Vīmeśvara-uḷlamudaiyān* etc. Tipparāja's name and titles appear here, like Goparāja-tanaya, kaṭhāri-Sāluva, Saṅgita-rasa-bhāvajña, Ubearādityasamvardhita, Camburāya-sthāpaka, Cāṇūramalla, Tālajña, and Kalyārapurendra. The colophon contains the name Sāluva-Tipparāja. Chapters I to VIII are a regular Tantra work, of course, following the *Sūrya-siddhānta* in its elements, methods and contents. But the work is only mediocre in merit.

The *Candravākyas* are expected to be used as an appendix to the first chapter taking the place of the so called *Vararucivākyas* used in the computation of the moon. But the *vākyas* go up to seconds, instead of stopping with minutes. At the end of the work there is some prose matter, in which an example to get Kali days is given. This is 16,74,709 days expired, Kali 4585 = 1484 A.D., the time of Tipparāja himself. His *Uparāga-darpaṇa*, on eclipses, is available manuscript form.

Next, in an astrological work by name *Vī (Bī)meśvara Uḷlamudaiyān*⁸, a small section on astronomy appears. An alternative name given is *Jodiḍagrahacintāmaṇi*. The main matter is what is found in the *Cuḍāmaṇi Uḷlamudaiyān*, i.e., mundane and electional astrology with a little horary astrology added. The author does

8. This was first printed in 1899 in the Viveka Vilakkam Press of Purasaipakkam (Madras), as edited by Mārglinga Jodisar, who calls himself a pupil of Kulandayānanda Swāmīgal of Mylāpore. It was reprinted in 1970 at Tirumagal Press, Madras-4, and published by Ratnanāyakam and Sons,

not give his name, but in every verse he mentions Vi(Bhi)meśvara, the Lord of Toḍukkāḍu in Tanjore District, often invoking the grace of Vimeśvara's consort, Nahaimuhavalli. The date of the work given in verse 10 is Kali 4728, i.e., 1627 A.D. The subtractive given for beginning the moon's computation is 16,35,565 days (1378 A.D.). The moon's anomalistic cycles are 3279 days (got by adding 248 and 3031) and 248 days. To bring the moon to true sunrise caused by the sun's equation of the centre, the change in daylight and the difference in longitude called *deśāntara* (the reduction to the equator being omitted, as in the original *Vākyakaraṇa* itself), subtractive or additive minutes being given for every 8 days of each month. The whole correction is wrongly called *deśāntara* correction by the commentator, though the actual work does not make this mistake. Since the average of all the corrections given is zero, we have to assume that either the *deśāntara* itself (about -7' for Tanjore) is not given, or that a correction is given to the mean moon, equal to 8'. This is the dawn of the *mānyadi* correction to the moon which includes a correction to the mean moon of about 26', used by the informants of Gentil and Warren, c. 1800 A.D., and later, upto this day. For finding the true sun within the month, the *Yogyādivākyaś* are given, appearing in this work for the first time instead of the *bhūpādivākyaś* of the *Vākyakaraṇa*, which are more difficult to use. Here too the commentary makes mistakes in the instructions to add or subtract, not to speak of the printing mistakes of the constants.

To compute the five planets, the subtractive to begin work is given as 1368 *Śaka*. Very rough values for the mean motions per annum are given, showing that the author intended them to be used for the author's own lifetime. But these values carry the *Parahita* corrections used by the *Vākyakaraṇa*. To correct the mean planets for the equation of the centre and equation of conjunc-

tion, values are given for each *rāśi*, a rough approximation instead of the intervals of $3\frac{3}{4}^{\circ}$ as in the *Parahita-vākyas* of Haridatta, repeated in the *Cūdāmaṇi Uḷlamuḍaiyān*.

During the last decades of the 19th century, several such astrological-cum-astronomical works appeared for the use of almanac makers. One is the *Muruga-Śekharam*, which gives the *Vākyakaraṇa* method for the sun and the moon and the corrected Parahita method for all. It mentions how to use different calendars for use by Muslims and others. It includes also Dharmaśāstra matter to determine dates for offerings to the manes, Vratas, fasts, feasts, etc. It is by one Muruhaiya Josyar, and reprinted by Ratnanaicker and Sons in 1932.

During the same period, a work called *Jōḍiḍagraha-cintāmnṇi*, popularly known as *Varṣādināl*, appeared, for the same purpose. It is practically the *Vīmeśvara Uḷlamuḍaiyān* with some ramifications of the astrological portion.

One Swāmi Iyengār of Karaiyūr issued a *Parahita Gaṇitam* in Tamil, with his own corrections, mostly following the *Vākyakaraṇam* constants and a subtractive from Kali days of 16,83,112 days, which is equal to eight times the sub-yuga used by Haridatta, etc. 2,10,389 days, the time being 1507 A.D. Why it uses this subtractive instead of a larger one, by which it could have obviated labour, is a mystery. It also gives methods to find day-time and ascensional difference for different latitudes.

A very important work c. 1880 is the *Jotiṣagaṇita-sāstram* of Mūṇāmpaṇṇai Kṛṣṇa Josyar of Nānguneri (Tirunelveli District). It is in Tamil and is extensively used by *Vākya* almanac makers, many of whom own copies of it. It follows the *Vākyakaraṇa*, but uses the *mānyādi* to bring the moon to true sunrise, with a *bīja*. The author's knowledge of astronomy is good, and he brings it to bear on his work. It is in five parts: (1) The sun, moon and

Rāhu; (2) The five Star-planets; (3) Eclipses; (4) Daylight, Ascension etc., (5) Miscellaneous matters. For eclipses he gives different ancient methods and *bija* corrections to secure tolerable accuracy. He also gives rules to determine dates for offerings to the manes and Vratas.

IV. Modern Times

We now come to our own century. A desire to know the correct positions of planets, for the sake of predictions, is evinced by the educated. A new type of almanac, called *Ḍṛgganīta*, whose calculation is based on modern Nautical almanacs and ephemerides is becoming popular. One such almanac-maker, C. G. Rajan, a Tahsildar in the Chinglepet district, compiled tables based on modern astronomical constants. Using these tables, fairly accurate positions of planets can be found for any day from 3000 B.C. to 3000 A.D., which may be used by astrologers and research scholars.

Rajan has named the work *Rāja-Jyotiṣa-gaṇitam*. It is in English, and there is also a Tamil translation by himself. It was published in 1935. In his explanation and discussion, he exhibits good knowledge and grasp of the subject. In 1959, he published three booklets for the benefit of *Vākya*-almanac-makers, based on the *Vākya-karaṇa*, one on the sun, moon and Rāhu, one on the computation of the five star planets, and one on calculating eclipses, using earlier cycles. He also gave the manner of computing the elements, in the usual method. In 1961, he published a booklet on the calculation of the solar eclipse, computing and using modern elements. Till he died a few years ago, he was preparing the *Rāṣṭrīya Panchāṅg in Tamil*, for the Government of India.

There was also one L. Nārāyaṇa Rāo, a native of Tanjore and a retired Central Government Officer, who was publishing a *Siddhānta Panchāṅgam*, till he died recently. He has collected and published the planetary

ephemerides for various groups of years from 1800 A.D. to 1950 A.D. Being a master of modern astronomical calculations as well, he has shown how the set of ephemerides given can be used for any time earlier or later outside the given periods, provided the time is not too far away. That is, he has shown how the sets of ephemerides can be used as perpetually repeating tables like the *Candra* and *Samudra-vākyas* of the *Vākya-karaṇam*. If he had given longer periods to prevent the accumulation of the inevitable small errors at the end of each given period, it would have been useful to research workers desiring to compute the correct positions of planets at very early times. Also, he could have minimised the labour of reducing the given positions to the times required earlier or later, provided they are not too far away, by giving the instruction for changing the equation of the centre by differentiation. He has also given some tables useful for astrologers.

Lastly, the author of the present paper too has made some modest contributions. He has translated the *Vedāṅga-Jyotiṣa* with an Introduction.⁹ He has also translated the *Vākya-karaṇa* in Tamil, with elaborate notes and worked examples. In the notes he has pointed out the merits and defects of the work, with instructions to bring the various items in line with modern astronomy. It is being used by almanac-computers and some of his friends.

He thought it would be good if, instead of the various instructions given in his notes to modernize the *Vākya-karaṇa* results, he added two appendices to his translation, one to compute the correct positions of planets, and the other to give the correct circumstances of eclipses. The former he put in the *Vākya-karaṇa* garb, just for the pleasure of it, though the planning and

9. Ed. by K. V. Sarma with a critical edn. of the text, Indian National Science Academy, New Delhi, 1985.

computation of the tabular values demanded a lot of ingenuity and enormous labour. In the case of the moon he has taken into account all the equations necessary for tolerable accuracy. The latter is a compendium in Tamil of a larger work he has written in English. He finished these two works by 1953 and 1956. They are still in manuscript form.

He has revised his old work (in English) on eclipses, making a better arrangement of the matter and the tables more easy to use. Its merit is that it is complete in itself, where the elements used in computation, normally taken from the almanac themselves, are computed by himself and given. To do it easily, he has devised methods to combine the large number of the respective equations and has given them in a smaller number of tables. He has given three methods of computing the circumstances, viz., the nonagesimal method used by the earlier astronomers, the right ascension declination co-ordinates method, and Bessel's method, explaining all of them fully.

In 1957, he critically edited the *Mahābhāskarīyam* of Bhāskara I with the commentary of Govindasvāmin and a super-commentary by Parameśvara, for the Government Oriental Manuscripts Library, Chepauk, Madras. In 1962 he brought out critical edition of the *Vākyakaraṇa*, with commentary jointly with K. V. Sarma.¹⁰

In 1962, he began the preparation of a critical edition of the *Pañcasiddhāntikā* with an elaborate Sanskrit commentary and worked examples and an English translation of both the text and the commentary.¹¹ He has also prepared a book on the theory and practice of modern astronomy, with tables and some basic mathematics and other matters useful to astronomers. This too remains in manuscript form.

10. Pub. The K. S. Research Institute, Mylapore, Madras-4.

11. English version being prepared for the press by K. V. Sarma.

THE AGE OF ŚAÑKARA

I. A Review of *The Age of Sankara*

by T. S. Narayana Sastry (TSNS)*

The Age of Sankara was first published in 1916, and a second edition has come out omitting certain parts of the Appendix. One of the important parts omitted is the 'Śakakāla or Śaka era', treating about the Śaka era mentioned as No. VI of the eras listed on page 22.

This work gives an interesting account of the life of the famous Ādi-Śaṅkarācārya, detailing the various incidents of his life, in the manner of a historical fiction. *Vimarśa*, a Sanskrit work on the same subject, written in 1898 by Rājarājeśvara Śaṅkarācārya Svāmī of the Dwaraka Mutt, has shown the author this method of treatment, together with many other things. The author says that the date of Śaṅkara is from 509 B.C. to 477 B.C. To see such an early date determined for Śaṅkara must be very pleasing to Hindus in general, and the Advaitins in particular. But when historians and research scholars determine from internal and external evidences that Śaṅkara could have lived only several centuries after this time, and it might be even more than a thousand years later, we have to accept it as true, because the evidences put forth by the author for his date are, I am afraid, untenable.

I shall set out the true position first. All the Mutts established by Śaṅkara have constructed their *paramparā*

* This is a detailed review of *The Age of Sankara* by T. S. Narayana Sastry, B.A., B.L. Second edition by T. N. Kumara swami, M.A., published by B. G. Paul & Co., No. 4, Francis Joseph Street, Madras, 1971.

('line of succession') from tradition and inadequate records, long after they were first founded, after they gained worldly importance, and felt the need for an uninterrupted pedigree from Śaṅkara downwards. There were several Vikramāditya-s in whose names records dated in *Vikramābda* are found, confusing them with the well-known Vikrama Era of 58 B.C., and this has led to several contradictions. The line of succession may be expected to be correct for a few pontiffs before the attempt at reconstruction was made, when memory would have been fresh. Generally speaking, uncertainty would increase as we go further up. The Sringeri Mutt, having had connections with the Vijayanagar and later empires and had become famous from the 13th and 14th century onwards, its line can be expected to be the most authentic. But mistaking Śaṅkara to have lived in the first century B.C., owing to the confusion in the Vikramāditya names, has resulted in giving 800 years to Śaṅkara's successor Sureśvarācārya, to connect him with the next in succession in that line, beginning from whom the succession has been quite authentic. If Śaṅkara is taken to have lived in the 8th century A.D., or a little earlier or later, on which there is an almost consensus of opinion among historians, not only will all discrepancies be resolved, but it will also fit in with the internal evidence of Śaṅkara having known Diñnāga, Bhartṛhari, Dharmakīrti and Kumārila, and having condemned the *Vijñāna* and *Śūnyavāda*-s of the Buddhists and several Jain tenets of later growth.

Like the line of succession of Śaṅkara, the writing of his history too must have been attempted long after he had lived, after the desire to write it had cropped up in peoples' minds. Therefore, some of the incidents mentioned therein may be authentic, but others that are impossible or seem improbable to the fair-minded historian, might not be true, and are the product of the

poet's *kāvya*-style of writing, a mixture of fancy and fact. That Śaṅkara was born on a Vaiśākha-śuddha-pañcamī must be true, and that the birth-star was *Ārdra* may also be true. But the planetary combinations and auspicious birth-times given variously by various authors must have been drawn from their imagination, fancying that such a great man must have been born under such a combination.

But one thing we can say. The *Śaṅkaradigvijaya*, popularly known as *Mādhaviya-Śaṅkaravijaya*, written by Vīra Vasanta-Mādhava,¹ better known as Mādhava-mantrin, of Aṅgīrasa gotra, minister of Immadi Bukka II, in the early 15th century, and published long ago with the two commentaries *Diṇḍima*² and *Advaitarājyalakṣmī*³ and referred to even by the *Suśamā*⁴ as *Śaṅkṣepa-Śaṅkara-vijaya*, being the most ancient extant text on Śaṅkara, may be taken to be the most authentic, saving the inevitable *mahākāvya* style.⁵

This work mentions a prior work, *Prācīna-Śaṅkara-vijaya*, of which this is a compendium, but that work does not seem to be available now, about which we shall see later. Some Sankara Mutts have caused to be written or patronised several *Śaṅkara-vijaya*-s, literary works and *Guru-paramparā*-s to establish each its own importance

1. Vide *The Age of Vidyaranya*, Part I, by K. R. Venkataraman, Calcutta.
2. This was written by Dhanapati Sūri in 1770 A.D. He also wrote a commentary on Sadānanda's *Śaṅkaravijaya* in 1783.
3. This was written by Achyutarāya in 1824.
4. Details about this will be given later.
5. The author TSNS tries to belittle the *Mādhaviya-Śaṅkara-vijaya*, saying that it is by one of his (TSNS's) own time. But the facts mentioned above and the fact that it had come in print long before the time of the alleged modern author, go against accepting his (TSNS's) words.

and, in some cases, when disputes arose about property or jurisdiction, and have even concocted new evidence like copper-plate inscriptions etc.

In thus concocting evidence, it is patent that contenders entering in recent times have certain advantages. They can use the better knowledge of history and science to make more plausible concoctions, and even hold up the earlier mistakes of their adversaries to ridicule. But if truth will triumph, the very excessive greed of these concoctors will expose them; the very knowledge that they use, being insufficient to cover them up, will expose them. The work which we are reviewing serves as a good example of what I am saying.

The author of the work reviewed here, T. S. Narayana Sastri, asserts in this work that Śaṅkara was born in 509 B.C. corresponding to Kali 2593, in the year Nandana, on Vaiśākha-śuddha-pañcamī, Punarvasu, Sunday, in Karkātaka-lagna. The evidence he adduces for this can be classified under four heads :

1. Bits of a *Byhat-Śaṅkaravijaya* (BSV) attributed to the famous Citsukhācārya, quoted in the commentary called *Suṣamā* (alleged to be by one Ātmabodhendra, a pontiff of the Kumbakonam Mutt), on the *Guru-ratna-mālikā*, giving the line of succession of the Mutt, alleged to be written by the famous Sadāśiva Brahmendra, together with a *Puṇyaślokamañjarī* also alleged to be written by a pontiff of the same Mutt.

2. Bits of a *Prācīna-Śaṅkarajaya* (PSJ) attributed to the famous Ānandagiri, alleged to be the seventh pontiff of the Mutt, belonging to the second century A.D.

Even according to the author, the *Byhat-Śaṅkara-vijaya* and the *Prācīna-Śaṅkarajaya* are not extant now but for the bits quoted in the *Suṣamā*. Of these two, regarding the former, only the name Citsukha is true as the author

of the famous *Citsukhtyam*. There is no evidence for the story that he was born in Śaṅkara's own village, had been his mate as a boy, lived always with him and long after his death, knew every detail of his life, became a pontiff of the Mutt and wrote the *Bṛihat-Śaṅkaravijaya*. All this is known only to the *Gurugranthamālikā-Suṣamā-Puṇyaślokamañjarī* group, and to nobody else. These have concocted, for the Mutt, a line of succession with precise dates of accession and death, from Śaṅkara downwards, using the names of authors of famous advaitic and other works, who have been established by research scholars to have lived in quite other times and at other places.

As for Ānandagiri, the name is well known as the writer of the gloss on Śaṅkara's works. *Prācīna-Śaṅkarajaya*⁶ may also be taken as having existed, if Mādhava had meant by the word the name of a prior work instead of an earlier *Śaṅkaravijaya* in general. But there is no evidence for believing that Ānandagiri wrote the work, or that he was the seventh pontiff of the line, and lived in the second century, excepting the *Gururatanmālikā-Suṣamā-Puṇyaślokamañjarī* set, written solely for the glorification of the Mutt. If what TSNS says about the *Bṛhat-Śaṅkaravijaya* and *Prācīna-Śaṅkarājaya* were

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6. In listing the various *Śaṅkara-vijayas*, TSNS mentions one by a certain Anantānandagiri, calls him an imposter and condemns him as being unreliable and confused, because the work, as the author knows it would do discredit to any Mutt that owned it. But since the TSNS's death, it has been emended by deleting or incorporating whole passages, and friends of the Kumbakonam Mutt now assert that Anantānandagiri is the old Ānandagiri himself and writer of the *Prācīna-Śaṅkarajaya*, (this, inspite of the patent super-imposition of two large layers exhibiting low taste, and written in halting Sanskrit, over an original good matter. They have also brought out a new edition of the work, trying to cover up the defects and presenting the work in the most favourable light to the Mutt).

true, then they would be the strongest evidence for Śaṅkara's life history.

In the *BSV* all incidents are given in a Yudhiṣṭhira Śaka (YS), Śaṅkara's birth-date being given as 2631 of the era, and in Ānandagiri's *PSJ* in the Kali era, the birth year being given as 2593. This YS itself, according to TSNS, began in 3140 B.C., (sometimes he says it is 3141 and sometimes 3139, according to his convenience), which date itself he has invented for his own purpose, no one else having mentioned it before as "beginning from the first coronation of Yudhisthira at Indraprastha." But the name itself is well-known and used by the Jains as beginning from 648 Kali. But the author asserts that the Hindus *all* use *his* YS of 3140 B.C. He is not speaking the truth here. While his YS *is not used by anybody else*, there are two other YS which have been in use among Hindus. The well-known *Jyotiṛvidābharaṇa*, in its list of eras current in Kaliyuga, says that the YS began with the Kaliyuga, in 3102 B.C., and ended with the Vikrama Era beginning in 58 B.C., which itself ended in 78 A.D., when the Śaka Era proper, later known as the Śālivāhana Śaka, began. Again, Varāhamihira in his *Bṛhat-saṃhitā*, in the context of the *Saptaṛṣi-cāra*, says that the era of Yudhiṣṭhira's reign began 2526 years before the Śaka Era of 78 A.D. :

ṣaḍ-dvika-pañca-dvi-yutaḥ (2526)
Śakakālas tasya rājyasya

The years of the Saptarṣi-cāra, omitting the centuries, and called the Laukika Era, was current in Kashmir and the nearby Himalayan regions. It is this that is used by Kalhaṇa in his *Rājatarāṅgiṇī*. If this is also taken as YS, since it began with Yudhisthira, we have it that all Hindus have been using these two, and none else was known. When this is the truth, TSNS asserts on page 22, Sec. V, that his concocted YS of 3140 B.C. was

the one used by all the Hindus. I shall explain why he was constrained to invent this, when dealing later with the Sudhanvā copper plate. For the present we shall accept what he says and proceed.

The *Gururatnamālikā* and the *Suṣamā* with the quoted portions of *BSV* and Ānandagiri, and the *Puṇyaślokamañjarī*, all state that Śaṅkara was born in Kali 2593 (the author's Y.S. of 2631) on Vaiśākha-śuddha pañcamī, Karkāṭaka-lagna, with the five heavenly bodies, the Sun, Mars, Jupiter, Venus and Saturn in exaltation (*ucca*), as shown in the following birth chart I, as given on page 243 and also 288 by the author.

Chart I (given)

Venus	Sun	Mer- cury	Moon
	Rasi		Jupi- ter Lagna
Mars			
		Saturn	

Chart II (actual)

Saturn	Sun	Mer- cury Venus	Moon
	Rasi		
Mars	Jupi- ter		

But actually on that date the positions were as in Chart II. Note how far away are the four planets Jupiter, Saturn, Mars and Venus from the exaltation positions given by the author. The differences are so great that they cannot be accounted for by the difference in the Siddhāntas used, or mistakes in them. Could the great Citsukha, alleged to be a contemporary, close friend and later a successor, have given that chart even by mistake, or Ānandagiri, who is alleged to have followed in the line close after? As the line of succession and dates of

accession have been constructed in the *Puṇyaśloka-mañjarī* and *Suṣamā* exactly following this date as origin, and as the line can neither be contracted nor expanded in time, giving as they do specific dates for each succession, they must all be equally unreliable, as also the existence of these two *Śaṅkaravijaya-s*, since they exist only in the quotations in these works. It is also obvious that works like the spurious *Vyāsācalīyam* quoted by the author on p. 247, giving for Śaṅkara's birth the same year 2593 Kali, *Nandana*, with the same five-planet exaltation, and for his death 2625 Kali, *Raktākṣa* on p. 230 must be considered as unreliable as the above works.

The probable genesis of these works is the publication of the *Vimarsa* in 1898, by the said Swamiji of the Dwaraka Mutt with a copper-plate inscription in support of Śaṅkara's date, alleged to have been issued by a contemporary king, Sudhanvā, to Śaṅkara himself, and said to have been preserved in the Mutt. (We shall examine the genuineness of this plate later.) In it the date of issue of the plate is given as 2663 YS, the wording showing that Śaṅkara was alive at that time. Our author resolved to use this for his purpose. Even if YS 2663 is the last year of Śaṅkara's life-span of thirty-two years accepted by all, he should have been born not earlier than 2631 Y.S. If this YS is taken as the accepted YS of *Jyotirvidābharaṇa*, Śaṅkara's birth must fall just after 2631 Kali, since both eras are synchronous. The author also wanted to give a birth-date for Śaṅkara, with five planets exalted, for the sake of plausibility, and his computers fixed a date in 2593 Kali (*Jyotirvid. Y.S. 2593*) thinking that that date answered to the specification, and no other date nearby. (This is the real meaning underlying the author's statement on page 245 to the effect, "We have ascertained from two of the greatest astrologers of South India that this particular combination of the

planetary bodies did actually occur on *Vaiśākha* (*Meṣa*) *Śukla Pañcamī* of the year Nandana in 2593 of the Kaliyuga, corresponding to 509 B.C.”). But since this is too low down from YS 2631, which is the lowest limit agreeing with the date of the copper-plate mentioned above, the author had to invent a YS beginning 38 years earlier, with the first coronation of Yudhiṣṭhira in 3140 B.C. according to him, and asserting without an iota of truth that that was the only YS known to the Hindus. But fortunately for truth, the concoctors have gone wrong, and the author’s edifice built upon their date has collapsed, as we have already mentioned.

Another mistake of the concoctors that exposes them is giving Nandana as the year of birth in both the *Śaṅkara-vijayas*, while actually it must be Dhātā for 509 B.C. according to the astronomical *Saṁhitās* and *Siddhāntas*. They have got it by counting backwards one year of the Jovian cycle for each solar year and arriving at Nandana for 2593 Kali, on the mistaken practice now current in the extreme south of India, since the solar calendar was adopted about a thousand and one hundred years back, (the rest of India following the correct procedure), as evidenced by the twelve-year advance there in the Jovian year from what is given in the South. (For details see the *Bulletin of the Institute of Traditional Cultures*, October 1967, Part I. Sect. ii, p. 45). The reason given for the lagging behind in the South, and advance elsewhere in the Introductions to some Tamil *Pañcāṅgas* is mere bluff, not supported either by the astronomical works or the *Dharma-śāstras*).

Further, historians and scholars agree that at such an early age as 509 B.C. weekdays like Sunday, names of *rāśis* like *Meṣa*, and ideas like exaltation, and giving numbers in *bhūtasāṅkhyā* were not in use among Indians. But these are freely used in the said quoted portions. Mistakes in the use of the *bhūtasāṅkhyā* indicate that

novices not conversant with it (like the people of the South accustomed to using mostly the *ka-ta-pa-yā-di-saṅkhyā*) are handling it. They are unaware that there is a practice governing its use. *Adhva* for 6, *vāra* for 7, *śāstra* for 6, and *aṅga* for 4 are not used in the *bhūta-saṅkhyā*. While the first three are simply not in use, the last one, *aṅga*, used for 6 by custom, cannot be used for 4, since the resulting ambiguity would play havoc with the astronomical constants. Thus too the concoctors stand exposed.

Against the argument of anachronism given in the last paragraph, the author may say that the historians have all gone wrong in dating the past events, that actually the early periods like the rise and fall of the Magadha kingdom etc., should be put back by about 700 years, and if done so what they fix for the second century A.D. will be possible for 509 B.C. The author has put forth his arguments for this in an Appendix to his first edition (omitted in the edition reviewed here), postulating that the real Śaka Era started in 576 B.C. (sometimes he makes it 575 B.C. and sometimes even 550 B.C. according to his convenience), and that the early records like the Aihole Inscription and astronomers from Varāhamihira to Bhāskarācārya, mean only this Śaka when they mention it. I have exposed the baselessness of his arguments in detail in a paper on the subject (see above pp. 255-87).

Another thing has got to be mentioned. In describing the planetary positions in verse 12, on page 273, we find the words, “षड्विंशे शतके श्रीमद्यष्टिरशकस्य वै”. This means that the birth was in the 26th century of the YS and the date would have to be taken as 2531 YS (equal to 609 B.C.) and not 2631 YS. But this year cannot be taken, for the following reasons: Mars cannot be in exaltation, being nearly four *rāśis* away. This will also go against the year of death given in the same work as

2663 YS and Kali 2593 for birth, and the cycle year Nandana given for it, as also the *Gurugranthamālīkā-Suṣamā-Puṇyaślokamañjarī* group, giving definite dates beginning from 2593 Kali alone. This would also contradict the concocted *Jinavijaya* agreement—(we shall be dealing with this presently)—and also the evidence of the Sudhanva plate. Another thing may also be added. In the quoted verses, there is a confusion that *Saugatas* and *Jainas* are synonymous. Do the concoctors mean that Citsukha and Ānandagiri did not know that Jainas are different from Saugatas, viz., Buddhists?

Another important point: On page 227 there is an alleged quotation from the PSJ. describing Śaṅkara's death, saying that in Kali 2625, Śaṅkara placed Sureśvara on the *Pīṭha* renowned as *Kāmakoṭi* to take care of Sarvajña, and leaving his body before *Kāmākṣi* attained *mokṣa*:

कलयद्दश्च शरेक्षणाध्वनयनैः सत्कामकोटिप्रथे
पीठे न्यस्य सुरेश्वरं समवितुं सर्वज्ञसंज्ञं मुनिम् ।
कामाक्ष्यास्साविधे स जातु निविशन्नुमुक्तलोकस्पृहः
देहं स्वं व्यपहाय देह्यसुगमं धाम प्रपेदे परम् ॥

This is an anachronism because the *Kāmākṣi* cult itself did not develop till after several centuries A.D. and archaeologists and historians agree that the *Kāmākṣi* temple itself arose on the ruins of the Jain and Buddhist temples situated in the same place, with many of the old *mūrtis* transformed into the present ones, (as seen from the old vestiges still visible), and the worship of *Kāmākṣi* commenced about the ninth or tenth century A.D., the temple itself being built later still. When such is the case, how could there be a Math called the *Kāmakoṭi Pīṭha* there or the mention of *Kāmākṣi* in the 5th century B.C.? (For details see the booklet, *Devī Kāmākṣhī in Kanchi—A historical study* by K. R. Venkataraman, Second edition, 1973). Thus we see the author's edifice collapsing at every line of examination.

III. A corroborative evidence advanced by the author is an alleged extract from a work called *Jinavijaya*, apparently written for the glorification of Jina, i.e., Vardhamāna Mahāvīra, the 24th Tirthaṅkara and founder of the Jain sect. It may be mentioned even at the outset that such a work is not found in the *Jinaratnakośa* a bibliography of known Jain works, nor in Aufrecht's *Catalogus Catalogorum*. There is a work of this name in the Madras University's *New Catalogus Catalogorum, of Sanskrit works*, but it deals only with the tenets of the Jain sect and nothing more. The author must have invented the name and concocted the quotations as therefrom, thinking that evidence from a rival faith's work would be more convincing. On p. 150, he himself says that he has no firsthand knowledge of this evidence, but saw it mentioned in an issue of the journal '*Sanskrita Chandrika*' of one Appa Sastri of Kolhapur that Śaṅkara was born when Kumārilabhaṭṭa was forty-eight years of age, and so he wrote to Appa Sastri for authority and that Appa Sastri sent him by post the verses quoted by the author, saying they were from the *Jinavijaya*. We have only to believe the author's words for this. Be it so. But why should he give references under the verses (like "vide p. 6, *Sanskrita Chandrika* etc.") as if these verses are taken from the journal itself? Further, an examination of the verses would show that the verses have been written by a person having very little knowledge, especially of Sanskrit. For example, but for the only statement that Kumānila was defeated in argument (by Jina himself!!) the gist of the verses glorifies Kumānila, against the ostensible purpose of the work. What is the relevance of Śaṅkara's year of death as *Raktākṣa* being given in this work, where even giving Kumānila's full life will be irrelevant? In the context of Śaṅkara meeting Kumānila, a verse (as translated by the author himself) says "when Sankara was fifteen years of age, *Siva* met Bhattacharya Kumarila". Did the Jains too consider

Śaṅkara as an incarnation of Śiva? Further, the wording is such that Śaṅkara and Śiva in the sentence are different persons. In mentioning Sudhanvan killing Jains, the verse says जि(?जै)नानां येन साधूनां चक्रे कदनमद्भुतम् ।, the word अद्भुतम् in the verse indicating that the narrator mentions the fact with exultation. Would a Jain speak like this? Here, too, in giving *Bhūtasāṅkhyā*, words not current then are used. For 'two' the author seems to be afraid of using *akṣi* and always uses the expression *martyākṣau* as if there would be some doubt in the number if *martya* is not used together. The same fear, bespeaking a modern novice of South India is seen in giving the instruction *vāmamelanāt*, again and again, as if otherwise the numbers would be used in the order he is accustomed to, beginning from the highest value, proceeding to lower and lower. Here, too, is the same mistake, in getting the Jovian cycle-year *Raktākṣa* as Śaṅkara's year of death. The mistaking of Saugata for Jain is also found, all pointing to the same person or group as concoctors. Again, while the *Śaṅkaravijayas* in general give as the opponents of Kumārila, the Bauddhas with whom he lived incognito, learnt their Śāstras and ultimately vanquished them, why should the Jains take their place here? Here is the hand of the Dwaraka Math, latterly obsessed with the Jains predominant in the region around it. The Sudhanva plate also exhibits this obsession and is perhaps the genesis of the substitution of the Jains for the Buddhists. In conclusion, since the alleged birth-date by the Jain reckoning also exactly coincides with the discredited 509 B.C., it is evident that the former has been concocted simply for corroboration with the latter.

IV. We shall now take up for scrutiny the evidence of the Sudhanva copper plate inscription, given on pp. 220-21. For a detailed study, I refer the reader to the speech, exhibiting deep critical acumen, delivered by Prof. V. Venkatachalam, Head of the Department of

Sanskrit, Vikram University, Ujjain, at the Seminar held at Sringeri, on the occasion of the Kumbhabhishekam renovating the *Adhiṣṭhāna* of Sureśvara on May 10, 1970, and printed in the *Commemoration Volume*, Seminar Section, pp. 86-105, under the title 'The Sudhanva copper-plate: A dispassionate re-appraisal'. I shall give the substance alone here, adding my own comments thereon.

(1) In 1898 A.D. the then Śaṅkarācārya of the Dwaraka Math published a Sanskrit work by name *Vimarśa* in which there was a copy in modern Devanagari of an alleged copper-plate inscription issued by King Sudhanvan to Śaṅkara in YS 2663. If genuine, the original should have been in the pre-Asokan Brāhmī script, examining which we can ascertain its genuineness. But it is not available at the Math now for examination. They say that it had been submitted as an exhibit in a Court of Law for evidence in a dispute, and was not taken back. It is not likely that they would have failed to get back such an important document unless it was a fake, and, so, was not claimed back, for fear of exposure by historians.

(2) Its language is different from that of c. 500 B.C. when it was alleged to have been issued, and contains modern provincialisms current in the Gujarat region. For example, the word *satta* is used in the sense of power or suzerainty.

(3) The word *Śaka* is used in the sense of era. This sense originated from the word *Śakābda* meaning the year in the Śaka Era of 78 A.D. used (like Vikramābda) to indicate the era named after the Śakas, and later extended by a semantic change to indicate an era in general, like the word எண்ணெய் = (எள் + நெய்) in Tamil, first meaning an oil expressed from sesamum, (எள்); and later used for any oil, so that we have words like தேங்காய் எண்ணெய், கடலை எண்ணெய் etc. (Even if the author's

plea for an older Śaka referred to already be accepted, a hundred years is too short a period for such a generalisation to happen.)

(4) The date of issue of the plate is given as YS 2663. Whether there was such an era current at that period or not, whether this plate is a fake or not, the Swamiji of the Math that published it in his book and used it for his *Life of Sankara* has taken the YS to mean the well-known one given in the *Jyotirvidābharaṇa*, beginning with Kali 3102 B.C. and dates the events of Śaṅkara's life and the accession of the subsequent pontiffs. This can be seen from the synchronism in the date of composition given by him at the end of the *Vimarśa* “श्रीमच्छंकर-भगवत्पूज्यगदाचार्याणां अवतारशकाब्दाः पौषशुक्ल-पूर्णिमायां.....” This gives Śaṅkara's year of birth as 414 years before the Vikrama Era of 58 B.C., i.e. as 472 B.C. Even if the plate had been issued in the last year of Śaṅkara's life, the birth year in YS should be 2631, i.e. the YS began in 3102 B.C., i.e., with Kali. Now, our author says that Śaṅkara was born in 509 B.C., 37 years earlier than 472 B.C. which we get by the YS of the *Jyotirvidābharaṇa* used in the *Vimarśa*. It is to make up for this that the author has invented his YS beginning 37 years earlier, in 3140-39 B.C. (about which we have spoken already).

(5) We shall now take up the contents of the plate for scrutiny.

(a) The only original authority for the existence and content of the plate is its publication in the *Vimarśa*. But in taking it and printing it in his work (p. 222) the author has silently changed the words “विश्वरूपापरनाम-सुरेश्वराचार्याश्च” into “मण्डनमिश्रापरनामधेय-सुरेश्वराचार्याश्च” for his own purpose. This is a trick to appropriate for the Kumbakonam Math the Sureśvarācārya, whom the Dwaraka Math claims for itself, and who was really in the Sringeri Math as evidenced by his ancient *adhiṣṭhāna*

(sacred tomb) there, for which the renovation Kumbhabhishekam ceremony was performed in 1970.

(b) Sudhanva states in the plate that Toṭaka was the least in knowledge among the four disciples, and therefore Śaṅkara appointed him to Jyotirmatḥa in the north where there would not be many controversialists to meet, and that Sureśvara was the greatest in knowledge and was appointed to the Dwaraka Math in the west, where there would be the greatest need to dispute with opponents. If Sudhanvan had this estimate of Toṭaka, would he mention it to Śaṅkara himself, and, even if Śaṅkara had estimated the ability of his pupils like this, would he have confided this to Sudhanvan, and even then would Sudhanvan mention this in a copper plate issued to him?

Such is the credibility of the plate, and our author Narayana Sastry gives this as an important piece of evidence for his date of Śaṅkara. I doubt if he himself believed it to be genuine, because his mind and that of the Dwaraka Śaṅkarācārya have worked on the same lines, and he has perpetrated all the tricks that the other has done. Both have fictitiously related most of Śaṅkara's poems with the events of his life. Both have created a line of succession with precise dates, though the Swamiji has stopped with Nṛsiṃhāśrami. Both have stated that biographers have confused the later pontiffs of the same name with the original Śaṅkara and mixed up the events of their lives, and both give authorities from the non-existent and concocted works. While the Swamiji creates a *Bṛhadrājataranṅiṇī* to give as authority, our author creates a *Jinavijaya* to support the created *Bṛihat-Śaṅkaravijaya* and *Prācīna-Śaṅkaravijaya*.

I must also mention here a concoction named *Kaliyugarāja-vṛttānta*, extensively used by the author Narayana Sastry, in the first edition, the name being chosen with the intention of creating in the minds of the

readers the impression that it has been taken from the *Bhaviṣya Purāṇa*, *Uttarakhaṇḍa*. (The authoritativeness of even this *Bhaviṣya*, *Uttara* can be seen from the fact that it contains the story of the *Bible* from Adam and Eve to Jesus Christ, the story of Mohamed and Islam and the history of Muslim India till Shah Alam, and the fact that the personages occurring in these stories are declared to be the good and bad kings and demons of the Dvāpara and earlier yugas that have re-incarnated themselves as the good and bad kings and tyrants and villains in the Kaliyuga). We can guess what this means when all attempts by research scholars to trace it to any source, puranic or otherwise, have failed.

To continue, both Narayana Sastry and the Swamiji of the Dwaraka Math have tried to appropriate for their own Math famous writers of advaitic and other works, though they belong to different places and times. But even as the 'perfect crime' was out, the author's false claims stand exposed by his own imperfect knowledge and trying to be too clever.

Besides what we have discussed in detail, there are sundry other points mentioned by the author that cannot be replied to in detail in this article, for fear of being made too long for a review. The reader is referred chiefly to the following work to get his doubts cleared : *The Kumbakonam Mutt and the Truth about it*, Parts I and II, (published 1965), by R. Krishnaswami Aiyar, M.A., B.L., Advocate, Tirunelveli (latterly known as Gnanananda Bharathi Swamigal) and Sri (now late) K. R. Venkataraman, Redt. Director of Public Instruction and Historical Records Officer, Pudukkottai, respectively.

But what is the correct age of Śaṅkara? I shall discuss this in the next paper.

THE AGE OF ŚAṆKARA: II

The genuine *Śaṅkaravijayas* extant are all late works, as we have stated in the previous paper, and some of them have given dates which are only guesses from traditional stories, and are found mostly untrustworthy on examination. The *Mādhavīya-Śaṅkaravijaya* is silent on the point of Śaṅkara's date of birth, but gives a planetary combination that can give a series of dates at intervals of about 300 or 400 years. Internal evidence from the *Śaṅkaravijaya* of Ānantānandagiri discredited by T. S. Narayana Sastry, as we have already referred to, points to a date later than even Rāmānuja and Madhva, and obviously absurd.

But there is plenty of internal evidence and some external evidence also, to show that Śaṅkara must have been born not earlier than the last part of the 7th century A.D. and not later than the first few years of the 9th century, on which point there is an almost consensus of opinion among historians and scholars of Indian philosophy.

Śaṅkara's writings show that he is well acquainted with the Purāṇas in their modern form, which were redacted during the Gupta period. He is said to have studied the *Sūtasamhitā* several times before he wrote his *Bhāṣyas* and the *Sūtasamhitā* forms part of *Skānda*, one of the latest of the purāṇas. He is also known to have purified and propagated the six Indian cults (*Ṣaṇmata-sthāpanācārya*) that were fully shaped only after several centuries A.D. In the Mahāyāna form of Buddhism, the two schools of Vijñānavādins and Śūnyavādins were perfected during the period from the 4th to the end of the 7th century A.D. and Śaṅkara discusses and refutes them in his *Brahmasūtra-bhāṣya*, II. 2. 18-36. His direct pupil Padmapāda refers to this in his *Pañcapādikā* thus :

“अतः स एव माहायानिकः पक्षः समाहितः”

In II. 2.28, Śaṅkara quotes the first half of a *kārikā* of Diṇṇāga, pupil of Vasubandhu, from his *Ālambana-parīkṣā* :

यदन्तर्ज्ञेयरूपं तत् बहिर्वदवभासते ।

सोऽर्थोऽवज्ञानरूपत्वात् न तत्प्रत्ययताऽपि च ॥

It is known that Diṇṇāga lived in the 5th century A.D.

Sureśvara, another direct pupil of Śaṅkara, mentions the Bauddha Naiyāyika Dharmakīrti, by name, in his *Bṛhadāraṇyaka-vārttika* :

त्रिष्वेव त्वविनाभावादिति यद् धर्मकीर्तिना ।

प्रत्यज्ञायि, प्रतिज्ञेयं हीयेतासौ न संशयः ॥

He also quotes him :

अभिज्ञोऽपि हि बुद्ध्यात्मा विपर्यसितदर्शनैः ।

ग्राह्यग्राहलसंवित्तिभेदवानिव लक्ष्यते ॥

And Ānandagiri, the author of the gloss, writes on this;

कीर्तिवाक्यमुदाहरति-अभिज्ञोऽपि हि etc.

This same verse occurs in Śaṅkara's *Upadeśa-sāhasrī* (XVIII. 142). Again, in his *Brahmasūtrā-bhāṣya* (II. 2. 28), in refuting the *Vijñānavāda*, Śaṅkara says,

इह तु यथास्वं बाह्योऽर्थ उपलभ्यमानः..... अत एव 'सहोपलम्भनियमोऽपि' प्रत्ययविषययो रूपायोपेयभावहेतुकः. नाऽभेदहेतुकः इत्युपगन्तव्यम् ।

This सहोपलम्भनियम is a reference to Dharmakīrti's

सहोपलम्भनियमादभेदो रूपाद्विद्वयोः ।

and

भेदश्च भ्रान्तविज्ञानैः दृश्यतेन्दाविवाऽद्वये ।

respectively, from his *Pramāṇanīścaya* and *Pramāṇavārttika*. This is again quoted by Vācaspati Miśra in the *Bhāmatī* under the पूर्वपक्षभाष्य of Śaṅkara :

“अपिच, सहोपलम्भनियमादभेदो विषयविज्ञानयोरपतति ।”

The Chinese traveller I-T'sing, who toured India during 673-695 A.D., says in his report that Dharmakīrti had been his own contemporary and pupil of Dharmapāla, head of the Nalanda University, and class-mate of Ācārya Śīlabhadra. Thus Dharmakīrti must have written his works during the second half of the 7th century. Śāṅkara also quotes in his *Bhāṣya* in II. 2. 22-24, several bits from Bauddha works, and one of them is from the *Abhidharmakośa-vyākhyā* by Guṇamati, who is placed in the middle of the 7th century.

The celebrated mīmāṃsaka, Kumārila Bhaṭṭa, who is said to be a contemporary of king Sranga-san-Gampo (629-698 A.D.) by Lama Tārānātha, refutes in his *Śloka-vārttika* Dharmakīrti's definition of *Pratyakṣa*, in the passage “कल्पनाऽप्योदमभ्रान्त” etc. He quotes in his *Tantra-vārttika* from Bhartṛhari's *Vākyapadīya* the verse,

अस्त्यर्थः सर्वशब्दानां इति प्रत्याय्य लक्षणम् ।

अपूर्वदेवतास्वर्गैः सममाहुर्गवादिषु ॥ (II. 121)

and I-T'sing in his report says that Bhartṛhari died in 651-52 A.D. Umveka Bhaṭṭa, who became famous as Bhavabhūti through his drama *Uttararāmacarita* is said to be a pupil of Kumārila. Kalhaṇa's *Rājatarāṅgiṇī* says that in the year 733 A.D. King Yaśovarman of Kanauj, who had Bhavabhūti and Vākpatirāja as his court poets, was vanquished by the Kashmir king Lalitāditya. Cf. the verse :

कविवाक्पतिराजश्रीभवभूत्यादिसेवितः ।

जितो ययौ यशोवर्मा तद्गुणस्तुतिवन्दिताम् ॥

These point to the second half of the 7th century and early 8th century for Kumārila. Śāṅkara, after writing his *Bhāṣyas* by his sixteenth year, met Kumārila, a well-nigh old man, and at his suggestion vanquished his pupil, Maṇḍanamīśra. While adopting the *pramāṇas*

established by Kumārila (व्यवहारशास्त्रनयः) and his conclusions (मीमांसात्यायाः) as norms, as far as possible, Śaṅkara refutes other views which are specifically Kumārila's, inconsistent with the ideas of the *Brahmasūtras*.

From all these, we can see that Śaṅkara must have been born not earlier than the last quarter of the seventh century A.D. Therefore, any date earlier than this is untenable, and the date c. 655-689 determined by T. R. Chintamani (*Journal of Oriental Research*, Madras, III) must be considered *slightly* too early.

We shall now proceed to discuss what authority we have to fix the date more precisely.

Shri Jayapura Vishvanatha Rajagopala Sharma in his *Śrī Jagadguru Śaṅkara-mata-vimarsa*, (pp 16-27), fixes 648 A.D. for Śaṅkara's birth, (which seems most plausible), giving the following reasons :

(1) The Sringeri Sarada Matha finding the 14th year of Vikrama mentioned in its records, mistook it for the Vikrama Era of 58 B.C. and determined a date corresponding to 44 B.C. But at that period this era itself was not called by the name of Vikrama, but went by the name of Mālavagaṇa Era or Kṛta Era. It was also a common practice in those days to reckon dates in the ruling king's regnal years. There were several kings bearing the title Vikramāditya and we have to determine from other factors which Vikrama is meant. From our discussion, the Vikrama referred to must be the Chalukya Vikramādiya I of Vātāpi (son of Pulikeśin II), who ascended the throne in 670 A.D. or thereabouts. This must have been meant by the records, and Śaṅkara must have been born in his 14th regnal year, c. 684 A.D. B. G. Tilak also has accepted this view and determined this date for Śaṅkara in his work on the *Gītā*.

(2) Śaṅkara must have finished writing his *Bhāṣyas* by about 700 A.D. (from his life stories) and become famous only after that time. (That explains why I-T'sing does not mention him). This agrees with certain other facts. Ramachandra Kak has concluded that "Ādi Śaṅkara visited Kashmir c. 700 A.D. to mark which the Sankaracharya Hill was named after him".

(3) The Hindu Religious Endowment Commission (Chairman, C. P. Ramaswami Aiyar), after personal examination of evidence tendered, says that "Ādi Śaṅkara reclaimed the temple of Badrinath in the beginning of the 8th century A.D. with the help of King Nanak-Pal of Gahrwal, and it is significant that the Gahrwal State still has control over Badrinath temple."

(4) Youthful Śaṅkara's meeting with Kumārila at the latter's old age also agrees well with this period.

(5) Cunningham considers that the *Śāstravāda* held at the Śārādā temple in Kashmir during the time of King Lalitāditya, already referred to, mentioned in Kalhaṇa's *Rājataranginī*, might well be by Śaṅkara, as it coincides with the first part of the 8th century.

(6) Rajendranath Ghosh in his work *Sankara and Ramanuja*, fixes 686 A.D. for Śaṅkara's birth, and D. R. Bandarkar fixes 680 A.D., both very near the date fixed.

(7) K. T. Telang fixes 688 A.D., using the verse,

युगमपयोधिरसो(642)न्मिषतशाक्रे
रोद्रकवत्सर ऊर्जकगसे ।
शकरलोकमगान्निजदेहं
हैमगिरौ प्रविहाय हटेन ॥

This verse gives 642 Śaka (720 A.D) as the date of Śaṅkara's abandoning his body among the Himalayas. (The verse is said to be taken from a work called *Śaṅkara-paddhati*, as quoted by another work called *Darśana-prakāśa* dated 1638 A.D. The year should

actually be *Pingala*, and *Raudra* is three years off, a mistake caused through back-reckoning usually committed by South Indians, as already referred to).

(8) To illustrate a point, Śaṅkara says in his *Bhāṣya*, “यो हि सुग्नान्मथुरां गच्छति मथुरायाः पाटलिपुत्रं च, स सुग्नान् पाटलिपुत्रं गच्छत्येव ।” From the present tense used, we can understand that the three places mentioned were important and existed at Śaṅkara's time. Mathura is as well known now, as then. and Srughna is now the village Sugghna near Mathura. But Pataliputra was destroyed by the floods of river son by the middle of the 8th century and ceased to exist in later times, the city of Patna having been built near the old site of Pataliputra by Sher-shah in 1541 A.D. This shows that Śaṅkara must have passed away by the end of the first quarter of the 8th century, agreeing with his date of birth 684 A.D.

But various other scholars have given various other dates, at the end of the 8th century or the beginning of the 9th, taking into consideration various other factors, which they consider more trustworthy,

(1) L. Rice in the *Mysore Gazetteer*, Vol I, p. 300, suggests A.D. 745-769 for Śaṅkara, I do not know on what grounds.

(2) J. F. Fleet, D. R. Bandarkar, Max-Muller, A. A. Macdonell, Buhler, M. Barth and Pathak give 788 A.D. as the date of birth, as also Nagamiah, (*Travancore State Manual*, Vol. II, Ch. VIII, p. 99), K. P. P. Menon (*History of Kerala*, Vol III. p 620), W. Logan, (*Malabar Manual*, Vol I, pp. 155 ff. and 187 ff.). Their authority for this seems to be two chronograms. The first is a string of floating verses, source not known, discussed by Pathak, and reading as follows :-

दुष्टाचारविनाशाय प्रादुर्भूतो महीतले ।

स एव शंकराचार्यः साक्षात् कैवल्यनायकः ॥

निघिनानोभवहयब्दे (3889) विभवे शंकरोदयः ।
 अष्टवर्षे चतुर्वेदान् (? वेदी) द्वादशे सर्वशास्त्रवित् ॥
 षोडशे कृतवान् भाष्य द्वात्रिंशे मुनिरभ्यगात् ॥
 (कल्गब्दे चन्द्रनेत्राङ्गवहयब्दे (3921) गुहाप्रवेशः ।)
 वैशखे पूर्णिमायां तु शंकरः शिवतामगात् ॥

Here Śaṅkara's date of birth is given as Kali 3889 (788 A.D.) and the date of his disappearance, Kali 3921 (820 A.D.) The second is a verse from *Śaṅkara-mandāra-saurabha* :

प्रासूत तिष्यशरदामतिगातवत्यां
 एकादशाऽधिकशतोनचतुस्सहस्राभ्याम् (3889) ।
 संवत्सरे विभवनाम्नि शुभे मुहूर्ते
 राधे सिते शिवगुरोर्गृहिणी दशम्याम् ॥

(Both give the Jovian cycle year Vibhava, by back reckoning, while it should properly be Prabhava).

(3) Shri (now late) K. R. Venkataraman, (Retired Director of Public Instruction, Pudukkottai), and Shri K. V. Venkataraman of Erode, have taken the planetary combination at Śaṅkara's birth given in the *Mādhaviya-Śaṅkara-vijaya* and the *Guruvamśa-kāvya* (Sun in Aries, Jupiter in Cancer, Saturn in Libra and Mars in Capricorn) as factual instead of mere poetic fancy, and decide 805 A.D. as the only year answering to the combination, during the 8th century or nearby, (also long ago fixed by Pichu Aiyer, Cochin State Astrologer). The verses are :-

लग्ने शुभे शुभयुते सुषुवे कुमारै
 श्रीपावतीव सुखिनी शुभवीक्षिते च ।
 जाया सती शिवगुरोर्निजतुङ्गसंस्थे
 सूर्ये कुजे रविसुते च गुरौ च केन्द्रे ॥
 (Mādhaviya., II. 71)

साध्वी सा किल नवमेऽथ मासि पूर्णे
 सल्लग्नोऽधिपसंस्थिते कुलीरे ।
 केन्द्रोच्चैर्गुरुशनिभूमिसूनुसूर्यैः
 प्रासोष्टामितमहसं सुखेन सुनुम् ॥
 (Guruvamśa., II. 59)

(The second verse here adds the Moon also in Cancer, which excludes the asterism Ārdrā for Śaṅkara). Shri K. V. Venkataraman adds for confirmation, a tradition in Kerala that a contemporary Kerala king, who was in danger of losing his kingdom, was established firmly on the throne, and peace and plenty came to the land by the blessings of Śaṅkara, and, in gratitude, the king started the Kollam Era in 825 A.D. from the day he met Śaṅkara. (But there is another tradition current among Kerala astronomers that there is an astronomical event connected with the starting of the Kollam Era).

K. R. Venkataraman also mentions this, but places more credence on a tradition recorded in the *Guruvamśa-kāvya*, that a Kerala king named Rājāśekhara (which is perhaps a surname of Bhāskara-Ravivarman of A.D. 798-834, of the Kulaśekhara line, who is reputed to have composed three dramas), read out his dramas to Śaṅkara for approval. Another evidence, most clinching in his view, is an inscription in Cambodia, belonging to the reign of Jayavarman II (A.D. 878-887) which mentions the royal preceptor Śivasoma, “who had learnt the Śāstras from Bhagavān Śaṅkara”,

येनाऽधीतानि शास्त्राणि भगवच्छंकराह्वयात् ।

निर्दोषसूरिर्मूर्धालिमालालीढाऽहघ्नियङ्गजात् ॥

(G. Coedes : *Inscriptions de Cambodge*, p. 40)

But we think that this does not go against an earlier date for Śaṅkara, since the term may refer to any later pontiff of the line, who are called Śaṅkarācāryas, as is the practice now. Anyhow, this sets the upper limit to Śaṅkara's age. The date 898 Samvat (841 A.D.) is given by Vācaspati Miśra in his *Nyāyasūcīnibandha* for its composition,

न्यायसूचीनिबन्धोऽयमकारि विदुषां मुदे ।

श्रीवाचस्पतिमिश्रेण वस्वङ्कवसु(898)वत्सरे ॥

He has written the commentary *Bhāmatī* on Śaṅkara's *Brahmasūtra-bhāṣya*. Thus, this also sets *the limit*.¹

1. I have since seen the article "On the Date of Sankara-charya and allied problems" by K. Kunjunni Raja, *Adyar Library Bulletin*, Vol. XXIV, parts 3-4, fixing 'the second half of the 8th century A.D. It reinforces my main conclusion that Śaṅkara's date must be in the neighbourhood of the 8th century A.D. That can fairly fix his place in the history of Hindu thought, though not exactly. As for exact determination, there are evidences for different dates, and we have to choose the most plausible, about which scholars will naturally differ. Nothing in this has made me change my opinion about what I have given in my thesis about the most plausible date. The extra evidence put forth, viz. that Kamalaśīla and Śāntarakṣita not being aware of Śaṅkara is only negative, and is not sufficient to make me change my conclusion.

ASTRONOMY

I. A Brief Historical Sketch*

The study of astronomy was pursued by the Chinese from very ancient times. Even from the third millennium B.C. we hear of professional astronomers in China, who enjoyed the patronage of the Chinese emperors and who brought the art of astronomical observation to a state of perfection surpassed only by the achievements of modern times. They discovered the sidereal periods of the sun, moon and the planets, and could predict their positions with tolerable accuracy. They discovered the metonic cycle and could even predict the occurrence of eclipses.

The ancient Egyptians were highly interested in astronomy and this interest was partly due to their being 'astrolators.' Plato says that they learnt their astronomy from the inhabitants of the submerged continent of Atlantis, believed by some to have been situated in and beyond the Sahara Desert position.

The Babylonians, who occupied the valley of the Tigris-Euphrates, observed independently all that the Chinese had done, and not stopping with that, they reduced their knowledge to a system. The practice of dividing the zodiac into twelve signs comes from them and it is supposed that they were the first people to give names for the days of the week. It was they who taught astronomy to the Greeks in the seventh and sixth centuries B.C.

To the Greeks should be given the credit for sublimating experience into theory and from their time begins the scientific study of the subject, however crude the

* *The Pudukotah College Magazine*, 1933, pp. 141-45.

basis of that science might be. Aristarchus of Samos who lived in the third century B.C. even attempted the determination of the distance of the sun from the earth by a method theoretically quite sound but having no practical value inasmuch as the errors of observation would be too many to yield any result in the neighbourhood of accuracy. In spite of the fact that he found the distance of the sun from the earth to be twenty times the distance of the moon—which is only a twentieth of the real distance—his very attempt is noteworthy in the history of astronomy. (The Hindu Siddhānta belief was only 13 times).

Next came Hipparchus who did more than anybody else in those days for astronomy. He computed the sidereal, tropical and synodic periods of the moon, the sun and the five planets, Mercury, Venus, Mars, Jupiter and Saturn. The value he found for the obliquity of the ecliptic was only five minutes in error of what it was then. He determined the moon's horizontal parallax. He located the sun's apogee, the position at which it is at its greatest distance from the earth. He discovered the phenomenon of precession by comparing the results of his observations with those of his predecessors, for which he gave the value of 36 seconds per annum, which is not very far from the true value when we consider the shortness of the period of authentic observation preceding him. He found with tolerable accuracy the values for the equation of the centre of the various heavenly bodies. He imagined with his contemporaries that uniform circular motion was the only form of perfect motion. He had to reconcile this idea with the apparent motion of the heavenly bodies, and discovered how their apparent motion can be represented by a series of eccentric circles; and this paved the way for the introduction of epicycles by his successors.

Ptolemy who came about 250 years after Hipparchus wrote the *Almagest* in which he placed the geocentric hypothesis on a firm footing; and the popularity of this work was responsible more than anything else for the lack of progress in astronomical science for several centuries to come. Those who came after him were either his expounders or his commentators.

Meanwhile the Hindus were not behind hand in the pursuit of this branch of knowledge. If in other countries it was curiosity and the desire for knowledge for its own sake, coupled with its use for finding time and guiding ships at sea, that led to a study of this subject, in India there was the additional impetus of the need of this knowledge for fixing the dates of the Vedic ceremonies and sacrifices; and astronomy became one of the 'Vedāṅgas.' As there was cultural contact between the Greeks and the Hindus, there should certainly have been some give and take. But it is wrong to imagine, as some do, that the Hindus borrowed their science of astronomy from the Greeks. The *Vedāṅgajyotiṣa*, the earliest astronomical work we know of, bears no traces of such borrowings. The great scholar and historian, P. T. Srinivasa Iyengar, has pointed out in his essay, 'Our Hellenic debt,' that it was the science of astrology that we borrowed from the Greeks, as the list of the names of the *Pūrvācāryas* or pioneers and the numerous technical terms found in our astrological works show.

Among the Hindu astronomers, Āryabhaṭa who lived at the end of the fifth century A.D. deserves special mention as one who gave the diurnal rotation of the earth as the explanation of the apparent diurnal rotation of the stellar sphere. He should have been a bold man to have said so, for almost everybody would have characterised it as being opposed to 'reason.' Varāhamihira who came after him refuted it in his works,

by an ingenious but fallacious reasoning based on an ignorance of the laws of kinematics; and so did others.

The Hindu astronomers had a knowledge of the theory of the lunar parallax, and by an application of that theory they found that the distance of the moon from us is about 65 times the radius of the earth. If this is the correct value in excess by 9 % it is not because their process of reasoning was faulty, but because their value for the horizontal parallax of the moon was erroneous, combined as it was with the effects of atmospheric refraction. Taking this result together with the geocentric hypothesis and their theory that the speed of the heavenly bodies in their orbits is constant, they tried to determine the distances of these bodies from the earth. Of course, the results were wrong, for the velocity is inversely as the root of the distance. If they had not been wedded to the geocentric hypothesis, they might easily have arrived at the correct values without having recourse to any physical laws; for the seeds of these values are embedded in the *śighraparidhi* of the inferior planets and *madhyaparidhi* of the superior planets given by them, and the problem is one of geometry.

In general the method employed by the Hindus was superior to that of the Greeks, for they had a better grasp than the Greeks, of the mathematical 'instruments' like algebra, geometry and trigonometry.

The 'Ptolemaic' system reigned supreme in the West for about fourteen centuries, and then came Copernicus, an original thinker, who broke away from the trammels of the old system. He taught that the sun is the centre of the solar system and the earth with the five planets Mercury, Venus, Mars, Jupiter and Saturn, moves round the sun while the moon moves round the earth. He did not advance any proof for his new hypothesis other than showing that the geometrical explanation of

the movements of the bodies would be simpler on this assumption, a smaller number of epicycles being sufficient to represent the movements. This simplicity itself may be taken as a proof, for all natural laws are characteristically simple, and any explanation based on a large number of *ad hoc* hypotheses is viewed by scientists with suspicion. But the substantial proof based on the phenomenon of 'aberration' was a long time coming, and Copernicus could only add to what he said an appeal to the emotion of the people by saying how beautiful it would be to have the glorious sun as the centre of the solar system.

His theory met with a good deal of opposition, especially from the church. To save himself from the wrath of the church he declared that his theory was given only as a geometric devise for ease in computation. But slowly laymen as well as scientists began tacitly to recognise its truth. The telescope was invented, better astronomical instruments were made, and better observations taken by Galileo etc. It was on the results of the careful observations of the astronomer, Tycho Brahe, that Kepler, his successor, based his famous laws of planetary motion.

By an instinct which is the privilege of the genius Kepler perceived that if the theory of uniform circular motion was abandoned, matters would become extremely simple. By examining the values given by Tycho Brahe for the position of Mars at different times, and trying hypothesis after hypothesis, Kepler discovered and formulated his laws of planetary motion :

(1) The orbit of a planet round the sun is an ellipse with the sun at one of the foci.

(2) The radius vector, i.e., the line joining the centres of the sun and the planet, sweeps equal areas in equal intervals of time. (It is interesting to note that he

arrived at this truth by a reasoning based on two wrong assumptions the effects of which fortunately cancelled each other).

(3) The square of the periodic time of a planet is proportionate to the cube of its distance from the sun.

Further advance was not possible in the then state of the science of dynamics, the laws of motion not having been clearly formulated as yet. Bold guesses were made that the gravity which manifests itself on the surface of the earth is responsible for retaining the moon in its orbit; and from the earth-moon system it was extended to the solar system. Some even guessed the inverse square law of the variation of gravitational force; but it was left to Newton, one of the greatest scientists the world has produced, for formulating the law of gravitation and giving a formal proof of it from the motion of the heavenly bodies.

Newton stated clearly the three laws of motion known after him; and, basing his reasoning on these laws, he investigated the import of Kepler's Laws. He showed that if Kepler's second and third laws must be true, as indeed they were, the gravitational force retaining the planets in their orbits must be directed towards the sun, and the gravitational force of the sun alone was responsible for this, no tangential force being necessary or admissible. He showed that if Kepler's first law must be true, then the gravitational force should vary inversely as the square of the distance; for it is only the inverse square law that can make bodies describe conic sections with the attracting body at one of the foci, (and an ellipse is a conic section.) He formulated his Law of Gravitation: $F = K \frac{mm^1}{d^2}$, where F is the gravitational force, m and m^1 , the masses of the attracting and the attracted bodies, d , the distance between them and K , a constant depending on the units chosen,

When he tried to verify this law with reference to the motion of the moon, his results were at first not satisfactory, for he did not know the correct radius of the earth, the received value of it being too small by 12 per cent. Later on he heard of a fresh determination of the size of the earth made by Picard in Paris. He made the necessary corrections and found close agreement. Then and then only did he publish the results of his investigations.

Prejudice dies hard even in the scientific world, and his opponents tried to disprove his Law of Gravitation by referring to the inequalities in the motion of the planets and the satellites, to the motion of the apsides and the nodes and to the phenomenon of precession of the equinoxes. Newton showed that the very existence of these is a proof of his law, and if they did not exist his law was not true. Slowly opposition died out, and the discovery of Neptune, whose existence in the very place it was found had been predicted from a study of the inequalities in the motion of Uranus, was a triumph for this Law. Astronomers who came after Newton showed this law to be universal and not merely restricted to the solar system. The method of investigation was perfected by the use of calculus and other branches of higher mathematics, and every observed inequality, except one or two, was tallied with the results of theoretical investigation with complete success.

But there were two disconcerting factors about this law which annoyed the scientists not a little. However much they tried they could not explain the residue of about 42 seconds a century in the angular motion of the apsides of Mercury. Secondly, while all other known phenomena of nature admitted of a mechanical explanation, the Law of Gravitation did not admit of such an explanation without necessitating the inadmissible

assumption of 'action at a distance;' and it remained an isolated fact.

It was left to A. Einstein to prove, on the strength of his 'Principle of Equivalence' and the postulates of the Theory of Relativity, that gravitation itself may be dispensed with and the complicated motion of the heavenly bodies may be due to their inertia and the curved nature of the 'space-time continuum' in the neighbourhood of matter.

From the above we should not conclude that Newton was superseded by Einstein and that his Law of Gravitation is of no use. The results obtained by an application of the law are still a very close approximation to the truth, Relativity giving us the additional security of a knowledge of the limits of error; and, in the words of J. Rice, Einstein stands on the shoulders of the greatest man of science ever born.

THE STARS*

On a clear moonless night every one of us has looked up at the stars and wondered what they are. Excepting a few that are called planets, (literally 'wanderers'), we see they occupy the same relative positions, so much so that we even fancy they form unchanging pictures of animals like the ram, the bull, the crab, the lion, the dog, the scorpion etc., and we name the groups according to the likeness they recall. We also know the stars by their peculiar twinkle—though this is not a sure way of distinguishing a star from a planet because some planets twinkle when they are near the horizon. The stars are small dots, we think, and the use of the most powerful telescope still shows them as mere points.

But really they are huge bodies about the great size of which we can talk but cannot form any idea. Most of them are as big as our sun—indeed they are suns—and many, bigger. But how big is the sun? It will take an express train, travelling sixty miles an hour day and night, seventeen days to go once round the earth along the equator. Such is the size of the earth. But a million earths can be packed into the sun, fancy that! And many stars are vastly bigger than even the sun. Some are so great in size that the whole orbit of the earth round the sun can be placed into them, an orbit whose radius is 93 million miles.

Like the sun too the stars emit their own light and heat and are called 'self luminous.' If we do not get any appreciable quantity of light or heat from them, their distance is responsible for that; for some of them are so

* *The Pudukkottah College Magazine*, 1934, pp. 94-95; 1935, pp. 148-51.

intensely hot that if they are placed at the distance the sun is from us, the whole of the earth will be evaporated away in a trice. Thank God, they are so far away.

The nearest star is at a distance of 25,500,000,000,000 miles, again something meaningless. Let us try to understand this. Vast as the size of the earth is, the speed of light is so great that it can travel more than seven times round the earth in a second. Travelling at this rate it takes about eight minutes for the light of the sun to reach us. That means if the sun goes out at this moment—which God forbid—we shall continue to see the sun for eight minutes more and then alone see it disappear. Now, it takes light about four years ($=4 \times 365 \times 24 \times 60$ eight-minutes) to reach us from the nearest star; which means it is at a distance of about 270 thousand times the distance of the sun. There are stars whose light reaches us after travelling for several hundred years through space. Though we think we see them as they are now, we only see them as they were when the light started from them, i.e. as they were several hundred years ago. If at this moment we transport ourselves to a star at a distance of 2500 light years from us, (i.e. a distance which light travels in 2500 years) and if we provide ourselves with some means of observing what goes on, say in India, from there, we can see India as it was at the time of the Buddha; and if we continue the observation for 2500 years we can 'see' the complete History of India from the time of the Buddha.

Situated at these inconceivably great distances, it is no wonder they look like mere pin-heads, with all their heat and light reduced to a glow-worm's twinkle. It is their distance also that makes us think they are fixed. They are no more fixed than we are, or the earth or the moon is. Many of them are moving about at a rate of several miles per *second*, but yet we see them fixed, they

are so far away. If they are observed at intervals of several years with the help of accurate measuring instruments we can see that they change their positions relative to one another. Even if they have not done so, we cannot be sure they are motionless, for they may be moving along the line of sight. There are ways of measuring even this kind of motion by the application of what is known as the 'Doppler's principle,' which is the same in its nature as the sudden fall in the pitch of the sound of the horn as the railway engine passes us. Scientists have measured the rate of the 'proper' motion of the stars as this is called, and given us the results for several stars. Even the sun, which we know is fixed with relation to the planets has been discovered to be moving at the rate of thirteen miles per second towards the star Vega (அபஜித்) with all its family of planets and moons.

It is known that the stars are not fixed though they appear to be so, that they have their own or 'proper' motion and that the rate of the proper motion has been calculated for several stars. This calculation is possible when their distances are known. Astronomers have found out the distances of several stars and given them in so many millions of miles or in so many radii of the earth's orbit round the sun (this is the most natural unit, as we shall see presently) or in so many light-years.

The method is easy. Thus, to find the height of a tower, or its distance from where you are, without going to the tower or climbing it, the following is the familiar method:—"A man finds the angle of elevation of the top of a tower to be 30° . Walking a hundred yards towards the tower on level ground, he finds the angle of elevation 60° . To find the height of the tower and its distance from the first place of observation." You draw a horizontal line, take a point on it and draw a straight line through it making an angle of 30° . You know the top of the tower is somewhere on

that line. Then you say to yourself. "Let me represent the hundred yards by 4 inches on the plan" and write "scale: 1"=25 yards." You mark a point at a distance of 4" from the original point on the side where the tower is. Through that point you draw a straight line making an angle of 60° on the side of the tower. You know the top of the tower is on this line too. As it is on both the 30° line and the 60° line, it must be at the point of their intersection. From this point you drop a perpendicular on to the horizontal line. That is the tower. You measure the length of the perpendicular and find it to be 3.464 inches which is the equivalent on the plan of $3.464 \times 25 = 86.6$ yards. The length from the first point to the foot of the perpendicular represents the distance required, which you find to be 150 yards.

You see it is possible now. Not only so, this is the only method available for measuring the heights of inaccessible peaks like the Everest; for the barometer or the hypsometer can be used only when we can get at the places of which we want to find the heights. As the method is capable of very great accuracy, it is used even in accessible places. For instance, countries have been surveyed for purposes of map-making not by going about dragging a chain all along, but by measuring a certain distance called 'the base line' with great accuracy and then measuring angles alone. This is known as the Great Trigonometrical Survey.

By observing the sun from two places on the earth its distance can be found out. It is 93 million miles which is the radius of the orbit of the earth round the sun. To find the distance of a star, the star, S, is observed and its position

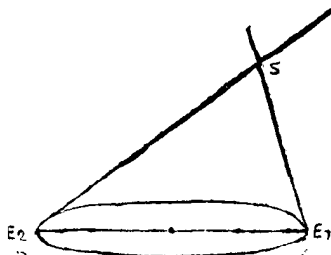


Fig. 1.

accurately found out with the help of a telescope when the earth is, say, at E_1 (see figure 1). After about six months the earth is at E_2 , the other end of the diameter. From there the same star is observed again and its position found out. $E_1 S$ and $E_2 S$ are the lines of sight. The star must be at their point of intersection; and as the diameter $E_1 E_2$ is known to be 2×39 million miles (The radius = 93 million miles) the distance of the star can be calculated. We see here that the diameter of the earth's orbit which is the same thing as saying the radius ($=\frac{1}{2}d$) is the natural unit in which the distance of a star is measured; and even if the length of the radius is not known, there is nothing to prevent us from finding out the distance of the star in terms of the radius.

Once the distance is known, it is easy to find the velocity of the star. Look at fig. 2. A star is observed, say, when it is at A. After some years the same star is observed to be at B. The point from which these observations are made is C. (C may be moving, but we can allow for that and assume it to be stationary.) Angle C



Fig. 2.

is measured. CA, the distance, is already known. If AB, the distance travelled by the star during the interval, is known, the velocity can be found by dividing it by the interval. The motion along AB can be 'resolved' into a motion along AD, perpendicular to the line of sight and a motion along DB along the line of sight. Angle C is small and is measured in seconds of arc, say, (3,600 seconds = 1 degree). The $AD = \frac{CA \times \angle C}{206,265}$.

The next step is to find DB, the motion along the line of sight. It can be found out, as stated, by using the 'Doppler's principle.' Let us try to understand how this can be done. We are standing near a railway-line and a railway-engine is whistling past us. The sound of its whistle has a constant pitch. But as the engine approaches us, more waves of sound reach our ear than

when the engine is stationary. So the frequency of the sound is increased and the pitch is apparently heightened. When the engine is moving away from us the number of vibrations per second decreases and there is an apparent lowering of the pitch. Now, if we can measure by exactly what amount the pitch has changed, we can find the speed of the engine. If this method is not used for finding the speed, it is because other easier and more exact methods are available. But in the case of the star this is very useful. The increase or the decrease from the normal of the frequency of a particular kind of radiation can be measured with the help of an instrument called 'Spectrometer,' and from the amount, the velocity along the line of sight calculated. From A D and D B, A B can be found, for D is a right angle (practically) and $A B^2 = A D^2 + D B^2$.

From the distance also we can find the mass of the star, if it happens to be a real double star. These double stars revolve about their common centre of mass. The distance of the bodies from the centre of mass can be calculated from their known distance from us; and the period of revolution observed. Then their combined mass can be found out by using the formula, $K (M_1 + M_2) = \frac{d^3}{T^3}$ where M_1 and M_2 are the masses, d is the distance from the centre of mass, T is the period and K is a constant depending on the units chosen. If the radius of the earth is chosen as the unit of distance, a year as the unit of time and the mass of the earth as the unit of mass, $K = \frac{1}{330,000}$ and the formula can be written in the form, $M_1 + M_2 = \frac{330,000 d^3}{T^3}$. Then the individual masses can be calculated from their distances from the centre of mass.

THE STRUCTURE OF THE ATOM*

There are two things in the Universe, matter and radiation. Gamma rays, x-rays, ultraviolet rays, light rays, infra-red rays, heat rays and wireless rays are all radiation. Alpha rays, beta rays, cathode rays and the 93 elements with their aggregates are all matter.

What is the distinction between matter and radiation? It was once believed by scientists that matter consisted of particles, and radiation of waves. It is now known that matter too behaves like waves and produces effects like diffraction and interference, characteristic of waves. Radiation, which has long been known to behave like waves, also behaves like discrete particles (photons). The possession or the non-possession of mass cannot be a distinguishing feature, for both matter and radiation have mass. But there is one thing by which we can easily distinguish matter from radiation. While matter can never acquire the velocity of light, radiation has always the velocity of light.¹

All substances are composed of matter. The ultimate constituents of substances as such, are called molecules. They are the smallest particles of a substance, having the properties of the substance. For example, the water molecule is the smallest particle of water

* *Pudukkottah College Magazine*, 1936, pp. 13 ff.

1. We owe this to the Theory of Relativity. According to that theory the equation for the mass of a body in motion, i.e. for the measure of the inertia of the body is $m/(1 - \frac{v^2}{c^2})^{\frac{1}{2}}$, where m is the mass of the body at rest, v is the velocity of the body and c is the velocity of light. The equation shows that as v approaches the velocity of light, m tends to infinity. Therefore no amount of force can accelerate the body so much as to give it the velocity of light.

having the properties of water. It is possible to break the molecule still further; but we no longer get particles having the properties of water, we get two particles of hydrogen and one of oxygen, quite different substances from water.

In general, molecules are built up of such smaller particles. They are called 'atoms,' which means, 'indivisible', for it was believed the atoms could not be broken. 92 different kinds of atoms are known to us at present, and at least one awaits discovery. Each of these represents an element and 92 elements are known to us at present.²

Are the atoms indivisible, as they were believed to be? Are they the ultimate building bricks of the universe? Some fifty years ago scientists put this question to themselves. They *felt* that the atoms could not be the ultimate particles. 93 is too large a number, in comparison with nature's simplicity. Further, a host of fundamental phenomena like the emission and absorption of light, the electrical and magnetic properties of matter, chemical affinity, valency etc. cannot be explained if the atoms are the ultimate particles; for the things to be explained are extremely complex, while the atoms can differ from one another only in their masses, sizes and elastic properties.

Investigation into the electrical properties of rarefied gases led to the discovery that very small particles in the atom are responsible for the conduction of electricity through matter. These were found to be particles of negative electricity and were named electrons. We now see that the atoms are made up of still smaller particles.

2. Till recently it was believed that there were only 92 elements. Fermi an Italian scientist discovered element No. 93 about two years back. No. 85 awaits discovery.

The electrons all carry the same amount of electric charge, 4.8×10^{-10} electrostatic units. On account of their charge, they can be attracted and repulsed by electric and magnetic forces. By subjecting a stream of electrons to these forces we can find the mass, the velocity and (indirectly) the radius of the electrons. All electrons have the same mass, 9×10^{-28} grams, and the same size, a sphere of radius 10^{-13} cm.

The electron is only one sort of the building-bricks of the universe. Why one sort? Why should not the whole atom be built up of electrons? That cannot be, for various reasons. The atom as a whole is electrically neutral, i.e. it is neither attracted nor repulsed by electric and magnetic forces. So, as the electron has a negative charge of electricity, there must be in the atom some other particle or particles having an equal positive charge to neutralize the negative charge. Secondly, as the mass of the electron is only a very small fraction of the mass of the whole atom, and as no atom contains more than 93 electrons, there must be something in the atom in which almost all the mass is concentrated.

This something is known as the nucleus. It forms the core of the atom. It too has been found to be a composite structure. The nucleus of the radium atom explodes spontaneously and shoots out helium ions. It can also be broken up by artificial means. From these it has been found that the nucleus is built up of particles called protons and neutrons. They are similar to the electrons in shape and size. But they are far heavier. Each proton and neutron has about 1840 times the mass of the electron. The proton carries a positive electric charge equal in quantity to the charge of the electron. It can be called a particle of the positive electricity. The neutron is electrically neutral, hence the name. It is now believed that these three kinds of particles, the

electron, the proton and neutron are the bricks out of which all the matter in the universe is built up.³

We shall now see how the different kinds of atoms are built up. Let us take them one by one in the order in which they occur in the Periodic Table of the

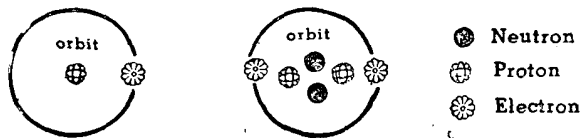


Fig. 1. Hydrogen atom.

Fig. 2. Helium atom.

elements. The simplest is the hydrogen atom, Fig. 1. It has a nucleus of one proton. To neutralize its single positive charge it has one electron moving round it. The next element is helium, Fig. 2. It has two electrons revolving round its nucleus. So it must have two protons in the nucleus to neutralize the charges. But the two protons can give the atom only an atomic weight 2. The atomic weight of helium is 4. So there must be 2 neutrons in the nucleus. Thus the nucleus has the positive charge 2, and the weight 4. The next element has 3 electrons revolving round the nucleus. So its nucleus contains 3 protons which can give it an atomic weight 3. The rest of the atomic weight is supplied by the neutrons. We find each successive element in the Periodic Table has one electron and one proton more than the previous one. If we take the N^{th} element, and if its atomic weight is Z , we see it has N electrons revolving round a nucleus containing N protons and $(Z-N)$ neutrons. The number of protons (or electrons) each atom has is the atomic number. The atomic

³ Two other particles, the positron and neutrinos are said to exist. The former has been discovered and the latter has been postulated to explain certain facts. Particles known as mesotrons, several times heavier than the electrons, are said to exist in the cosmic rays.

number of the elements increases one by one in the order of their arrangement in the Periodic Table. As the arrangement is according to the chemical and other properties of the elements, we see at once that the atomic number is the fundamental thing and the outer electrons may in some way be responsible for the chemical behaviour of the elements.

There was a time when scientists thought that the atomic weights were fundamental, on account of the almost perfect agreement between the arrangement according to the atomic weights and that the Periodic Table. So they were disconcerted when they found disagreement in the case of Argon, Cobalt, Iodine and Thorium. They have now found out that the atomic weights do not matter and it is the atomic numbers that count.

What is it that binds the electrons to the nucleus and makes them revolve round it in orbits? The electrons possessing negative charges are attracted by the protons in the nucleus, having positive charges. It is this attraction that keeps them in their orbits.⁴

There is a close and striking similarity between the atom and the solar system. The atom is a miniature model of the solar system. In the place of the sun is nucleus. In the place of the planets are the electrons which revolve round the nucleus in orbits as the planets revolve round the sun. The only difference is the nature of the forces keeping the planets on the one hand and the electrons on the other, in their orbits. The one is gravitational and the other, electrical.

⁴ It may be thought that the gravitational attraction between the electrons and the nucleus is the force keeping the electrons in their orbits. But the gravitational force is nothing when compared with the electrical force which is 2.3×10^{39} times as great as the gravitational force.

In another respect also there is similarity. The solar system is practically empty of matter when we consider its size, the planets forming a few particles of dust, and the sun, a small pellet of stone, on a vast field. It is so with the atom. Let us picture the atom which has a radius of 10^{-8} cm, as a football. The electrons, protons and neutrons having a radius of the order of 10^{-13} cm would each be a thousandth of a millimetre across; and there are not more than about 330 of these invisible specks even in the densest atom. All but a millionth of a millionth of the whole space occupied by the atom is empty. If all the matter composing the body of thousand men is collected together, it would not be sufficient to fill a mustard seed. How small we are! Yet how great, being able to comprehend the whole universe!

NEWTON AND THE LAW OF GRAVITATION*

Introduction

Newton was the formulator of the Law of Gravitation, on which is based the edifice of modern astronomy. That we may have a better appreciation of his genius we shall examine the work of his immediate predecessors, Tycho Brahe and Kepler; and that we might grasp the true import of his discovery, we shall examine the uses this law is put to and also how far Einstein's Theory of Relativity affects the importance of the law.

Tycho Brahe was a Danish mathematician and astronomer of the time of Queen Elizabeth. He came after Copernicus who was the first man in Europe after the Greeks to assert that the sun is the centre of the Solar System and that the planets revolve round it, with the satellites moving round them. But Tycho did not accept what all he said. He propounded, instead, that the earth was fixed and the sun, with the other planets going round it, revolved round the earth. This is known as the Tychonian System.

His services to astronomy consist chiefly in his accurate observations of the positions of the stars and the planets, made with the telescope just coming into use. But for his observations, Kepler, who was his assistant for sometime and who succeeded him in the field of astronomy, could not have formulated his laws governing the movements of planets. Kepler used his observations of Mars for his purpose, for the orbit of Mars has the greatest ellipticity next to that of Mercury, which itself could not be used owing to observational difficulties.

* From *Pudukottah Raja's College Magazine*, 1938, pp. 19-24,

As we have seen, Kepler was a junior contemporary of Tycho Brahe and was his assistant for some time. After Tycho's death he succeeded him as Royal Astronomer to the German Emperor and completed his work, the construction of a set of astronomical tables. Before his time the calculation of the position of a planet was a very difficult affair. It was thought that perfect motion meant moving in circles and astronomers believed, on metaphysical grounds, that heavenly bodies must have this kind of motion alone. As their theory did not agree with actual observation they said that the bodies moved in circles the centres of which moved on the circumference of their circles, the centres of which moved on the circumference of still other circles and so on. The explanation is ingenious, and the beauty of it is that it can yield the desired result. In its nature it is only a geometrical representation of the development of a function in series. But in the case of planets with orbits of great eccentricity a very large number of these circles will be required, which will render the calculations laborious.

Kepler felt—herein lies his genius—that nature's ways are not so involved and elaborate as this. Nature's laws are characteristically simple, and he wanted to simplify the theory of planetary motion. He tried hypothesis after hypothesis and at last arrived at the truth that the orbits of planets are ellipses with the sun at one of the foci. Of course, the ellipse includes the circle as a particular case, the case where the eccentricity is zero. Next he discovered that the radius vector, *i.e.*, the straight line joining the sun and the planet sweeps equal areas in equal time. Long after he discovered this law he discovered his third law that the square of the periodic time of the planet's revolution round the sun is proportionate to the cube of its distance from the sun.

Newton's Work

Kepler arrived at these laws empirically, and incidentally they proved true. He did not know the why of them. It was left for Newton to formulate the Law of Gravitation, and demonstrate mathematically that Kepler's Laws are a result of the Law of Gravitation, and as such, a proof of the law. Not that people did not know anything of gravitation before Newton. For the matter of that, from very ancient times people have explained the falling of bodies towards the earth as being due to the pull of the earth on the bodies. Bhāskarāchārya, the famous Indian astronomer, in discussing how the earth stands poised in space, says that the earth pulls bodies on its surface, that 'towards the earth' is 'down' and therefore the question why the earth does not fall down is meaningless. Kepler had a vague idea that gravitation is responsible for maintaining the planets in their orbits and making them follow his three laws. But he did not know what exactly was the amount the effects which he supposed must be attributed to it. Some had even surmised, before Newton's days, that gravitation varies inversely as the square of the distance. A friend of Newton's, Halley, actually proposed to him the problem, how a body moving round another under the influence of the inverse square law would behave. Newton had already solved the problem and was ready with the answer: the moving body would describe an ellipse, a parabola or a hyperbola relative to the central body which itself would be at a focus.

Newton's contribution to science in this connection is the exact statement of the Law of Gravitation, first with reference to the pull of the earth and afterwards generalised to include all matter. Some of his predecessors had guessed that the moon is kept in its orbit by gravity which is the same in its nature as the gravity acting on the surface of the earth. But they did not

care to verify if it was so; they were not sure of their ground. Newton saw that without being sure of the laws of motion these astronomical problems could not be solved with any degree of certainty. Some spade work had been done in this direction by Galileo. Newton continued his work and clearly stated his three laws of motion, familiar to every student of science. Then, on the supposition that the acceleration due to gravity varies inversely as the square of the distance, he proceeded to verify if the moon is kept in its orbit by the pull of the earth. He knew the amount of the pull on the surface of the earth, 32 feet per second per second. He knew the distance of the moon from the earth in terms of the radius of the earth (the radius of the earth is the natural unit in measuring the distance of the moon). At first he did not know the correct value of the radius of the earth, the accepted value being in error by 12%. This gave an error proportionate to this in the result. Any other man but Newton would have been baffled at this and discarded the thing as hopeless. But Newton felt that there might be an error in the value of the radius of the earth. Happily for him, a fresh determination of the radius was being made in France, and when the result reached him he tried a fresh verification with complete success.

The next step was to extend the law to the Solar System as a whole. Here the sun is the attracting body and the planets are the bodies whose motions are to be investigated. By a process of pure mathematical reasoning he arrived at the conclusions arrived at empirically by Kepler. Thus he supplied the why of Kepler's Laws. He showed that if there is no other force acting on a planet except the central force due to the attraction of the sun, Kepler's second law must hold good, whatever be the nature of the central force. The second law is, as stated earlier, that the radius vector sweeps equal areas in equal time. Provided the body is accelerated only

towards the sun, never mind whether the acceleration is inversely proportionate or directly proportionate or anything else, it must sweep equal areas in equal time.

The first law of Kepler, as we have stated, is: (a) that the orbit is an ellipse, and (b) that the sun is at one of the foci. Newton showed that elliptical orbits can be described under two kinds of forces, a force proportionate to the distance, and a force inversely proportionate to square of the distance. But if the attracting body is to be at one focus, the force must be inversely proportionate to the square of the distance. Thus the first law of Kepler is a proof, by verification, of the law of gravitation formulated by Newton.

What is the significance of Kepler's third law, viz., the square of the periodic time is proportionate to the cube of the distance? Newton theoretically arrived at the following result: $T^2 = 4\pi^2 d^3 / \mu$. If as Kepler states $T^2 \propto d^3$, $4\pi^2 / \mu$ must be a constant. $4\pi^2$ is obviously, a constant, and μ is the mass of the attracting body multiplied by the gravitational constant (if we neglect the mass of the attracted body, as we can do in the case of the planets). If this is also to be a constant, the attracting body must be the same for all planets, as indeed it is, viz., the sun. Thus Newton showed that the Third Law of Kepler is a proof that the sun is the parent, as it were, of the planets revolving round it.

If the Law of Gravitation holds good in the earth-moon system and in the Solar System, there is no reason why it should not be true beyond the Solar System, in the region of the stars. Better telescopes were required before this could be verified and Newton could not do it with the crude telescopes (crude in comparison with the modern telescopes) of his days. Later on double stars were discovered which revolve round each other obeying Newton's Law of Gravitation. Thus the law has been

found to be universally true and it can be stated as follows:— Every particle of matter attracts every other particle with a force proportionate to the product of the particles and inversely proportionate to the square of their distance from each other [$F = Gm_1 m_2/d^2$]. Newton thus laid the foundations of modern astronomy.

Prejudiced and jealous scientists of Newton's days tried their best to disprove the law. They pointed to the inequalities in the motion of the moon and said they should not exist if the law was true. Newton said that his law required the existence of the inequalities. They would not exist if there were only two bodies, the attracting and the attracted. But there is a host of other bodies, besides, which must affect the body in motion, however small the effect is. Newton actually worked out the major inequalities in the motion of the moon called 'evection' and 'variation'. Gradually all opposition died out and the law was recognised by one and all.

The Law of Gravitation in use

A triumph for this law was soon to follow. The planet Uranus was discovered and the elements of its orbits were calculated. From these elements and from the equations for the perturbations of the planet by others the future positions of the planet were calculated. It was seen to occupy a position slightly different from the predicted position. Either the law has failed or there is some other planet not discovered yet, disturbing the planet. Astronomers calculated where the new planet must be found if it existed. Powerful telescopes were directed towards that spot in the heavens, and lo! there it was, hitherto mistaken for a star; it was christened Neptune. The discovery of the planet Pluto later was another triumph for the law.

The Law of Gravitation has enabled scientists to determine the mass of the earth. The pull of the earth

on a known mass is compared with the pull of another known mass on the first mass and as the pulls are in proportion to the masses, the earth's mass can be found out. It is about 6×10^{21} tons. From that the mean density of the earth is calculated to 5.5 grams per c.c.

The determination of the exact shape of the earth has been made by using this law. The equation for the time of oscillation of a pendulum is $t^2 = \pi^2 l/g$, where t is a half-vibration period, l is the length of the pendulum and g is the acceleration due to gravity at the place. Fixing l , we find t^2 varies inversely as g , and by measuring t we can find the acceleration at the place. The acceleration will give the distance of the place from the centre of the earth. Many such determinations will give the shape of the earth.

By using the law we can theoretically determine whether there can be an atmosphere on the moon (or any other body). There is a critical velocity connected with every body, exceeding which an object on the body will leave it altogether overcoming the gravitational pull of the body. It is given roughly by the equation $v = 7\sqrt{\frac{m}{r}}$, where v is in miles, m is the mass of the body (taking the earth's mass as unity) and r is the radius of the body (taking the earth's radius as unity). For the earth it is about 7 miles. For the moon it is $7\sqrt{\frac{1}{81} \div \frac{1}{4}} = 1.5$ miles, roughly. Now, the Kinetic Theory of Gases says that all gases consist of molecules moving about at speeds depending on the molecular weight of the gas and the temperature. At the temperature that obtains on the moon, all gas molecules must exceed the critical velocity of 1.5 miles and fly off into space.

Conclusion

After the advent of the Theory of Relativity there is a tendency among some people to minimise the signi-

ficance of Newton's work. The Theory of Relativity has shown that Newton's Law of Gravitation is only an approximation to the truth. But the approximation is so close that Newton's Law suffices for all purposes of astronomy. Colossal as the masses of the astronomical bodies are; the closer formula of Einstein gives practically no measurable difference. It is only in the case of the motion of the apsides of Mercury that an accumulated difference of 42" per century is detectable. Also, Newton's Law is simple in form and easy to apply. Thirdly, but for Newton Einstein cannot have propounded and verified in a determinate manner the Theory of Relativity at all, for Newton's work is a necessary step towards the progress achieved by Einstein. The very solution of Einstein's gravitational equation is effected by taking Newton's equation as a first approximation.

We can compare Newton with Euclid in this respect. Geometries rival to Euclid's have sprung up, the Hyperbolic Geometry of Johann Bolyai and Lobatschewsky and the Elliptic Geometry of Riemann and Hilbert. It is also probable that the geometry of our space is elliptic. But does it mean we can discard Euclid's geometry? The geometry of Euclid suffices for practical purposes. It has the advantage of being simple; simple in its nature as a particular case of those geometries, and also because of our mental habits and the kind of intuition we have of space—flat. Euclid will live, as also Newton.

*THE EVOLUTION OF THE UNIVERSE ACCORDING TO SIR JAMES JEANS**

Before seeing how the universe has been evolved, we shall understand what constitutes the universe. The primitive man thought that his immediate surroundings with the things in them formed the universe. Gradually, as his sphere of activity increased and knowledge expanded, the Earth came to be supposed the centre and the most substantial part of the universe. The Earth, with the celestial sphere just reaching beyond it on all sides with the Sun, the Moon, the planets and the stars fixed on to the sphere, was thought to be the entire universe.

As knowledge increased man learnt that the earth is only one of the many planets moving round the sun. First there is Mercury (Budha) nearest to the sun, at a distance of 36 million miles. It is about a sixteenth of the size of the earth. Next comes Venus (Śukra) at a distance of 67 million miles from the sun. It is almost the same size as the Earth. Next to Venus comes the Earth, which is at a distance of 93 million miles from the sun. Its radius is about 4,000 miles and mass, about 7×10^{31} tons. It has one satellite, the Moon. Beyond the Earth is Mars (Kuja) at a distance of 142 million miles. It is small, compared with the Earth, being about a seventh in size. But it has two moons. Beyond the orbit of Mars a vast number of bodies, only a few miles in diameter, called the asteroids, move round the Sun. Next comes Jupiter, (Guru) the Giant

* *Pudukkottah Raja's College Magazine*, 1940, pp. 4 - 7.

** A new theory has been propounded now after exposing the draw backs in this theory, and the new theory is now generally accepted.

Planet. It is the biggest planet and can contain more than a thousand Earths. It has 9 moons. Its distance is 483 million miles, more than five times that of the Earth. Next there is Saturn (Śani) almost the size of Jupiter, at a distance of 886 million miles. It too has nine moons. It has a beautiful belt of 'rings,' really small stones close together moving round in a belt. Next comes Uranus and Neptune, of size about 60 times and distance about 19 and 30 times that of the Earth. Uranus has 4 moons and Neptune, one. About ten years back a new planet was discovered to which the name Pluto was given. It is at a distance of 40 times that of the Earth and has a period of revolution of 250 years. Besides these planets there are a large number of comets and swarms of meteors, many of them mere pieces of stones, moving round the Sun. The Sun is the parent, as it were, of all the planets and the grand-parent of the satellites. All these constitute the Solar system.

Yet the Sun is only one of the millions of stars that form the star-city—a galaxy as it is called—to which we belong. The number of stars in our galaxy is a little over a hundred thousand million. This is the approximate number of stars in each galaxy. From this we should not think that the galaxy is crowded. Even the Solar system is practically empty of matter when we consider its size, the planets forming a few particles of dust, and the Sun a small pellet of stone, on a vast field. The galaxy is emptier still. It takes 8 minutes for light to reach us from the Sun, but 4 years to reach us from the nearest star. If this gives the mean distance between star and star, we can have an idea of the emptiness of a star-city, in spite of its hundred thousand million stars. Such great emptiness can be compared only with the emptiness of what we call matter, which, a great physicist says, is all holes with nothing between.

If we leave our star-city extending over a space of two hundred thousand light-years (a light-year is the distance which light travels in one year), we must travel four times the extent of our city to reach the nearest star-city; and there are millions of star-cities in the universe. This does not mean they are infinite in number. Space, which is co-extensive with the universe, is finite. There is no meaning for scientists to ask what is beyond, for the question does not arise. [If we think it is infinite, we have erred in our judgement, as the primitive man erred in his judgement as regards the shape of the Earth and conceived it flat. Experience only tells us that space must be unbounded, and the assumption that it is also infinite is unwarranted. Infinitude belongs to measure relations, while unboundedness belongs to extent relations; and experience confirms the latter and not the former.] The radius of the universe (space) is 10^{13} times the radius of the Earth's orbit.

We shall try to picture the universe. If the Earth represents the universe, the towns on the Earth may represent the star-cities. The stars that are the citizens of these star-cities are the size of a mustard seed, about a hundred thousand million in each town. If the star is a Solar system—the chance for this is very little—the planets, on our model, are specks of dust, a few in number, placed near the seed at a distance of a hair's breadth. And where are we men on one of the smallest of the specks of dust? If we can visualise this, we have obtained an excellent cure for our megalomania.

Let us now see how this universe, as it is at present, has been evolved. Was it ever so, or was there a time when the different orbs in it did not exist? There was. We have travelled through space and seen its extent. Let us now travel through time, backwards, and see what the universe was millions and millions of years ago,

It was a vast globe of incandescent gas. What it was before that, we cannot even guess. The particles of the gas were moving to and fro with enormous speeds, now attracted, now repelled, by one another, but in general attracted by gravitation. Ages rolled on, and the gas, at first evenly distributed throughout space, condensed and formed smaller globes of gas. Each globe contained the material for a star-city, the material for about a hundred thousand million Suns. Further condensation took place. At the same time the globes began to rotate on account of gravitational attraction between the particles. Each globe became flattened like an orange. The flattening increased, at the same time condensation taking place in the globe; and individual stars began to appear. The degree of flattening represents the age of the gas-globe, the nebula, as it is called. The more flattened the nebula is the more condensed it is, and the more the number of stars in it.

One such flattened nebula is our galaxy. The sun is at a distance of a third of the radius of the galaxy from the centre, and takes part in the rotation on the galaxy. It is this rotation that keeps the stars from coming together at the centre by mutual attraction. Incidentally, it is this rotation that enables us to calculate the mass of a galaxy.

Then millions of years passed, and a strange thing happened about 3,000 million years ago. Our Sun then was a young maiden, as it were, brighter and more glorious than it is to-day. It had a companion, another star, a maid in attendance. They had no work to do, and spent their time in chasing each other round. At the period I referred to, a new star, during his wanderings happened to go near them. The attraction of the stranger was too strong for our Sun maiden to resist; and the ultimate result was that the planets were born,

This is how it happened. As the new star approached the Sun, great tidal forces were set up in it and it became elongated towards and away from the new star. But the elongated mass could not fall back when the new star receded, because of the velocity of the mass, and it ultimately condensed into the planets. The star moved away after generating the planets, the Sun's old companion eloping with him.

This is the story of the birth of a planet. But such celestial matrimony is rare, in spite of the great number of stars in the universe and the long, long period of time elapsed, because space is vaster still. "Leave ten bees on the continent of Asia at different places. What chance is there of their meeting each other? There is the same chance for stars to visit one another."

Let us continue the story. The planets were globes of gas at first. As they cooled they became liquid and then a solid crust was formed on the surface. On one of these life originated; and by gradual evolution man came upon the earth.

Rare as a Solar system is, a planet with life on it is rarer still; for special conditions are necessary for life to exist. In the Solar system, it is now evident that the Earth alone has life on it, life as we know it. It is just at the range of temperature within which life can exist. The other planets are either too cold or too hot. Mars perhaps has very low forms of life.

One day the Earth too will become a cold, desert planet like many others, unable to support life. In an insignificant corner of the great universe man will have existed for an infinitesimal fraction of time and vanished.

THE DURATION OF ECLIPSES*

Section 1: Lunar Eclipses

The total duration of a lunar eclipse is given in hours, by the formula $2 \left[D^2 - P^2 \left\{ 1 - \dot{p}^2 / (\dot{p}^2 + \dot{m}^2) \right\} \right]^{\frac{1}{2}} / (\dot{p}^2 + \dot{m}^2)^{\frac{1}{2}}$ (1), where D is the distance between the centres of the moon and the shadow at first or last contact, P is the latitude of the moon at the time of opposition of the sun and the moon in longitude, \dot{p} is the increase in P per hour and \dot{m} is the motion per hour in longitude of the moon, relative to the sun.

This is clearly 0 when P^2 etc. $= D^2$

i. e., when $\pm P \left\{ 1 - \dot{p}^2 / (\dot{p}^2 + \dot{m}^2) \right\}^{\frac{1}{2}} = D$, i. e., when P is numerically greater than D by $D \dot{p}^2 / 2(\dot{p}^2 + \dot{m}^2)$ approximately. This comes to about $14''$ on the average. Thus it is wrong to say that there is no eclipse when P is greater than D , for even when P is within about $14''$ greater than D at opposition we have an eclipse. When $P=D$, the duration of the eclipse is not 0, but $2P\dot{p} / (\dot{p}^2 + \dot{m}^2)$, which is about 22 minutes, the eclipse commencing or ending at opposition.

When is the duration a maximum?

Clearly when P , the latitude of the moon at opposition is 0, i. e., the maximum occurs when at opposition the sun and the moon are also at the nodes. It is equal to $2D / (\dot{p}^2 + \dot{m}^2)^{\frac{1}{2}}$ hours.

But D , \dot{m} and \dot{p} are functions of l , and l' , the moon and the sun's mean anomalies, respectively. Therefore

* *Pudukkottai Raja's College Magazine*, 1949, pp. 24 ff.

the maximum duration itself varies between limits, whose maximum and minimum we shall evaluate now.

In the neighbourhood of the sun and the moon's conjunction or opposition in longitude near a node, we have the following equations retaining functions of l, l' alone, where l and l' are the moon and sun's anomalies at *sthūla-parva*. (This is the time of a fictitious conjunction or opposition with the true moon = mean moon + $315' \sin l$, and the true sun = mean sun + $127' \sin l'$)

The moon's Equatorial Horizontal parallax

$$\pi'' = 3447''.9 + 224''.4 \cos l.$$

The sun's Do. Do. parallax $\pi' = 8.8'' + 2'' \cos l'$

The moon's semi-diameter $r = 939''.6 + 61''.1 \cos l$

The sun's semi-diameter $r' = 961''.2 + 16''.1 \cos l'$

The radius of the shadow $s = 2545''.4 + 228''.9 \cos l$
 $- 16''.2 \cos l' (\dot{m}^2 + \dot{p}^2)^{\frac{1}{2}} = 1875''.6 + 260''.1 \cos l$
 $- 5''.0 \cos l'.$

Now the distance between the centres of the moon and the shadow at first or last contact = $D = s + r = 3485.0 + 290.0 \cos l - 16.1 \cos l'$. Therefore $2D / (\dot{p}^2 + \dot{m}^2)^{\frac{1}{2}} = 2(3485.0 + 290.0 \cos l - 16.1 \cos l') / (1875.6 + 260.1 \cos l - 5.0 \cos l')$. This is a maximum when $l = l' = 180^\circ$ and not when $l = 0$, though it makes the semi-diameters of the shadow and the moon a maximum, for the increase in the numerator is outweighed by that in the denominator. Thus the max. is $2(3485 - 290 + 16.1) / (1875.6 - 260.1 + 5) =$ about 238 minutes. The lower limit occurs when $l = l' = 0$ and it is, $2(3485 + 290 - 16.1) / (1875.6 + 260.1 - 5)$

hours=about 212 minutes. If we do not neglect functions of 21 the maximum is 237.4 minutes.

The maximum duration of the total phase of a lunar eclipse can be found by making $D=s-r$, and finding the maximum value. This too occurs when the sun and the moon at opposition are at the nodes, and when $l=l'=180$; and it is $2(1605.8-167.8+16.4)/(1875.6-260.1+5)$ hours=about 108 minutes.

Section 2 : Solar Eclipses

The formula for the duration of a solar eclipse in general on the earth (as opposed to the duration at any particular place) is the same as for the duration of a lunar eclipse, with this difference; that here $D=\pi-\pi'+r+r'$, and p is the latitude of the moon at conjunction of the sun and moon in longitude. Here too, as in the case of the lunar eclipse, we can see that the duration is 0, not when $\pm p = D$, but when P is numerically greater than D by about $20''$. When $\pm P = D$ the duration is about 33 minutes.

The maximum duration of a general solar eclipse occurs when $P=0$, i.e., when the conjunction in longitude is at a node. It is given by $2D/(\dot{p}^2+\dot{m}^2)^{\frac{1}{2}}$, in hours= $(5339.9+285.5 \cos l+15.9 \cos l')/(1875.6+260.1 \cos l-5 \cos l')$ hours. This is a maximum when $l=180^\circ$ and $l'=0$ and not when $l=0$, though that makes both the parallax and the semi-diameter of the moon a maximum, because the increase in the numerator is outweighed by the increase in the denominator. It is $2 \times (5339.9-285.5+15.9)/(1875.6-260.1-5)$ hours = 6 hours 18 minutes nearly. It is easily seen that under this condition the eclipse is annular. Therefore the maximum duration on the earth as a whole happens at an annular eclipse. (If we do not neglect 21 , the maximum is 6 hours 16 minutes).

The duration of a solar eclipse at a given place on the earth is given by $(r + r') / (\dot{p}^2 + \dot{m}^2)^{\frac{1}{2}}$ corrected for parallax, which changes rapidly and varies from place to place. So the work of finding it is a bit difficult. But the maximum duration is easy to determine. This occurs when the central eclipse is at apparent noon. At this time the apparent semi-diameter of the moon is $r + 16''$. Also, the nearer to noon, the greater is the retardation in the relative hourly motion of the moon, owing to parallax, with the result that the greater is the increase in the duration of the eclipse. For an hourangle of 34° on both sides of noon the average retardation is $850''.3 + 55''.4 \cos l$ per hour.

The total duration is given by $2(r + 16'' + r') / \{(\dot{p}^2 + \dot{m}^2)^{\frac{1}{2}}\} -$ (the hourly retardation due to parallax). So the maximum duration is given by $2(1917 + 61 \cos l + 16 \cos l') / \{(1875.6 + 210 \cos l - 5 \cos l') - (850.3 + 55.4 \cos l)\} = 2(1917 + 61 \cos l + 16 \cos l') / (1025.3 + 204.7 \cos l - 5 \cos l')$, when $l = 180^\circ$ and $l' = 0$. The maximum, we can easily calculate is about 4 hours 35 minutes, and it occurs when there is a combination of the most favourable circumstances, viz. the conjunction occurring at a Node, the central eclipse falling at noon, and l being equal to 180° and l' , 0.

The maximum duration of the Annular or Total phase at a given place is also at apparent noon for the same reason. As the period is very short we shall take the motions per minute for purposes of calculation. The duration of an Annular eclipse near noon is given by $2(r' - r - 16) / \{(31''.3 + 4''.3 \cos l) - (15'' + 1'' \cos l)\} = 2(5.7 - 61 \cos l + 16 \cos l') (16.3 + 3.3 \cos l)$. This is a maximum when $l = 180^\circ$, $l' = 0$. Calculating, we

can find it to be about 13 minutes. The minimum, of course, is 0, as is patent from the formula.

The Total phase is given by $2 (r+16''-r') / \{ (31''.3 + 4.3 \cos l) - (15' + l'' \cos l) \} = 2 (61.1 \cos l - 16.1 \cos l' - 5.7) / (16.3 + 3.3 \cos l)$. This is a max when $l=0$ and $l'=180^\circ$; whose value is $2 \times 71.5 / 19.6 =$ about seven minutes. The minimum is clearly 0. Comparing the results we find that, other things being equal, an annular eclipse has in general a greater duration than a total eclipse.

THE LUNAR ECLIPSE IN HINDU ASTRONOMY*

1. Introduction

It is a well-known fact that a lunar eclipse occurs when, in the neighbourhood of a moon's node, the sun and moon are in opposition, i.e., when the moon and the earth's shadow are in conjunction. At the time of such a conjunction, say T , let p be the latitude of the moon (north latitude being considered $+$), p' , the hourly increase in latitude (increase towards the north being considered $+$), m' , the excess of the hourly increase in longitude of the moon over that of the sun, M , the angular radius of the moon, and S the angular radius of the shadow, at the moon. Then, at any time t hours after the time of conjunction, T , the distance between the shadow and the moon, in longitude, is $m' t$ and the latitude of the moon is $p + p' t$; and so the distance between the centres of the shadow and the moon is $\{ m'^2 + (p + p' t)^2 \}^{\frac{1}{2}}$. The eclipse begins or ends when the moon's rim just touches the rim of the shadow in entering it or leaving it.

The distance between them at such a time is $S + M = D$, say. Then $\{ m'^2 t^2 + (p + p' t)^2 \}^{\frac{1}{2}} = D$ gives the time of the beginning or end of the eclipse. Solving this for t , we get. $t = -pp' / (m'^2 + p'^2) + \{ p^2 p'^2 / (m'^2 + p'^2)^2 + (D^2 - p'^2) \}^{\frac{1}{2}}$; in which, obviously, the upper sign gives the beginning, and the lower, the end of the eclipse. The total phase of the eclipse begins or ends when the rims touch the moon being inside the shadow, i.e., when the

* *Pudukkottai Raja's College Magazine*, 1951,

distance between them is $S - M$. If we substitute this instead of $S + M$ for D , in the above solution, we get the times of the beginning and ending of the total phase of the eclipse.

Thus we arrive at the following results :

- (1) The eclipse begins at $T - pp'/(m'^2 + p'^2) - \left\{ p^2 p'^2 / (m'^2 + p'^2)^2 + (D^2 - p^2)/(m'^2 + p'^2) \right\}^{\frac{1}{2}}$, hours.
- (2) The eclipse ends at $T - pp'/(m'^2 + p'^2) + \left\{ p^2 p'^2 / (m'^2 + p'^2)^2 + (D^2 - p^2)/(m'^2 + p'^2) \right\}^{\frac{1}{2}}$, hours.
- (3) The middle of the eclipse falls at $T - pp'/(m'^2 + p'^2)$, hours. From this we see: (a) that if p and p' are both positive or both negative, the middle of the eclipse is before the time of conjunction, and if one is positive and the other negative, the middle is after conjunction; (b) only when the latitude at conjunction, p , is 0, the middle falls at T , the time of conjunction, (for p' cannot be 0 near a node).
- (4) If $D = p$, (1) and (2) reduce to $T - pp'/(m'^2 + p'^2) \mp pp'/(m'^2 + p'^2)$, from which we see: (a) that the eclipse begins or ends at conjunction, and (b) that the duration is $2pp'/(m'^2 + p'^2)$, which may amount to about 22 minutes.
- (5) The duration is 0 when the expression between the double brackets is 0, i.e., when p is greater than D by $Dp'^2/2(m'^2 + p'^2)$, (neglecting fourth powers of p'/m'); which may amount to about 14'' in the mean.
- (6) If t is not real, there is no eclipse or total eclipse according as D is taken to be $S + M$ or $S - M$.

For e.g., let us take the eclipse that is going to occur on the 11th February 1952. The shadow and the moon

will be in conjunction in longitude at 5.58 a.m. (I. S. T.) $p = +3083''$, $p' = -164''$, $m' = 1641''$, $S = 2339''$, $M = 888''$. Using these we get $t = +11$ minutes ∓ 36 minutes, i.e., the middle of the eclipse falls at 6-9 a.m. The eclipse begins at 5-33 and ends at 6-45. Putting $D = S - M = 1451''$, t is not real, and therefore there is no total phase.

2. Calculation of the Lunar Eclipse in Hindu Astronomy

So far as we know, the *Vedāṅgajyotiṣa* (about 1220 B.C.) is the most ancient Hindu astronomical work. Two recensions of it are extant, the Rigvedic and the Yajurvedic, neither of which makes any mention of eclipses; as also the Jain and Buddhistic works the *Sūryaprajñapti*, the *Jyotiṣkaraṇḍa* and the *Kālāloka-prakāśa* and also the Brahminical works like the *Pitāmaha-Siddhānta*, which all came a few centuries later.

The next period forms a transition from this to the time of the advanced siddhāntas of later times. Three of the siddhāntas condensed by Varāhamihira in his *Pañcasiddhāntikā*, viz., the Vāsiṣṭha, the Pauliśa and the Romaka belong to this period. In them we find the beginnings of an attempt at finding the true longitudes of the sun and the moon and calculating eclipses. Varāhamihira's Romaka does not give the rules for calculating a lunar eclipse, though the more difficult calculation of the solar eclipse is given. The methods of the Vāsiṣṭha and the Pauliśa are mixed up. This is what they say in effect. Rule (1). Find the difference n in longitude at the time T of the moon's conjunction with the shadow, between the near node and the moon. If n is within 13° , there is a lunar eclipse. (2). $T \mp \frac{3}{20} \sqrt{169 - n^2}$ hours gives the beginning and end of the eclipse. (3). Find the latitude of the moon p at that time, in minutes. Then, if m' is the difference in the hourly

motion in longitude of the sun and the moon, $T \mp \sqrt{55^2 p^2} / m'$ hours gives the beginning and the ending of the eclipse. (4). $T \mp \frac{21}{5m} \sqrt{25 - n^2}$ hours gives the beginning and end of the total phase.

From (1) and (3) we see that 13° difference between the longitudes of the node and the moon is equivalent to $55'$ of latitude: i.e., near a node the latitude changes at $55/13$ minutes per degree and the maximum latitude is 4° . From (3) we see $S+M=55'$; from (4), $S-M=21'$; and so according to these siddhāntas, $S=38'$ and $M=17'$ which are taken to be constant. (2), (3) and (4) neglect p' , so that the middle of the eclipse falls at T , and the duration is 0 when $S+M=p$. (2) neglects m' as well, so that the duration is only mean, as given by this rule.

Applying these rules to the example given in 1, taking T and m' as given there, and n equal to $10^\circ 2'$, we have according to (3), $T \mp 77$ minutes and according to (4), $T \mp 74$ minutes. In each case the duration is more than double what it is.

The *Sūryasiddhānta* condensed by Varāhamihira and the later Siddhāntas in general give fairly accurate methods and constants for the calculation of the eclipse. Their methods are practically the same, though the constants vary a little from one another's. They furnish the basis for finding π' and π , the parallaxes of the sun and the moon, and R and M , their radii. π' , π , R and M in minutes are according to the 'condensed' *Sur. Sid.* 3.8, 51.4, 16.1 and 16.1; according to the *Sūryasiddhānta* 4.0, 53.5, 16.2 and 16.0, and according to the *Siddhānta Śiromaṇi* of Bhāskarāchārya, 3.9, 52.7, 16.2 and 16.0. All three give 270 minutes for the maximum latitude of the moon.

The following, in effect, are the rules they give for computing the eclipse: (1) Find T . Find p , moon's

latitude at T, from: $\text{lat} = 270' \sin (\text{Moon} - \text{Ascending Node})$. (3) Find m' . (4) Correct π' and R by multiplying each by the sun's daily true motion in longitude and dividing by the mean motion, $59.1'$. Correct π and M by multiplying by the true daily motion of the moon and dividing by the mean motion, $790.6'$. Using these values, $\pi' + \pi - R = S$. $S + M = D$ for the first and last contacts, and $S - M = D$, for the beginning and ending of the total phase. (5) $T \mp \sqrt{D^2 - p^2}/m'$ hours is a first approximation for the beginning and the ending of the eclipse. Repeat, using for p the latitude at the approximate time of beginning. A more correct time of beginning is thus obtained. If the latitude of this time is substituted and computation again made, a still more correct time is got, and so on, till we have the necessary accuracy. The same process is to be followed to find the correct time of ending.

It is to be noted that this process of successive approximation is necessary because the change in p caused by p' is neglected in the formula in (5) above. Perhaps the authors feel that this way of finding the times is easier than finding them once and for all by solving the quadratic with p' included; for, even Bhāskarāchārya gives this rule though he knows how to solve a quadratic and interpret the solution. But this method will fail when $p = D$, for then $\sqrt{D^2 - p^2}/m'$ is 0, and no time before or after T is got for a second approximation. (We have already seen that in this case the eclipse begins or ends at T and the duration will be about 22 minutes). But one thing must be mentioned here, that for the Siddhāntas the difficulty will not arise in the matter of calculating the first or last contact, because they do not require us to calculate such eclipses. The *Siddhānta Śiromaṇi*, for instance, says that if $S + M - p$ is less than a sixteenth part of the moon's diameter, i.e. $2'$, the eclipse should not be calculated

because its visibility will be marred by the brightness of the moon.

Another matter to be noted is that the parallaxes and radii are mentioned as varying proportionately to the true motions, which is not strictly correct.

Let us now apply the above rules to the example in 1, assuming T and m' there, and also given the moon's true longitude $141^{\circ} 14'$, the longitude of the Ascending node $331^{\circ} 16'$, the moon's true daily motion $717.3'$ and the Sun's $60.7'$. Taking the constants of *Siddhānta Śiromaṇi*, (1) $T=5.58$ a. m. (2) $p=270' \sin 169^{\circ}58'=47'$. (3) $m'=27.4'$. (4) $4'$ and $16.3'$, each multiplied by 60.7 and divided by 59.1 equal $4.1'$ and 16.7 . $52.7'$ and $16'$ multiplied by 717.3 and divided by 790.6 make $47.8'$ and $14.5'$. Thus $S=35.2'$ and $M=14.5'$. $S+M=D=49.7'$. (5) $\sqrt{49.7^2-47^2}/27.4$ hours = 35 minutes. So the approximate time of the beginning is 35 minutes before T and of the ending, 35 minutes after T. The latitude decreases at $2.32'$ per hour. So at $T-35m$ the lat. is $48.35'$ and at $T+35m$ it is $45.65'$. Using these we get $T-25m$ for the beginning, and $T+43m$ for the ending, as a second approximation. Repeating the process, we finally find that the eclipse begins at $T-28m$, and ends at $T+45m$. (5-30 a.m. and 6-43 a.m.) The agreement with the correct values is close, as we see.

THE THEORY OF RELATIVITY*

PART I

At the beginning of the present century, a new scientific theory, known as the Theory of Relativity was propounded by the great German scientist, Einstein. Its consequences in the field of science were far-reaching : it necessitated a change in the very outlook of the scientists. The Theory was at first restricted in its scope to uniform motion in a straight line and therefore called the Restricted Theory. In 1916, ten years after the Restricted Theory, the General Theory of Relativity was given to the world.

The Restricted Theory is based on two hypotheses which are really scientific truisms : (1) the velocity of light in a vacuum is constant, i.e., every observer finds it has the same value, and (2) it is impossible to find the absolute velocity of a body through the ether, relative velocity alone is observable, i.e., of two persons on two frames in relative motion 'to each other, if each asserts that he is at rest and the other is in motion, there is no test to find out who is right. (It is from this second hypothesis that the theory gets its name). The General Theory rests on only the second of the above-mentioned hypotheses, generalised to include accelerated motion. Before we proceed to see what circumstances led to the framing of the Theory and how it is developed from the given hypotheses, we shall understand the import of the Theory, i.e., the results that follow from it.

A man stands on a railway track and observes a train moving away from him with a velocity v . A passenger in

* *Pudukkottah Raja's College Magazine*, 1943, pp. 63-66; 1944, pp. 18-23; 1945, pp. 87-90; 1946, pp. 18-22.

the train is running from the guard's van towards the engine with a velocity, u . With what velocity is the passenger moving away from the man? $v+u$, we say. For instance, if the train is moving at 30 miles an hour, and the passenger is running at 8 miles an hour, he must be leaving the man behind at 38 miles an hour. But, no, says the Theory of Relativity. The formula according to the Theory is $(u+v) / \left(1 + \frac{v}{c^2} u \right)$, where the new quantity c is the velocity of light. If, as in the example given, u and v are small in comparison with c , (which is about 186,000 miles per second), there is practically no difference which formula we take—the reader may work out the difference if he is curious, 0".017. But if u and v are large, the difference is considerable. Suppose a nebula is receding from us at 18,600 miles a second and an electron is moving on it in the same direction at 93,000 miles a second. The velocity of the electron as observed by us is not $18,600+93,000=111,600$ miles, according to the familiar formula. It is $(18,600+93,000) / \{1+(18,600 \times 93,000) / 186,000^2\} = 106,286$ miles. There is a difference 5,314 miles, nearly a third of the velocity of the nebula.

There is something very interesting to be said in this connection. Long before the Theory of Relativity was thought of, a scientist by name Fresnel, while experimenting on the behaviour of light, measured the velocity of light through the water flowing in a tube. Allowing for the drag of the flowing water, the result showed not a velocity c/μ , (μ is the refractive index for water), as must be expected. It was $c/\mu + u(1-1/\mu^2)$, where u is the velocity of the flowing water. He was puzzled at this result. The experiment was repeated, but the result was the same. At last scientists came to accept the discrepancy $u(1-1/\mu^2)$, as an empirical fact and came to call $(1-1/\mu^2)$ the Convection Coefficient. The Relativity formula for the composition of velocities explains this

easily, furnishing an indirect proof of the Theory of Relativity.¹

Let us now examine the equation, $(u+v)/(1+uv/c^2)$, for its other implications. We shall take the case where u and v are both positive. If they are each less than c , the velocity of light, the composed velocity is always less than c . If one of the velocities is equal to c , the total is c , i.e., no velocity can be added to the velocity of light. (This result is inherent in the first hypothesis). If at least one of the velocities is greater than c , the resultant is less than at least one of them, which result is absurd, because we have taken both u and v positive. Therefore we have to conclude that no velocity can exist greater than that of light. This result is confirmed by another consideration as we shall see later on.

As another consequence of the Theory of Relativity, we have to give up our old idea of mass. We have been defining mass as the amount of matter in a body and thus identifying mass with matter. But, Relativity teaches that mass has nothing specially to do with matter. Radiation, as well as matter, has mass; and even though the amount of matter in a body does not change, its mass may change. The mass M is given by the equation $M=m/\sqrt{(1-v^2/c^2)}$, where m is the mass when the body is at rest, the rest mass or proper mass, as it is called; v is the velocity of the body, and c , the velocity of light. Usually v is small in comparison with c , and therefore neglecting v^4 and higher powers, we can write $M=m+\frac{1}{2}mv^2/c^2$. Now, we know that the kinetic energy of a moving body is $\frac{1}{2}kmv^2$ where k is a constant depending upon the units chosen. So we can conclude that the additional mass may be due to the kinetic energy of the body, i.e., the energy E , has mass equal to E/c^2

¹ Thus: neglecting squares of u , a small quantity, $(c/\mu + u)/(1 + c/\mu \times u/c^2) = (c/\mu + u)(1 - u/c\mu) = c/\mu + u(1 - 1/\mu^2)$.

grams, if E is given in ergs and c in cm per second.² This additional mass riding on energy is usually very small because ordinary velocities are small and also because the divisor c^2 is enormous ($=9 \times 10^{20}$). But it increases rapidly with the velocity. For an electron with a velocity of 149,000 miles per second, the increase in mass is 60 per cent. For still higher velocities the mass increases very rapidly, tending to infinity as v approaches c . But as the mass increases, increasingly greater forces will be required to increase the velocity by a given amount, with the result that no body can acquire the velocity of light. Thus we return to the conclusion that the velocity of light is absolute and cannot be exceeded.

At the end of the last century, J. J. Thomson, in his experiments with electrons, observed that the mass of the electrons increased with the velocity, and he arrived at the empirical formula for the mass, $M = m + \frac{1}{2}mv^2/c^2$. This result is explained by the Theory of Relativity, which itself is proved thereby.

We saw that the additional mass is due to the kinetic energy. May it not be that the rest mass itself is due to the energy inherent in matter in a potential form? Considerations of convenience recommend this assumption. This is also highly probable for the following reasons. Matter has been known to be annihilated, exhibiting itself in the form of energy, the mass of the matter appearing as the energy mass. Secondly, matter which to all appearances is static and contains no energy, really possesses a large quantity of energy, the kinetic energy of its molecules and atoms in motion, (which exhibits itself in the form of temperature), the energy that binds the atoms together to form the molecule, the energy that keeps the electrons and protons together

² The sun radiates energy at the rate of 3.8×10^{33} ergs a second. So, every second it loses 4.2×10^{12} grams of its mass which is equal to about 4 million tons a second.

in the atoms and the energy in the electrons and protons themselves owing to their spin and electric charges, energy being required to charge them.³ So we conclude that all mass is due to energy, with which it is inseparably associated. Conversely to what we said about the mass of energy, the energy in a given mass is enormous ($E=mc^2$) because the mass has to be multiplied by $c^2(=9\times 10^{20})$ to get the energy. For example, an ounce of matter is equal to $28.35\times 10^{20}\times 9$ ergs $=2.55\times 10^{22}$ ergs which amounts to about 700 million kilowatt-hours or 940 million horse-power-hours. This is greater than the energy generated by a thousand horse-power engine working incessantly for 108 years. Only, there is no known means of making this energy available for use.⁴

The Theory of Relativity has revolutionized our ideas of space and time and at the same time given us an insight into their true nature. We derive our ideas of space and time indirectly through our sense-impressions. This is what philosophers mean when they say space and time are modes of perception. But time enters our consciousness also directly, without the intervention of the senses. It is this time that is perceived as flowing from the past to the present and from the present to the future. By identifying this subjective time with the objective time obtained through our senses, we get the

3 It is interesting to note that the mass of the electron is of the order of the mass of the energy required to charge the particle with electricity. If e is the charge and r the radius of the electron, the energy is e^2/r and the mass is e^2/rc^2 . Substituting their respective values for the electron we have $4.774^2\times 10^{20}/9\times 10^{20}\times 1.85\times 10^{13}=13\times 10^{28}$ gr. which is of the order of 9×10^{28} gr, the mass of the electron.

4 The article was written before the Atom bomb was used upon Hiroshima and Nagasaki. It was only then that people came to know that it was possible to release atomic energy so as to be available for use.

idea of the flow of time in general. It is this subjective time that gives the world its dynamic character. We shall revert to this later on.

We know that every event is four-dimensional, i.e., we require four qualities to specify each event. We say an event has happened at a particular place at such and such a time. To specify the place we require three quantities, co-ordinates as they are called: so much in length, so much in breadth and so much in height. In other words, space is three-dimensional. But we always separate the four dimensions into two sets, three of space and one of time, never mixing the space and the time. Between two events there is, we feel, a definite interval in space as well as in time. If the interval in space is zero we say they happen at the same place. If the interval in time is zero we say they take place at the same time; i. e., the events are simultaneous.

According to the classical theory, everyone measures the interval in space as being equal, as also the interval in time (provided an ideal measuring rod and an ideal clock are used). By time I mean physical time, not subjective time, i. e., time as sensed by each individual. Subjective time is notoriously different for different persons: a person awaiting his love, and another, the execution of his death-sentence, do not have the same appreciation of time. True, there is a classical theory of relativity which says that if a person A measures an interval x in space and t in time, a person B, moving uniformly away from A at u units of length per second, measures the interval in time as t , but the space as $x - ut$. But the classical theory does not take this difference seriously, for it takes it for granted that it can be known who is *actually* in motion, and a correction may be made accordingly; so that there is agreement between A and B in their measurement of space as well.

But Einstein's Theory of Relativity has changed the possibility of all this. It says that neither the interval in space nor that in time is the same for two different persons in motion relative to each other. (As for absolute motion, it is denied by the postulates of Relativity.) If A measures an interval x and t in space and time respectively, B measures a space-interval of $(x - ut)/\sqrt{1-u^2}$ units and a time interval of $(t - ux)/\sqrt{1-u^2}$ seconds. (For the sake of simplicity the distance which light travels in one second is taken as the unit of length and the second is taken as the unit of time).

Let us now proceed to examine the formula for their implications. Suppose A has a clock with him and observes an interval t seconds. As the clock is with him always, x , the interval in space, is zero. According to the formula for time given above, B measures the time as $(t - u \times 0)/\sqrt{1-u^2} = t/\sqrt{1-u^2}$. B says A's clock is slow, and as there is a correspondence between A's acts and his clock, he says A is a lazy person. But A accuses B of being lazy, the relation being reciprocal. Sometimes each envies the other that the other's *jilebi* lasts longer than his own, but finds a consolation in the fact the other's visit to the dentist lasts longer. If we do not experience this difference in time it is because u , the relative velocity, is very small (remember, the unit of length is 186,330 miles. Even if u is 400 miles an hour, the difference is only $1/10^{11}$ percent). Another thing to be noted is that $\sqrt{1-u^2}$ becomes imaginary if u is greater than 1. From this we see the velocity of light is the limit to which relative velocity can attain. But there is nothing very remarkable in this, for we have only taken out now what we have put into our postulate as the constancy of the velocity of light.

Let us now go to their measurement of space. While A measures x , B measures $(x - ut/\sqrt{1-u^2})$ (accor-

ding to the classical theory he must measure $x - ut$) His measuring rod as well as his time has shortened. A thinks that it is natural because B is in motion. But B says A is in motion and his rod has shortened. As we cannot find absolute motion we cannot say who is right. As x cannot be greater than t (to measure a length greater than t in time we should move faster than light) no one can see events happening in an order reverse to what another sees they happen. In my article on the stars I humorously remarked that if, at this moment, we transplant ourselves on to a star distant 2,500 light years, we can see the Buddha preaching *Ahimsā* to the world. But we now see we can never have that joy, poor, mortals, and the Buddha must preach without us as audience! This is evident from the following consideration. You start now moving away from the world, all the while observing it through a telescope. As long as you do not move faster than light you cannot catch up with the light that left before you and perceive the event again. But nothing can make you move faster than light. So, all that you can do is to see the processes of the world slowing down and have the satisfaction of accusing the world of laziness, because you are there to whip it up to activity!! Thus the world is saved from topsyturvy-dom by the constancy of the velocity of light. Time is furnished with its arrow, as it were, by the limit set to attainable velocity, and shown which way it should move.

The Theory of Relativity has thus compelled us to throw overboard our concepts of absolute velocity, absolute space, absolute time and simultaneity, the bedrock on which we have built our universe of sense-impressions. But Minkowski, a great mathematician, has shown how to recover the absolute by going back to the four-dimensional origin of our perceptions of space and time. Though A and B calculate the interval in

space and in time to be different, they agree that their interval in space-time is the same. If A finds the four-dimensional interval $x^2 - t^2$, B finds it $(x - ut)^2 / (1 - u^2) - (t - ux)^2 / 1 - u^2 = x^2 - t^2$. Thus events are points in a four-dimensional world of space-time which is absolute, and their interval apart for every person is the same. But each carves out his own interval of space and interval of time. The accident that we move with a small velocity relative to one another has masked the unity of the physical entity, space-time, because if we had considerable relative velocities, we would have obtained different values for physical space and physical time, and the conclusion would have been forced upon us that space and time are only points of view, space-time alone being real. Thus Relativity has given us an insight into the true nature of space and time.

We shall now examine some results of the General Theory of Relativity. The force of gravitation as given by Newton is $G m m' / d^2$, where m and m' are the masses of the bodies attracting each other, d is their distance apart, and G is a constant depending on the units of mass and length. From what we have said about space and time it is evident that the law cannot be in this form in the Theory of Relativity; for, mass and distance are variable according to the frame of reference with respect to which they are measured. The Theory of Relativity gives a law of gravitation essentially different from Newton's but practically agreeing with it. One difference is remarkable. According to Newton's law it can be shown that as long as only two bodies are involved (say, the sun and a planet) the apsides of the orbit of revolution lie fixed in space. But in fact the apsides of the orbits of the planets are revolving in the plane of the orbits, though the revolution is very slow (for the earth it is 1,163 seconds of arc per century). This is due to the disturbance, known as perturbation, caused by a

third body. Let us take the case of Mercury. If there were no other planets, its orbit would be fixed in space and there would be no revolution of the apsides. But owing to the disturbance caused by the other planets there is a revolution of the apsides. The contribution of each planet to the revolution can be calculated from theory and compared with the results of observation. While observation gave a revolution of 574 seconds of arc per century, calculation gave a difference of 43". At first it was thought that some other planet, as yet undiscovered, was responsible for this. The existence of Neptune and Pluto were thus predicted from theory before they were discovered at the very places predicted. It was thought that there was a planet between Mercury and the Sun affecting the orbit. Astronomers had even a name ready for it, Vulcan. But it was not discovered and the 43" remained unexplained, until the Theory of Relativity accounted for it.

The law of gravitation according to the Theory of Relativity shows that even when there are only two bodies there must be a slow revolution of the apsis equal to $6\pi m/l$ radians per revolution of the body, where m is the gravitational mass of the attracting body and l is the semi-latusrectum of the elliptical orbit, the unit of length being the *lux* (186,330 miles). For the planets of the solar system it is about $\sqrt{16}/r^5$ seconds of arc per century, where r is the distance of the planet from the sun, the distance of the earth from the sun being taken as the unit of length. Substituting the value of r for Mercury, we get 43", the exact amount of discrepancy between the old theory and observation. Thus an experiment performed in Nature's laboratory has vindicated the Theory of Relativity.

According to the Theory of Relativity light rays passing near a body are bent towards the body in the same manner as the path of material bodies. Supposing

the light comes from a star, and the ray of light passes very close to the body in question, the amount of bending is $4m/r$ radians, where m is the gravitational mass of the body and r is its radius in *lux*. It is not possible to say how waves of light could be attracted by a body, if we use Newton's law, which applies to material particles. Even supposing that the law applies to non-material particles as well, or supposing, as Newton himself did, that light rays consist of material particles, Newton's law gives only half the value given by Einstein's law. Here is a crucial test for the Theory of Relativity. Einstein predicted from his theory that light rays from a star passing close to the sun would be deflected towards the sun by $1.78''$ of arc. This can be verified by observing the position of a star very close to the sun; but such an observation is possible only during a total eclipse of the sun, when the light of the sun cannot obscure the light of the star. The first verification of this prediction was made during the total eclipse of 1919, and this brought Einstein and his Theory of Relativity into the limelight. When Einstein made epoch-making discoveries like the photon (light-quantum) or the two Theories of Relativity, nobody took any notice of him outside scientific circles. But when the above-mentioned verification was made, he caught the imagination of the people and there was a general desire to know him and his discoveries. I distinctly remember how all sorts of things (not excluding nonsense) were talked about the Theory of Relativity, and how meaningless comparisons were made between Einstein and Newton, of course to the disadvantage of the latter.

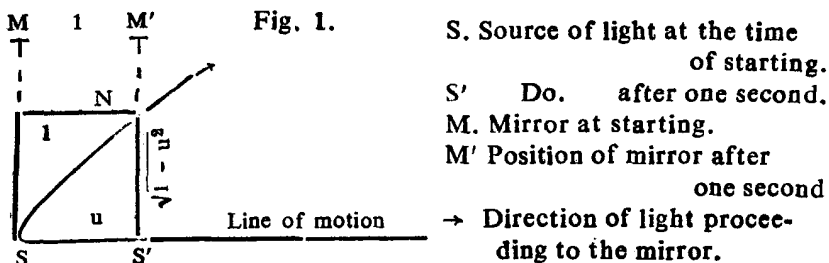
Einstein made another important prediction from his Theory. He showed that the natural period of vibration of a particle is shorter on a light body than on a heavy body. The relation is given by the equation $(1 - 2m_1/r_1) t_1^2 = (1 - 2m_2/r_2) t_2^2$, where m_1 , r_1 and t_1 refer

to one body (its gravitational mass and radius and the time of vibration on it) and m_2 , r_2 and t_2 refer to another. If we suppose the left-hand member refers to the earth and the right-hand member to the sun, as m_1/r_1 is negligible in comparison with m_2/r_2 , and as $m_2/r_2 = .000002$, we can say t_2 (the period on the sun) = $.000002 t_1$. So the frequency of vibration is greater for the terrestrial particle than for the solar particle. If the spectral lines of a terrestrial atom, say an atom of hydrogen, are compared with the same lines in the solar spectrum, we must find the lines in the solar spectrum shifted by a small amount towards the red-end. But the shift is so small (for the H_B line the increase in wave-length is only $.01\text{\AA}$) that it cannot be verified as it is masked by errors of observation. There are certain stars called white-dwarfs in which, owing to peculiar conditions, tons of matter are packed into a cubic inch of space. One such white-dwarf is the companion of Sirius in the constellation, Canis Major. Examination of the stellar spectrum of this dense body has established the truth of this prediction, thus proving the Theory of Relativity.

In the discussion above we took the equations arising from the Theory of Relativity for granted and compared them with the corresponding equations of classical (also called Newtonian) mechanics. We also tried to understand their implications which gave a shock to our accepted ideas. Now we shall derive the equations used above from the hypotheses of the Theory of Relativity. We shall first take the Restricted Theory. The two hypotheses of that theory are: (1) The velocity of light is constant as measured by all observers, whether they are relatively at rest or in motion with *uniform* velocity as regards one another. (2) Of two men having *uniform* relative motion, if one asserts that he is at rest and the other is in motion, and if the other asserts that it is he

that is at rest, there is no means of determining who is right.

Let us take two men, A and B, having such relative motion. A thinks, naturally, that he is at rest and B is in motion, say with a velocity u . For the sake of convenience let us take as our unit of length the distance light travels in one second. Suppose B phones to A, "I sent at the same instant two beams of light, one along the line of our relative motion and the other at right angles to it. They were reflected back to the source of light by two mirrors placed at a distance, d , from the source. I found that they reached the source at the same instant, i.e., $2d$ seconds after they started." A says to himself, "How is it possible? I made the same experiment and got the same result. But B is in motion and so he must get a different result. Evidently there is something wrong with his measurement of time or measurement of length or possibly both. Let me find out by calculation the amount of his error once for all, so that in future I can correct his results for agreement with mine." Let me first take the reflection at right angles to his line of motion.

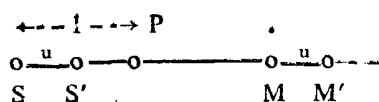


Though he had placed his mirror on a line at right angles to his motion (see fig. 1), it was the oblique ray of light, marked by the arrow that was reflected by the mirror, (though he was not aware of it), for by the time it reached the mirror, the mirror would have moved along with B. As light travels one unit in one second, and as in one second B moves a distance u , the light will

approach the mirror a distance $\sqrt{1-u^2}$ in one second. As the distance of the mirror is d , the time taken to reach it would be $d/\sqrt{1-u^2}$ seconds. As the same argument holds for the return journey to the source, the total time must be $2d/\sqrt{1-u^2}$. Evidently his clock is slow and gives a shorter interval of time, $2d$. So let me hereafter divide the time given by him by $(\sqrt{1-u^2})$, so that, his result may agree with my 'correct' result.

"Then, for the beam of light along his line of motion. (See fig. 2). In one second light travels one unit and is at P. But the source S has moved a distance u and is at S'. The mirror M has moved a distance u and is at M'. Thus in one second the light nears the mirror by $1-u$ units of length. So the mirror must have been reached in $d/(1-u)$ seconds. During the return journey the light approaches the source at one unit per second while the source approaches the light at u units per second. So the distance is lessened $1+u$ units every second. Therefore the distance, d , from the mirror to the source is done in $d/(1+u)$ seconds. Adding the two we have $2d/(1-u^2)$, the time from the source to the mirror and back. But the fellow says it is $2d$. As his clock is slow I divide $2d$ by $\sqrt{1-u^2}$, as I have already resolved to do. But it makes $2d/\sqrt{1-u^2}$ but not $2d/(1-u^2)$ which I have found to be the correct time. Evidently there is something wrong with his measurement of lengths *along the line of motion*. It must have shortened. I must divide his lengths *along the line of motion* by $\sqrt{1-u^2}$ so that I may get the correct time $2d/(1-u^2)$."

Fig. 2.

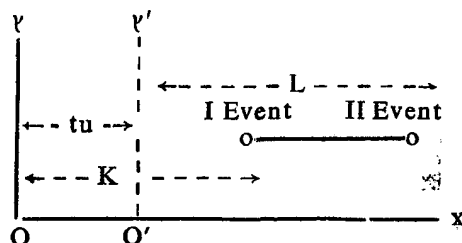


S: The position of the source when
light started.
M: do. of the mirror do.
S': The source after one second.
M': The mirror after do.
P: The position of the light after
one second.

So says A, and we appreciate his standpoint. But we must remember that the second hypothesis allows B to claim he is stationary and A is moving away from him with a velocity, $-u$. That is no special privilege of A's. Arguing exactly like A, but putting $-u$ in the place of u in A's argument, B comes to the conclusion that it is A's clock that is slow and \therefore he must divide A's time by $\sqrt{1-u^2}$ (for $\sqrt{1-(-u)^2} = \sqrt{1-u^2}$) to get his own correct measure of time. In the same way, he says it is A's length that has contracted and divides that too by $\sqrt{1-u^2}$ to obtain agreement with his own correct measurement of time. These two results are fundamental in the Theory of Relativity.

Bearing these two results in mind let us proceed to derive the formulae used earlier. Suppose between two events B measures a time interval, t' , and a space interval along the line of motion, x' . What space interval will A find between the same two events? As marked in fig. 3, let B measure the first event to be distance K from his origin, O . Let him measure the second

Fig. 3



event to be at a distance L from the second position of his origin O' . (For the origin has shifted according to A). B thinks that his frame of reference is stationary. So he does not distinguish between O and O' , and measures $x' = L - K$. But A says that at the time of the second event B has moved a distance $u t$ or which is the same to him, $u t' / \sqrt{1-u^2}$. B's measurement of length x'

itself should be divided by $\sqrt{1-u^2}$, according to A, to compensate for the contraction of B's lengths along the line of motion. Making these two corrections we find A's measure of the distance between the events,

$$x = \frac{x'}{\sqrt{1-u^2}} + \frac{ut'}{\sqrt{1-u^2}} = \frac{x' + ut'}{\sqrt{1-u^2}} \dots \dots \dots (1)$$

But the argument being reciprocal, as we have seen, if A measures a distance x , and a time interval t , between two events, B will calculate his distance, $x' = \frac{x - ut}{\sqrt{1-u^2}}$. (Note: for B, u is negative).....(2).

Eliminating x between the above two equations, we get an expression for the interval in time between the two events, as measured by A, (expressed in B's measurements, t' and x'), $t = \frac{t' + u x'}{\sqrt{1-u^2}} \dots \dots \dots (3).$

Eliminating x' between equations (1) and (2) we get B's measurement of time in terms of A's measurement of time and space, viz, $t' = \frac{t - ux}{\sqrt{1-u^2}} \dots \dots \dots (4).$

We can also derive (4) from (3) directly by substituting $-u$ for u , and t , x , for t' , x' , which the second hypothesis permits us to do.

If u is small, so that we may neglect u^2 , equations (1) and (2) reduce to, $x = x' + ut'$, $x' = x - ut$, respectively. This is the same as the equations of Classical (or Newtonian) relativity, $x = x' + ut$, $x' = x - ut$, if we bear in mind that in Classical Relativity $t = t'$, i.e., there is no difference in the measurement of time of two people in relative motion. If we neglect u^2 in equations (3) and (4), they reduce to $t = t' + ux'$, $t' = t - ux$. There is nothing equivalent to this in Classical Relativity, according to which these equations are absurd. Only in

the case when $u=0$, these equations acquire a meaning for Classical Relativity, viz., $t=t'$. But then there is no relative motion and the equations are outside the pale of the Theory of Relativity.

The four equations (1) to (4) show that two persons in motion relative to each other do not have the same measurement either of distance or of time. The implications of this fact have already been discussed earlier. We have seen there that though space and time are different for each, the space-time interval between two events is the same for all. Let us call this interval 'separation' or s . We have seen, $s^2=t^2-x^2=t'^2-x'^2$.

Suppose B reports a velocity, v^1 , of a body. With what velocity does the body move as observed by A? Velocity is defined as displacement (i.e. distance travelled) per unit time. For A, $v=x/t=\left(\frac{x^1+ut^1}{\sqrt{1-u^2}}\right)\bigg/\left(\frac{t^1+ux^1}{\sqrt{1-u^2}}\right)=(x^1+ut^1)/(t^1+ux^1)=\left(\frac{x^1}{t^1}+u\right)\bigg/\left(1+u\frac{x^1}{t^1}\right)=(v^1+u)/(1+uv^1)\dots\dots\dots(5)$.

It may be remembered this is the equation given earlier for the composition of velocities, c^2 having disappeared from the formula, because we have taken c as the unit of length. Thus we see that all the results mentioned there follow from the Theory of Relativity.

We can, in the same way, derive the velocity v^1 for B if we know v , the velocity for A. Substituting $-u$ for u , on the strength of the Relativity hypothesis, in equation (5) and v for v^1 and vice versa, we get $v^1=(v-u)/(1-uv)\dots\dots\dots(6)$

Ordinarily u and v are very small, for our unit of length is the enormous distance that light travels in vacuum in one second. If we therefore neglect the product

of small quantities uv or uv^1 , the equations (5) and (6) reduce to $v=v^1+u$ and $v^1=v-u$, respectively, which are the equations for the composition of velocities according to Classical Relativity. Also, we have seen earlier how Fresnel's Convection Co-efficient can be derived from equation (5).

There are three conservation laws that are fundamental in Newtonian or Classical mechanics: (a) the Law of Conservation of Mass, (b) the Law of Conservation of Momentum and (c) the Law of Conservation of Energy. Most of our work in Physics and Chemistry is based on these laws. So we should examine how far these laws are affected by the Theory of Relativity, especially by the new equation for the composition of velocities.

The Law of Conservation of Mass states that as long as no matter (which is identified with mass) is added to or taken away from a body or a number of bodies, the total mass remains the same. For example, if two molecules of masses m_1 and m_2 unite to form a new molecule, the mass of the new molecule is m_1+m_2 .

The Law of Conservation of Momentum states that if a number of bodies are in motion and if no external force, which alone is capable of adding new momentum, acts on the system of bodies, the total momentum remains constant, though the momentum of the individual bodies may change by their interaction or collision. Let us take, for example, two masses m_1 and m_2 , with velocities v_1 and v_2 , respectively. Momentum = mass \times velocity. The law affirms that $m_1 v_1 + m_2 v_2$ is constant, under the given condition, though v_1 and v_2 may change by their collision.

The Law of Conservation of energy states that energy can be neither created nor destroyed. There are

several forms of energy, kinetic energy, potential energy, heat energy, chemical energy, electrical energy and energy of radiation. The law says that one form of energy may be transformed into another form, but it cannot be destroyed or created. For example, if there are two masses, of velocities, v_1 and v_2 respectively, their total kinetic energy is $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$. By the impact between the two bodies, part of the energy may be converted into heat, but the remaining kinetic energy plus the energy in the form of heat is equal to the original kinetic energy.

Let us take the conservation of momentum, first. It is easy to see that according to Classical Relativity momentum is conserved. If B observes velocities v_1 and v_2 , respectively, A will observe velocities v_1+u and v_2+u . If B finds a total momentum $m_1v_1+m_2v_2$, A finds a total momentum, $(m_1v_1+m_2v_2)+u(m_1+m_2)$. As $(m_1v_1+m_2v_2)$ is a constant, because B finds it conserved, and as (m_1+m_2) is also a constant by the law of conservation of mass and u is a constant, A's total momentum too is conserved, though it may have a different value from B's.

We shall now examine whether momentum is conserved if in the place of $u+v$ we use the Relativity formula for the composition of velocities, $\frac{u+v}{1+uv}$.

The total momentum for A is:—

$$\frac{m_1(u+v_1)}{1+uv_1} + \frac{m_2(u+v_2)}{1+uv_2} = u\left(\frac{m_1}{1+uv_1} + \frac{m_2}{1+uv_2}\right) + \left(\frac{m_1v_1}{1+uv_1} + \frac{m_2v_2}{1+uv_2}\right)$$

In this expression although u , (m_1+m_2) and $(m_1v_1+m_2v_2)$ are constant, the value of A's expression for total momentum will not in general be constant, because v_1 and v_2 may vary in all sorts of ways. A numerical example will make this clear. Let m_1 and m_2 be 2 and 5

units respectively and v_1 and v_2 be $\cdot 3$ and $\cdot 4$ units. The total momentum for B is $2 \times \cdot 3 + 5 \times \cdot 4 = 2.6$ units. Now let v_1 become $\cdot 8$ units after an impact. As the total momentum for B should still be 2.6 units, v_2 must be $(2.6 - 2 \times \cdot 8)/5 = \cdot 2$ units. The total momentum for A under the first condition is, if $u = \cdot 6$ units,

$$\cdot 6 \left(\frac{2}{1+\cdot 6} \times \cdot 3 + \frac{5}{1+\cdot 6} \times \cdot 4 \right) + \left(\frac{2 \times \cdot 3}{1+\cdot 6} \times \cdot 3 + \frac{5 \times \cdot 4}{1+\cdot 6} \times \cdot 4 \right) = 5.6$$

Under the second condition it is

$$\cdot 6 \left(\frac{2}{1+\cdot 6} \times \cdot 8 + \frac{5}{1+\cdot 6} \times \cdot 2 \right) + \left(\frac{2 \times \cdot 8}{1+\cdot 6} \times \cdot 8 + \frac{5 \times \cdot 2}{1+\cdot 6} \times \cdot 2 \right) = 5.5.$$

The total momentum has changed. The Law of conservation of momentum does not hold for A. But it must hold, for he too, like B, has discovered the law from the experiments. So there must be a mistake either in the definition of mass or in the definition of momentum. The fact that in the Theory of Relativity space and time have merged to form a single entity, 'separation', suggests the following definition: momentum = mass \times displacement per unit separation, in the place of the usual definition, mass \times velocity, i.e., momentum = $m \times x/s$, where $s^2 = t^2 - x^2$. Then $s^2/t^2 = (t^2 - x^2)/t^2 = 1 - x^2/t^2 = 1 - v^2$. Therefore

$$t/s = 1/\sqrt{1-v^2}. \text{ Therefore } m \times x/s = m \times \frac{x}{t} \times \frac{t}{s} = mv / \sqrt{1-v^2}.$$

This new formula for momentum is tantamount to taking the old formula $m v$ with the mass defined as $m/\sqrt{1-v^2}$, i.e., taking the mass as varying with the velocity in the specified manner.

Let us now see if the new definition of mass saves for A his Law of conservation of momentum. The total momentum for B is $m_1 v_1 / \sqrt{1-v_1^2} + m_2 v_2 / \sqrt{1-v_2^2}$, and this is conserved. The total momentum for A is

$$\left\{ m_1 \left(\frac{u+v_1}{1+uv_1} \right) / \sqrt{1-\left(\frac{u+v_1}{1+uv_1} \right)^2} \right\} + \left\{ m_2 \left(\frac{u+v_2}{1+uv_2} \right) / \right.$$

$\sqrt{1 - \left(\frac{u+v_2}{1+uv_2}\right)^2}$, which reduces to $\frac{1}{\sqrt{1-u^2}} \left\{ \sqrt{\frac{m_1}{1-v_1^2}} + \sqrt{\frac{m_2}{1-v_2^2}} + u \left(\frac{m_1 v_1}{\sqrt{1-v_1^2}} + \frac{m_2 v_2}{\sqrt{1-v_2^2}} \right) \right\}$. Here, as u and $1/\sqrt{1-u^2}$ are constants, as long as $\left(\sqrt{\frac{m_1}{1-v_1^2}} + \sqrt{\frac{m_2}{1-v_2^2}} \right)$ and

$\left(\frac{m_1 v_1}{\sqrt{1-v_1^2}} + \frac{m_2 v_2}{\sqrt{1-v_2^2}} \right)$ are constant, the whole expression will be constant. Now $\left(\sqrt{\frac{m_1}{1-v_1^2}} + \sqrt{\frac{m_2}{1-v_2^2}} \right)$ is the total mass according to B and $\left(\frac{m_1 v_1}{\sqrt{1-v_1^2}} + \frac{m_2 v_2}{\sqrt{1-v_2^2}} \right)$ is the total momentum. They are both conserved, and therefore constant. So the total momentum according to A is also conserved. From this discussion we conclude that though two persons in relative motion will not find the momentum equal, yet if it is conserved according to one, it is conserved according to the other as well.

We shall now proceed to examine how the new definition of mass affects the conservation of mass for

A. The total mass for A is $m_1 / \sqrt{1 - \left(\frac{u+v_1}{1+uv_1}\right)^2} + m_2 /$

$\sqrt{1 - \left(\frac{u+v_2}{1+uv_2}\right)^2}$ which reduces to

$$\frac{1}{\sqrt{1-u^2}} \left\{ u \left(\frac{m_1}{\sqrt{1-v_1^2}} + \frac{m_2}{\sqrt{1-v_2^2}} \right) + \frac{m_1 v_1}{\sqrt{1-v_1^2}} + \frac{m_2 v_2}{\sqrt{1-v_2^2}} \right\}$$

i.e., u times B's total mass is added to his total momentum and the whole is divided by $\sqrt{1-u^2}$ to get the total mass for A. As B's total mass and momentum are conserved and as u is constant, the total mass according to A is conserved, though A does not agree with B about the actual value. It is interesting to note

that mass and momentum occur together in both the above formulae showing an underlying unity. Of course this is expected, for when B says he is at rest and measures only mass, A says B is in motion and measures momentum as well. We thus see that a new definition of mass has been forced upon us by the Theory of Relativity. $m/\sqrt{1-v^2}=m$ when $v=0$, i. e., when the body is at rest, the mass is m . Let us call this the rest mass. As v is usually small, $m/\sqrt{1-v^2}=m+\frac{1}{2}mv^2$, neglecting v^4 and higher powers. The additional mass acquired by motion is $\frac{1}{2}mv^2$. Now the kinetic energy of a body due to its motion is $\frac{1}{2}mv^2$. This suggests that the additional mass is due to the kinetic energy. This leads us to conclude that the proper mass, m , itself may be due to the energy inherent in the body. We have seen earlier that $E=mc^2$, (where if m is in grams, and c in centimetres per second, E is the velocity of light), E is in ergs. (For a full discussion see p. 417 ff.). Thus as all mass is due to energy, conservation of mass turns out to be merely conservation of energy. In this way, all the three conservation laws are saved for us and in addition to this, our investigation has unmasked the close relation between them.

COMPUTATION OF THE SOLAR ECLIPSE IN HINDU ASTRONOMY*

I Introduction

It is a well known fact that a solar eclipse occurs, when in the neighbourhood of a moon's node the sun and the moon are in conjunction.¹ The computation of the solar eclipse for the world in general without reference to any given place is like that of the lunar eclipse. Let T be the Indian Standard Time of conjunction in longitude.² p the latitude of the moon, P the hourly change in latitude, (north latitude and motion towards the north being considered positive), M the excess of the hourly motion of the moon in longitude over that of the Sun, L the angular radius of the moon, and S , the angular radius of the Sun. Then at any time t hours *after* conjunction, the distance between the sun and the moon's longitude is Mt and the moon's latitude is $p+Pt$. So the distance between their centres is $\{M^2t^2+(p+Pt)^2\}^{1/2}$. The eclipse begins or ends when their rims appear to touch. This can happen even if the distance between them is greater than $L+S$, for the moon's parallax may push it towards the sun. The maximum of this effect is $\pi-\pi'$ ($=\pi$), π being the equatorial horizontal parallax of the moon,

* *Pudukkottah Raja's College Magazine*, 1952, pp. 17-27.

1 By conjunction is meant conjunction in Right Ascension or in Longitude. Modern astronomers use the Rt. As. Declination Co-ordinates of the Sun and the moon in the computation of eclipses, while the ancients used the Latitude-Longitude co-ordinates. Because I here give the modern method for comparison with that of the ancients, I use the latter throughout.

2 The apparent longitude of the Sun, i.e., the true longitude minus $20''$ for aberration, is to be used in eclipse work.

π' that of the sun. Thus the rims can appear to touch when the distance between the centres is $\pi + L + S (=d)$ at the most. Then $\{M^2 t^2 + (p + Pt)^2\} = d^2$ gives the times of the beginning and end of the general eclipse. Solving for t , we get, $t = -pP / (M^2 + P^2) \mp \{ [p^2 P^2 / (M^2 + P^2) + d^2 - p^2] / (M^2 + P^2) \}^{\frac{1}{2}}$, in which, obviously, the upper sign is to be taken for the beginning, and the lower for the end. $T+t$ is the I.S.T. of the beginning or end.

At any given place the eclipse begins or ends when the rims appear to touch at that place, i.e. when the *apparent* distance between the centres is $L+S$. Now at any time T (I.S.T.) near the times of conjunction in longitude, let the apparent distance in longitude between the centres be m , the apparent excess of the moon's hourly motion in longitude over that of the sun M , the apparent difference in latitude p , the apparent excess of the moon's hourly motion in latitude over that of the sun P , the sum of the apparent angular radii of the sun and the moon d , and its variation per hour D [By 'apparent' is meant here 'as affected by parallax.' Apparent $m = \text{real } m + \pi \cos A \cos B \times (1 + \pi \cos A \sin B)$.

Apparent $p = (\text{real } p + \pi \sin A) (1 + \pi \cos A \sin B)$. Apparent $(L+S) = S + L (1 + \pi \cos A \sin B)$, where A is the zenith distance of the Nonagesimal, given by $\sin A = \sin w \cos \phi \sin v - \cos w \sin \phi$, and B is the Orient ecliptic point minus the longitude of the moon.

The orient ecliptic point $= \tan^{-1} \{ \tan \frac{1}{2} (90^\circ + v) \cos \frac{1}{2} (90^\circ + \phi - w) / \cos \frac{1}{2} (90^\circ + \phi + w) \} + \tan^{-1} \{ \tan \frac{1}{2} (90^\circ + v) \sin \frac{1}{2} (90^\circ + \phi - w) / \sin \frac{1}{2} (90^\circ + \phi + w) \}$, where ϕ is the latitude of the place, w is the obliquity of the ecliptic and v is the sidereal time in degrees at the moment, given by, $v = 97^\circ 30' + \text{the Greenwich East longitude of the place} + \text{the mean longitude of the sun} + \text{the I.S.T. at that moment in degrees}$. For strict accuracy, the geocentric

latitude and the horizontal parallax at that latitude should be used.

If T is the time for which we have found m , p and d , the apparent distance between the centres of the sun and the moon at any time t hours *after* T is $\{(m+Mt)^2 + (p+Pt)^2\}^{\frac{1}{2}}$. When this is equal to $d+Dt$, the eclipse begins or ends. Thus it begins or ends at $T + (dD - mM - pP) / (M^2 + P^2) \mp [\{(mM + pP - dD)^2 / (M^2 + P^2) + d^2 - p^2 - m^2\} / (M^2 + P^2)]^{\frac{1}{2}}$.³ The middle of the eclipse, i.e. the maximum eclipse, occurs at $T + (dD - mM - pP) / (M^2 + P^2)$.

The total eclipse begins or ends when the rims apparently touch, the sun being within the moon. The distance between them at such a time is $(L-S)$. So by substituting for d in the above formula another d equal to $(L-S)$, we can find the times of the beginning and end of the total phase. Another thing should be noted here. S may be greater than L , so that the moon may be immersed in the sun, leaving a circle of light all round. This is called an annular eclipse. Obviously, the beginning or end of the annular phase is got by making $D=S-L$.

For e.g., let us compute the solar eclipse that occurred on 9-5-1948, for the world in general, and for Pudukkottai ($78^\circ 4' 17''$ E L and $10^\circ 18' 51''$ N. Geocentric Latitude) given: time of conjunction in longitude $T = 7^h - 59^m.9$ (I.S.T.), True longitude of the moon = apparent long. of the sun = $48^\circ 22' 24''$. $P = +1425''$. $\pi = 3439''$. $\pi' = 8.7''$, $M = 937''$, $S = 950''$.

3. As P , M and D vary rapidly the time T should be taken close to each of the beginning, middle and end, and each found separately, using P , M , D not varying per hour but a shorter period, say, 12 minutes.

Hourly change in Sun's longitude $+ 145''$,

„ „ Moon's longitude $+ 1992''$

$M = 1847''$, Hourly change in $p = P = +183.5''$.

Hourly change in $L = +.4''$.

From these, for the general eclipse, $d = \pi - \pi' + d + S = 5318''$. Substituting these in the formula for the general eclipse we have ${}^h 59.9 - 4.6 - 2 {}^h 45.6 = {}^h 59.7$ for the beginning, ${}^h 59.9 - 4.6 + 2 {}^h 45.6 = {}^h 1040.9$ for the end.

As at Pudukkottai the sun will be seen already eclipsed when rising, we shall compute the middle and end alone for that place.

T. (i.e. the time near middle or end)	${}^h 6 19.7$	${}^h 6 31.7$	${}^h 7 19.5$	${}^h 7 31.5$
Real m.	$- 3082''$	$- 2713$	$- 1242''$	$- 873''$
Parallax in m.	$+ 3085$	$+ 3091$	$+ 3027$	$+ 2990$
Apparent m.	$+ 3$	$+ 378$	$+ 1785$	$+ 2117$
Real p.	$+ 1119$	$+ 1155$	$+ 1302$	$+ 1338$
Parallax in p.	$- 1462$	$- 1407$	$- 1172$	$- 1107$
Apparent p.	$- 343$	$- 252$	$+ 130$	$+ 231$
Real d	$+ 1886.5$	$+ 1886.5$	$+ 1887$	$+ 1887$
Parallax	$+ 1.5$	$+ 2.5$	$+ 5$	$+ 6$
correction	$+ 1888$	$+ 1889$	$+ 1892$	$+ 1893$
apparent d.				

Taking ${}^h 6 19.7$ as T for the middle, $m = +3''$, $M = +375''$, $p = -343''$, $P = +91''$, $d = +1888''$. $D = +1''$, from which we find the middle occurs at ${}^h 6 19.7 + 2.6 = {}^h 6 22.3$.

Taking ${}^h 7 19.5$ as T for the end, $m = +1785''$, $M = +332''$, $p = +130''$, $P = +101''$, $d = +1892''$, $D = +1''$, from which we find the end is at ${}^h 7 19.5 - 1 {}^h 0.1 + 1 {}^h 3.7 = {}^h 7 23.1$.

THE COMPUTATION IN HINDU ASTRONOMY

(A) According to the Paulisa Siddhanta

Though there is mention made of the eclipse in Vedic literature, the ancient astronomical works like the Vedanga Jyotisha, Garga Samhita, Paitamaha Sid, etc., do not give methods for its calculation. The Paulisa Sid. seems to be the most ancient work dealing with the solar eclipse. We shall here give its method as condensed by Varahamihira in his Panchasiddhantika.

1. Find the time of conjunction in longitude.
2. Find the time of noon.
3. Find the interval between conjunction and noon.
4. Find the hour angle of the Sun at conj., in degrees, calculating at the rate of 6° for one nadika or 24 minutes.
5. Multiply the sine of the hour angle by 4 nadika or 96 minutes.
6. If the conj, is before noon deduct the time got, from the time of conj. If it is afternoon, add. This is the time of the middle of the eclipse.⁴
7. Find the longitude of the moon and the mean longitude of Rahu at this time.⁵
8. Make the following corrections in Rahu.
 - a) Multiply the degrees of latitude of the place by 5 and divide by 27. Add the resulting degrees to Rahu if it is the ascending node. If not subtract.

4. This work will roughly compensate for the parallax in longitude.

5. Here Rahu is taken to be the node near which the moon is situated. The corrections will roughly compensate for the parallax in latitude.

- b) Find the declination in degrees of the point of the ecliptic 90° from the Sun. Multiply this by the result of (5) in nadikas, and divide by 22. The resulting degrees are to be added to the ascending node if the sun's longitude lies between 270° and 90° , and the time of conj. is forenoon, or if the sun's longitude lies between 90° , and 270° , and the time of conj. is afternoon. Otherwise the degrees are to be subtracted from the ascending node. If the conj. is at the descending node, the opposite of this is to be done, i.e., addition should take the place of subtraction and vice versa.
- c) Take the time in nadikas of conj. elapsed from sunrise in the morning or to elapse for sunset in the evening. Multiply this by the degrees of declination of the moon and divide by 80. If conj. is at the ascending node the resulting degrees are to be added if the moon is between 180° and 360° , and subtracted if between 0° and 180° . For conj. at the descending node, subtract and add respectively.⁶
9. Find the difference in longitude between this corrected Rahu and the moon. If it is less than 8° , there is a solar eclipse.
10. Square the difference in degrees and subtract from 64. Take the square root. This multiplied by 18 is the duration of the eclipse in minutes.

6. Of the above rules (a, b, c), (a) does duty for $-\cos w \sin \phi$, (b) and (c) take the place of $\sin w \cos \phi \sin v$, for this is equal to $\sin w \cos \phi \sin O \cos h - \sin w \cos \phi \cos O \sin h$, taking $v = O - h$, roughly. (O is the longitude of the sun, h is the hour angle). The first part of this is taken by (c), and the second, by (b),

11. Half the duration subtracted from the middle is the beginning, and added in the end.

Let us compute the eclipse of 9-5-1948 according to these rules, using the elements given already ; and also given Rahu (here the ascending node) = 44° , time of noon $12^h 11^m$ and the time of sunrise $5^h 58^m$.

1. The time of conj., is $2^h 2^m$ after sunrise.
2. Noon is $6^h 13^m$ from sunrise.
3. Conj, is $4^h 11^m$ before noon.
4. The hour angle of the sun at conj., is $62^\circ 48'$.
5. $\sin 62^\circ 48' \times 96^m = 86^m$ or $3 \cdot 57$ nadikas.
6. This deducted from 8^h gives $6^h 34^m$ (I.S.T.) for the middle.
7. The longitude of Rahu is 44° .
8. a) The geographical lat. of Pudukkottai is $10^\circ 4'$. This $\times 5/27 = 1^\circ 9'$. This is to be added to Rahu which becomes $45^\circ 9'$.
 b) $15^\circ 7' \times 3 \cdot 57/22 = 2^\circ 5'$. As Rahu is the asc. Node, the sun lies between 270° and 90° and conj. is before noon, this is to be added, making Rahu $48^\circ 4'$.
 c) The time of conj. from sunrise is $5 \cdot 1$ nadikas. The declination of the moon is $18^\circ 9'$.

7. As our purpose here is to compare the methods alone, we are assuming that these elements have been found correctly and so use these as correct ones.

8. This includes the moon's latitude, $+18'$, according to this Siddhanta.

This $\times 5.1/80 = 1.1$. As the conj., is at the asc. node, and the moon is between 0° and 180° , this is subtractive. The corrected Rahu is $47^\circ.3$.

9. The difference of this and the moon is $1^\circ 1$. As it is less than 8° , there is a solar eclipse.
10. $18 \times \sqrt{64 - 1.1} = 143^m = 2^h 23^m$.
11. $6^h 34^m \mp 1^h 11^m = 5^h 23^m$ and $7^h 45^m$ are the beginning and end.

(B) According to the Romaka Siddhanta

The method of the Romaka is more like the modern one, and more accurate. The following is Varahamihira's condensation of it.

- 1—7. The same as in the Paulisa.⁹
8. At conj., as affected by parallax, (C. A. P.), find the orient ecliptic point. Deduct 90° from this. This is the Nonagesimal.
9. Find the declination of the Nonag. The declination is North if the Nonag. lies between 0° and 180° , and South otherwise.
10. Deduct the longitude of the asc. node from this point and multiply its sine by $280'$. This is North if the result of the subtraction is 0° to 180° , and South otherwise.
11. If the result of (9) and (10) are both north, add, and call it North. If they are both south, add, and call it South. If one is North and the other South, deduct one from the other and call it by the direction of the larger.

9. This does not mean that the same elements will be got according to both.

12. If the result of (11) is North and greater than the latitude of the place, deduct the latitude and call it North. If it is North and less, subtract it from the latitude and call it South. If it is South, add it to the latitude and call it South.¹⁰
13. Multiply the sine of this by the daily motion of the moon in minutes and divide by 15. The result is the parallax in latitude. It has the same direction as (12).
14. Find the latitude of the moon by multiplying the sine of the difference between the moon and Rahu by 280'. If the moon is greater near the asc. node, or less near the desc. node, it is North. Otherwise it is South. (The moon at C.A.P. is to be used for this).
15. Add the latitude and the parallax in latitude if they have the same direction. If not, subtract one from the other. This is p in minutes.
16. The mean angular radius of the sun is 15', and that of the moon 17'. The radius of each multiplied by its true motion and divided by its mean motion gives the true radius. Let us call them S and L.
17. $\sqrt{(L+S)^2 - p^2} \div M$ gives the half duration of the eclipse in hours if M is the difference between the true motions in minutes of the sun and the moon, per hour.

10. It is obvious that only North latitude is envisaged by this rule.

. Applying these rules to the given example, we have :

- 1—7. The same as for the Paulisa.
8. At C.A.P., $6^{\text{h}} 34^{\text{m}}$, the Orient Ecliptic point for Pudukkottai can be found to be $57^{\circ} 44' \cdot 4$. This $-90^{\circ} = 327^{\circ} 44' \cdot 4 = \text{Nonagesimal}$.
9. The declination of this point is $12^{\circ} 16' \cdot \text{S}$.
10. $327^{\circ} 44' \cdot 4 - 44^{\circ} = 283^{\circ} 44' \cdot 4$. The sine of this is $\cdot 971$. $\cdot 971 \times 280' = 272' \cdot \text{S} = 4^{\circ} 32' \cdot \text{S}$.
11. $12^{\circ} 16' \cdot \text{S} + 4^{\circ} 32' \cdot \text{S} = 16^{\circ} 48' \cdot \text{S}$.
12. The Geographical lat. of Pudukkottai is $10^{\circ} 24'$. $16^{\circ} 48' + 10^{\circ} 24' = 27^{\circ} 12' \cdot \text{S}$.
13. The daily motion of the moon $= 797'$. Sine $27^{\circ} 12' \times 797' / 15 = 24' \cdot 3 \cdot \text{S} = \text{parallax in latitude}$.
14. The moon at C.A.P. is $47^{\circ} 35'$. The difference between this and Rahu $= 3^{\circ} 35'$. $280' \sin 3^{\circ} 35' = 17' \cdot 5 \cdot \text{N}$.
15. $24' \cdot 3 - 17' \cdot 5 = 6' \cdot 8 = p$.
16. $15' \times 58 / 59 \cdot 2 = 14' \cdot 7 = \text{S}$. $17' \times 797 / 790 \cdot 6 = 17' \cdot 1 = \text{L}$.
17. $M = (797' - 58') / 24 = 30' \cdot 79$. $\sqrt{31 \cdot 8^2 - 6 \cdot 8^2} / 30 \cdot 79 = 1^{\text{h}} = \text{half duration}$. The middle being $6^{\text{h}} 34^{\text{m}}$, the eclipse begins at $5^{\text{h}} 34^{\text{m}}$ and ends at $7^{\text{h}} 34^{\text{m}}$.

The correct times of the middle and end, as we have seen, are $6^{\text{h}} 22^{\text{m}}$ and $7^{\text{h}} 23^{\text{m}}$. The following are the inaccuracies in this method. The maximum mean parallax is assumed to be $53'$ instead of the correct $57'$. The max. mean latitude of the moon is taken to be $280'$. The declination of the Nonagesimal minus the latitude of the place is taken to be the zenith distance of the

Nonagesimal. The formula for the correction of this declination (10) is wrong. We are not instructed that the times obtained are only approximate, and that for a closer approximation, the calculation should be repeated taking the latitude and parallaxes pertaining to the first approximate times.

(C) According to the Later Siddhantas

The method of the Surya Siddhanta condensed by Varahamihira in the Panchasiddhantika is typical of that of the later Siddhantas, and fairly correct. Only in their constants do these differ from one another. I shall here give the method, and the constants of Varaha's Sur. Sid., the New Sur. Sid. and the Siddhanta Siromani of Bhaskaracharya, in order. The reader will do well to compare this method with the modern one, given in the first part.

1. Directly or indirectly each Siddhanta gives the mean angular diameter of the sun and the moon. For the Sun it is $32'1$, $32'4$ and $32'5$ ($=2S$), and for the moon $32'2$, $32'0$ and $32'0$ ($=2L$). These are to be made true by being multiplied by the true motion and divided by the mean motion. (The correct mean angular diameters of the sun and the moon respectively are $32'0$ and $31'3$,¹¹ and it will be better to multiply by half the sum of the true and mean motions).

2. The mean horizontal Parallax of the moon is $51'6$, $57'5$ and $52'9$, and of the sun $3'8$, $4'3$ and $3'9$. These too are to be made true by being multiplied by the true motion and divided by the mean motion. (The correct figures are $57'5$ ¹¹ and $0'1$, respectively). Let their difference be written π .

11. These are the mean for syzygies. The mean diameter for any time is $31'1$ and the horizontal parallax $57'0$.

3. For any time near enough to the event sought the zenith distance of the Nonagesimal, A, is found. The orient ecliptic point or Lagna is found from which the Lagna minus the longitude of the moon at the time, B, is found. (Though different methods are given for this, they ultimately reduce to what we have given in Part I.)

4. $\pi \cos A \cos B$ is the parallax in longitude. Let M be the difference between the true motions of the Sun and the moon per hour. $\pi \cos A \cos B/M$ are the hours to be deducted from the time of conj. to get the conj. as affected by parallax (C.A.P.). (Note that for B greater than 90° the hours will be negative and so C.A.P. will be later.)

5. Using the A and the B of the C.A.P. got, (4) is to be repeated which will give a more correct C.A.P. If repeated once again, the process will give a still more correct value. (The explanation of this method of successive approximations is this. To find the time of any event the parallax etc. at that time, which themselves depend upon the time, are required. To get over this difficulty successive approx. is resorted to.) This time is the time of the maximum eclipse, technically called the Middle. •(Really the Middle will occur within a few minutes of this).

6. Find p, the latitude of the moon at the last but one C.A.P., because it will be found convenient if we take the last but one instead of the last. $p = 270' \sin$ (Moon's longitude the asc. node).

7. $p + \pi \sin A$, is the latitude as affected by parallax where A pertains to the last but one C.A.P. and is already found.

8. Square (7) and deduct from $(L + S)^2$. Take the root, divide this by M. The result is in hours.

9. Deduct this from the last C.A.P. This is the provisional beginning. Added, it is the provisional end.

10. Take this time of beginning and find A, B and p. Find $p + \pi \sin A$. Repeat (8) which gives the half duration in hours. Using the A and B here, find the C.A.P. Deduct the half duration from this. This is the first approximate time of beginning.

11. Take the provisional time of ending and do everything mentioned in (10) up to finding the C.A.P. Add the half duration to the C.A.P. This is the first approximate time of ending.

12. If in the place of the provisional times the first approx. times are used and (10) and (11) repeated, the second approx. times are got, and so on. (But going beyond the second approx. is useless, in view of the error in the constants).

We shall compute the solar eclipse of 9-5-1948, according to this method, for Pudukkottai, using the constants of the New Surya Siddhanta.

$$1. \text{ The true diameter of the sun} = 32'4 \times 58/59 \cdot 1 = 31'8.$$

$$\text{The true diameter of the moon} = 32'0 \times 797/790 \cdot 6 = 32'3.$$

$$2. \text{ The true horizontal parallax of the}$$

$$\text{sun} = 4'3 \times 58/59 \cdot 1 = 4'2.$$

$$\text{moon} = 57'5 \times 797/790 \cdot 6 = 58' \pi = 58' - 4'2 = 53'8.$$

$$3. \text{ Taking the time of conj. } v = 343^{\circ}7', \cos A = \cdot 9606 \text{ and } B = 30^{\circ}10'.$$

$$4. \pi \cos A \cos B = 44'7. M = 30'79. 44'7/30'79 = \frac{h}{l} \frac{m}{27} \text{ deductive. So C. A. P.} = 6-33 \text{ as a first Approx.}$$

5. v for 6-33 is $321^{\circ}22'$, $\cos A = .9126$ and $B = 9^{\circ}46'$
 $\pi \cos A \cos B = 48.4$. This divided by M gives
 $1 \overset{h}{35} \overset{m}{}$. Deducting this from conj., the approx.
 time of C.A.P. is 6-25. This is the approx
 time of the middle. We stop here.
6. The last but one C.A.P. is 6-33. The lat. of the
 moon at this time is $270' \sin (47^{\circ}35' - 44^{\circ}) =$
 $+16'9$.
7. $\sin A$ for this time is $-.4088$. $\pi \sin A = -22.0$
 $p + \text{this} = -5'1$.
8. $L + S = (32^{\circ}3' + 31^{\circ}8' / 2 = 32^{\circ}05'$. $\sqrt{32^{\circ}05'^2 - 5^{\circ}1'^2} /$
 $30.79 = 1 \overset{h}{2} \overset{m}{}$.
9. The provisional beginning is 5-23, and the
 provisional end is 7-27.
10. We shall compute the time of the end alone.
 $v = 334^{\circ}52'$, $Lagna = 70^{\circ}40'$, longitude of the
 moon $= 48^{\circ}4'$, $B = 22^{\circ}36'$, $\log \sin A = (-) T.$
 5191 , and $\log \cos A = T.9749$. $p = +19'14$ π
 $\sin A = -17'78$. $p + \pi \sin A = -1'36$. $\sqrt{32^{\circ}05'^2 -$
 $-1'36'^2} / 30.79 = 1 \overset{h}{2} \overset{m}{}$, half duration. $\pi \cos A \cos B$
 $= 46.88$. $46.88 / 30.79 = 1 \overset{h}{31} \overset{m}{}$. The new C.A.P.
 $= 8-1 \overset{h}{31} \overset{m}{}$ = 6-29. This + the half duration
 $= 7-31$, the first Approx. time of the end of
 the eclipse. The second approx. time should be
 found in the same way.

The method of the later Karanas or manuals are based on the Siddhantas, and they generally sacrifice accuracy for ease of computation. There is nothing new in them.

HINDU ASTRONOMY THROUGH THE AGES— A SHORT SKETCH*

1. The Vedic Period

The history of Hindu Astronomy goes back to a very ancient period. Evidence of the astronomical knowledge of the Brahmans is found in the Vedas. Even in the age of the Mantras it was known that the Moon returns to its position among the stars once in 27 days. Each day it was observed to be in conjunction with a single star or group of stars, and the day was designated by that asterism. Thus arose the later division of the ecliptic into 27 asterismal segments. They knew that once in about $29\frac{1}{2}$ days the Moon is in conjunction with the Sun and this period they used as the measure of their month. (The word *mās* means (i) the month, (ii) the Moon, and (iii) a measure.) They knew that the solar year marked by the cycle of the seasons consists of 365 days and that this is in excess of 12 lunar months by about 11 days. The months were named after the asterisms at or near which the moon became full, like Phālguna, Chaitra etc. The shortest days were noted to be when the noon-sun was low down in the sky at Winter Solstice (W.S.) and the days were noted to become longer as the Summer Solstice approached. The year began with the first day of the light fortnight of the month of Phālguna near the W. S. During the age of the Brāhmaṇas it was observed that the Pleiades (Krittikās) rose due east, and this fact was used in the orientation of the sacrificial halls. (From this we can compute the age of the observation to be about 3000 B.C.) Professional star gazers called *nakshatra-darśa-s* are mentioned and in the Chāndogya Upanishad a lore of

* *Pudukkotta Raja's College Magazine*, 1953, pp. 20-25.

stars, called Nakshatra-vidyā, is mentioned. The planets Jupiter and Venus were known. The Atri family was credited with the knowledge of eclipses.

2. The Immediate Post-Vedic Period

The astronomical knowledge of this period is found the Vedāṅga Jyotiṣa (V.J) of Lagadha which is the most ancient Hindu astronomical work extant. This deals with the computation of the ending moments of the Tithi and the Nakshatra, the Sun's Nakshatra etc., things usually found in almanacs, useful for the performance of the Vedic sacrifices. But these Tithis etc. are mean, as distinguished from the true Tithis etc. given by the modern almanacs. The elements from which the computations were made are the following: In each period of 1830 days called a yuga, it is taken there are 5 solar years, 60 solar months, 62 lunar (synodic) months, 1860 lunar Tithis, 1809 lunar Nakshatras and 135 solar Nakshatras. Supposing the Sun and the moon have only mean motions, (i.e., move uniformly on the ecliptic), the computations are made. This would give only the approximate Tithis etc., but true Tithis etc. were fixed by observation. The method of computation is peculiar and very interesting.

The V. J. says that the winter solstice began with the sun at the asterism Sravishṭhā, from which we can calculate the date of the work to be about 1200 B.C. Close upon the V. J. followed several other works, the Garga Samhitā, the Paitāmaha Siddhānta, the Sūrya Prajñapti, the Jyotishakaraṇḍa, the Kālāloka Prakāśa etc., all of which dealt only with the mean motions of the Sun and the Moon, like the V. J. We do not find anything very much special or noteworthy in them.

3. The Period of Transition to the Regular Siddhāntas (Circa 100 B.C. to Circa 300 A.D)

This period is represented by the Vāsisṭha, Pauliśa and Romaka Siddhāntas condensed by Varāhamihira in

his Pancha-Siddhāntikā. In the works of this period the influence of Greek culture on Hindu astronomy, especially astrology, is visible for the first time. The names of week-days like Ravi-vāsara, Indu-vāsara etc., and the names of the 12 solar signs composing the Zodiac, like Mesha, Rishabha etc., occur for the first time now. Many scholars are of opinion that these originated in Babylonia and reached India via the Greeks.

The Vāsishttha seems to be the oldest of the three. In it we find, for the first time, methods for computing the true positions of the Sun and the Moon. The time taken by the Sun to traverse each Rāśi of 30° has been found empirically and given. An ingenious formula for the Moon's Equation of the centre, on the assumption that it increases or decreases uniformly, has been given by using the summation of series, the maximum being $347'$. A method for finding the day-time from the shadow is given, and another for computing the lunar eclipse. Herein also are the planets dealt with for the first time.

The Paulīśa Siddhānta closely follows the Vāsishttha. The name, as also the fact that Alexandria is taken to be the 0° longitude, shows that it must have been affected by Greek influences. In this we find a rough method for the computation of the Solar eclipse.

The Romaka Siddhānta uses a tropical year of 365·2466 days in the place of the Sidereal year used by the other Siddhāntas. This, the name Romaka, and the fact that it gives $143'$ as the maximum equation of the centre for the Sun, and $296'$ for the Moon, all show its Greek origin. For the first time we find the Solar eclipse treated in a scientific manner in this work.

A host of astrological works, many of them claiming Greeks as their Pūrvāchāryas, seem to have been written in this period, followed later by Hindu authors with zest. It is for this concoction, this bane of our culture, that we are indebted to the Greeks in a large measure. In astronomy proper there seems to have been very little 'borrowing'.

4. The Period of the Regular Siddhāntas (300 A.D. to 1200 A.D.)

A succession of astronomical works like the Old Sūrya Siddhānta condensed by Varahamhira, the Āryabhaṭīya of Āryabhaṭa, the Brāhma - Sphuṭa - Siddhānta of Brahmagupta, the New Sūrya - Siddhānta etc, down to the Siddhānta - Śiromaṇi of Bhāskarāchārya mark this period of the heyday of Hindu astronomy. The following are some of the salient features of these works.

(a) The earth is a sphere of diameter 8000 miles, poised in space with nothing for its support. Bhāskara, in a beautiful discussion, refutes arguments advanced against this idea. For instance the Puranic cosmogony is that the earth is supported by eight elephants which are supported by the Great Tortoise, which, in turn, stands on the Great Serpent Ādi-Śesha. In order to avoid the fallacy of infinite regress the Puranas say Śesha supports himself. "Then why not the earth support itself" asks Bhāskara; "If it is also divine, why go so far as Śesha and stop there?" But the layman's question "Will not the earth fall?" should be answered. He says, "This question arises from our experience that things fall towards the earth, attracted by it. So falling means moving *towards the earth* and so the question of the *earth* falling does not arise at all".

(b) The stellar sphere revolves round the earth once in 24 hours (exactly in 23 hrs. 56 minutes). There

were some astronomers like Āryabhaṭa who asserted thus. The verse is अनुलोमगतिर्नोऽस्यः पश्यत्यचलं etc. (गोलपाद 9.). It is made out by commentators of Āryabhaṭa that this is not so. The next śloka gives the usual theory. Are the commentators correct or is the next śloka an interpolation? It has been tampered with changing सूः into म्, that this revolution is only apparent, and is due to the rotation of the earth. But the generality of astronomers from Varāhamihira to Bhāskara argued that this rotation theory could not be accepted because “then birds leaving their nests cannot return to their nests again, and flags will fly ever pointing westwards on account of the eastward rotation of the earth”. We now know that these arguments exhibit an ignorance of the laws of Kinematics, and that there is a direct proof of the earth’s rotation in the behaviour of the ‘Focault’s Pendulum’. But in those days these arguments were considered unanswerable.

(c) The Sun moves round the stellar sphere once in $265\frac{1}{4}$ days along the ecliptic, which cuts the celestial equator at an angle of 24° . (The inclination at the present day is $23^\circ 27'$ but in those days it was very nearly 24°).

(d) There is a retrograde movement of the point of intersection mentioned in (c), along the ecliptic called Ayana-chalanam (precession). Varāhamihira is the first astronomer who has reported this. According to some it is one degree in 67 years, and others, in 60 years. It is noteworthy that these values are very much nearer the modern value of 72 years, than the value given by the Greeks, 100 years. According to some, like Muñjāla, the precession is continuous but others assert that the precession will stop after some time and change into a processional movement. We know now that Muñjāla’s view is correct, but it could only have been a conjecture in those days.

(e) The Moon and the planets move in orbits of their own, each inclined to the ecliptic by a small angle. Their motion is not uniform but varies, being slowest at apogee and quickest at perigee. This varying motion was geometrically represented by eccentric circles or epicycles, and epicycles were also used to represent the equation of conjunction of the planets. The Greeks did the same. But the Hindu constants were generally more accurate.

(f) From the horizontal parallax of the Moon, which was tolerably accurate, the Hindus calculated the distance of the Moon to be about 65 times the earth's radius. If this in excess of the correct value by about 9 per cent, it is because their value for the horizontal parallax of the Moon was a little erroneous, combined as it was with the effects of atmospheric refraction. They held the theory that the Sun the Moon and the planets had a uniform speed. From this, and from the distance of the Moon they calculated the distances of the Sun and the planets, but these were bound to be wrong, because their theory of uniform speed was wrong. Really the velocity is inversely as the root of the distance.

(g) A table of sines and another of versines are generally given from which all the other trigonometrical functions are derived wherever they are required. The Greeks gave also a table of chords. The New Sūrya Siddhānta has an interesting method for constructing the table of sines, based on the fact that the second differences are proportionate to the sines. Bhāskara-charya's treatment of the trigonometrical functions, addition and subtraction formulae etc., have a surprisingly modern air.

(h) Generally a separate chapter is set apart called the Tripraśnādhyaya (Chapter dealing with the three

types of questions, relating to time, place, and direction) for problems solved by spherical trigonometry. Bhāskara is seen at his best here, giving methods of solving problems that baffle even the modern minds.

(i) Separate chapters are devoted to eclipses, and conjunctions of planets among themselves or with stars. For this purpose a list of the principal stars with their coordinates are given.

(j) A chapter called the Yantrādhyāya is generally devoted to astronomical instruments.

(k) The longiture of Ujjain passing through Rohitika and Kurukshetra on to the N. Pole is taken as the prime meridian, for Ujjain was the Greenwich of India in those days. Mean sunrise or mean midnight at Ujjain is taken as the beginning of the astronomical day. A certain boldness and freedom from prejudice characterises the discussions in these works, especially in the Siddhānta Śiromaṇi. In this matter, the works of this period are in pleasing contrast with several works of the next age, which are authorityridden and in which very little originality is seen.

It seems that during this period two schools of astronomers flourished in this country. Ujjain was the seat of one school and there was a long hierarchy of astronomers there from very ancient times. Varāha (c. 500 A.D.), Brahmagupta (c. 600 A.D.) and Bhāskara (c. 1150 A.D.) all belonged to this school. The other school was on the west coast of South India and the Āryabhaṭīya (c. 500 A.D.) has been followed in this school. Certain peculiarities in his work are found in the system of almanac-making in vogue in the Tamil and Malayalam country.

5. The Period of Decline

After Bhāskara there was a decline. This was due to several causes. The Muslim occupation deprived the science of much valuable patronage. With the astronomical instruments which were at their disposal, the astronomers had come almost to the end of their tether, and new discoveries would be possible only with new and more powerful instruments. (This is true for discoveries in all sciences.) Where new discoveries are not made the tendency is to lay stress on ancient authority, which itself lays a strangle-hold on further discovery. The result was that very little that was original came in this period, though here and there some remarkable persons enjoying the patronage of some remarkable princes could be found. Prejudice and subservience to authority reigned supreme and reason was blinded. For example, Kamalākara, the author of the *Siddhānta-Tattva-Viveka*, a very able man, condemns many correct formulae given by Bhāskarāchārya, simply because his rival had taken Bhāskara for his authority.

As a result, the chief astronomical works were *Karaṇas* or manuals, based upon some ancient *Siddhāntas*, intended for use by almanac-makers. Not accuracy but ease of computation was the aim of these works and they were intended to be used only for a few hundred years, not more, because error would accumulate. Not knowing this the present-day almanac-makers still use *Karaṇas* that have long ago ceased to be serviceable, and the result is a deplorable state of affairs. The so-called '*Siddhānta*' and '*Vākya*' almanacs are examples.

The *Siddhānta* almanac, in vogue throughout India except the Tamil and Malayalam parts, are most of them based on a *Karaṇa* called *Grahalāghava* by Gaṇeśa

Daivajña (circa 1500). The Vākya almanacs followed in the Tamil country are based on a work called Vākya-karaṇa or Vākya Panchādhyāyī commented upon by Sundararaja. (about 1300 A.D.) The karanas called Panchabodha used in Kerala are prepared on the basis of the Parahita-gaṇita of Haridatta following Āryabhaṭīya. Is it not time we change our Karana ?

Recently, thanks to the study of European astronomy, many works have been written by Indians, using which the correct positions of the Sun, the Moon and the Stars can be computed. Some have taken pains to put them in a form very much serviceable to almanac makers. Even the writer of this article has done some work in this direction. He will be only too glad to help people make correct almanacs.
